AF Relaying for Millimeter Wave Communication Systems with Hybrid RF/Baseband MIMO Processing

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***Abstract*—Due to the high cost and power consumption of radio frequency (RF) chains, millimeter wave (mm-wave) communication systems equipped with large antenna arrays typically employ less RF chains than the antenna elements. This leads to the use of a hybrid MIMO processor consisting of a RF beamformer and a baseband MIMO processor in mm-wave communications. In this paper, we consider amplify-and-forward (AF) relay-assisted mm-wave systems with the hybrid MIMO processors over frequency-selective channels. We develop an iterative algorithm for jointly designing the receive/transmit (Rx/Tx) RF/baseband processors of the relay based on the orthogonal matching pursuit (OMP) algorithm for sparse approximation, while assuming orthogonal frequency division multiplexing (OFDM) signaling. Simulation results show that the proposed method outperforms the conventional method that designs the baseband processor after steering the RF beams.**

I. Introduction

Recently, millimeter wave (mm-wave) wireless systems are emerging as a promising technology for next generation cellular communication [1], [2]. Although mm-wave systems experience higher propagation loss compared to the existing cellular systems, they can utilize the vast bandwidth available in mm-wave spectrum and, in addition, employ large antenna arrays packed in a very small area thanks to the short wavelength. These characteristics allow the design of multi-Gbps mm-wave systems, as demonstrated in indoor systems such as wireless LAN [3] and wireless PAN [4].

In mm-wave systems equipped with large antenna arrays, it is difficult to provide each antenna element with a radio frequency (RF) chain, consisting of amplifiers, mixers and analog to digital converters (ADCs)/digital to analog converters (DACs), because of its high cost and power consumption. Typically, mm-wave systems employ less RF chains than antenna elements and performs beamforming in RF domain for linearly combining the antenna outputs. As a result, MIMO processing for such a system is realized by a cascade of the RF beamformer and baseband MIMO processor, which will be referred to as the hybrid MIMO processor.

Various techniques have been proposed for hybrid MIMO processing. Use of RF beamformers based on phase shift networks, called *phased array* beamformers, has been proposed for hybrid MIMO [5], where phase shifters are designed by extracting the phases of singular vectors of the MIMO channel state information (CSI) matrix. In [6]-[10], more sophisticated design methods for RF beamforming have been proposed, including the optimal RF beamforming for clustered subarrays maximizing the capacity [6], [7], joint minimum mean square error (MMSE) design of RF/baseband beamformers at the receiver [8], and the joint transmit/receive (Tx/Rx) beamforming maximizing a capacity lower bound [9], [10]. More recently, in [11]-[13] the RF and baseband processors are jointly designed by applying the *orthogonal matching pursuit* (OMP) algorithm [14]. This method approximates the optimal MIMO processor with full complexity RF chains (one RF chain per antenna) by iteratively selecting a beamforming vector from the set of array response vectors and determining the corresponding baseband processor. The OMP-based technique leads to the use of simple phased array beamformers (array response vectors can be realized by phase shift networks) and can outperform the *beam-steering* method that designs the baseband processor after steering the RF beamformer to desired directions.

In this work, we develop an OMP-based sparse approximation algorithm for jointly designing the Rx/Tx hybrid MIMO processors of a half duplex amplify-and-forward (AF) relay in mm-wave communications. Furthermore, in contrast to most of the previous work on hybrid MIMO that assumes frequency flat fading, we consider orthogonal frequency division multiplexing (OFDM) over frequency selective channels and design the RF beamformer that is the same for all subchannels of OFDM signaling.

The organization of this paper is as follows. Section II presents the system model. The proposed algorithms for AF relaying of OFDM signals in mm-wave systems are developed in Section III. The simulation results and the conclusion are presented in Sections IV and V, respectively.

Notations: We use the following notations in this paper. $||\mathbf{A}||_F$ is the Frobenius norm of a matrix $\mathbf{A}$. $\mathbf{A}^T$ is the transpose and $\mathbf{A}^H$ is the complex-conjugate transpose of $\mathbf{A}$; $\mathrm{diag}\,(\mathbf{A})$ is a vector consisting of diagonal entries of $\mathbf{A}$; $(\mathbf{A})_{i,j}$ is the (i, j)-th entry of $\mathbf{A}$; $\mathbf{A}^{(j)}$ is the j-th column of $\mathbf{A}$; $[\mathbf{A}\,|\,\mathbf{B}]$ denotes horizontal concatenation; and $\mathbf{A}^{-1}$ denotes the inverse of a square matrix. The $N \times N$ identity matrix is denoted by $\mathbf{I}_N$, and the expectation of a random variable X is denoted by $\mathcal{E}\,[X]$.

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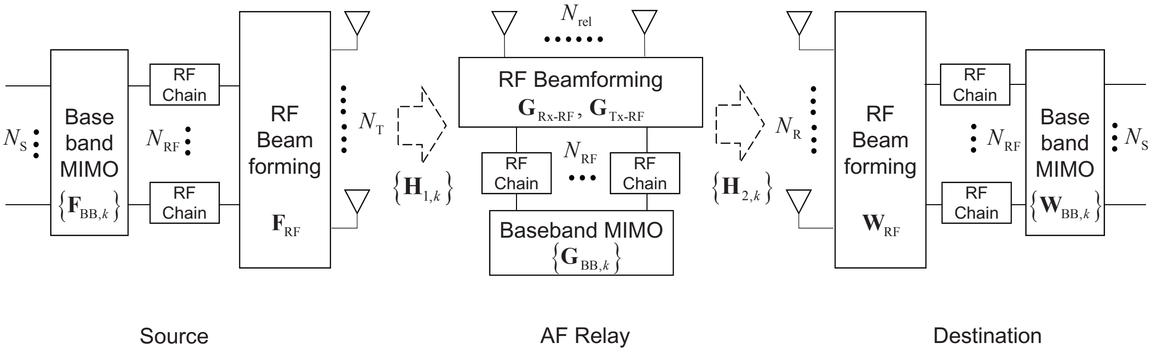

Fig. 1. A half duplex AF relaying system where each node is equipped with a hybrid MIMO processor.

II. System Model

Consider the relay link consisting of two terminals and one AF relay (Fig. 1). All nodes are equipped with hybrid MIMO processors employing phased array beamformers in the RF domain. It is assumed that there is no direct path between the source and the destination, and that all nodes operate in half-duplex mode. The number of antennas at the source, the relay, and the destination are denoted as $N_{\rm T}$, $N_{\rm rel}$, and $N_{\rm R}$, respectively. All nodes have the same number of RF chains, denoted as $N_{\rm RF}$, which is less than the minimum of $(N_{\rm T}, N_{\rm rel}, N_{\rm R})$. The number of data streams delivered through this relay link is denoted as $N_{\rm S}$, where $N_{\rm S} \le N_{\rm RF}$.

We consider a wideband system whose channel impulse response from the source to the relay and from the relay to the destination, denoted as $\bar{\mathbf{H}}_1[t]$ and $\bar{\mathbf{H}}_2[t]$, respectively, are given by [15], [16],

$$\bar{\mathbf{H}}_p[t] = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_{p,l}\delta[t-l], \tag{1}$$

where $p \in \{1,2\}$, t is the time index, L is the number of taps corresponding to the scattering clusters, $\delta[t-l]$ is the Kroneker delta function. Using the parametric channel model [16], the channel matrix for the l-th tap, $\bar{\mathbf{H}}_{p,l}$ is given by

$$\bar{\mathbf{H}}_{p,l} = \frac{1}{\sqrt{N_{\rm ray}}}\sum_{i=1}^{N_{\rm ray}} \alpha_{l,i}^{(p)} \mathbf{a}_{\rm r}\left(\phi_{l,i}^{{\rm r},(p)}, \theta_{l,i}^{{\rm r},(p)}\right) \mathbf{a}_{\rm t}\left(\phi_{l,i}^{{\rm t},(p)}, \theta_{l,i}^{{\rm t},(p)}\right)^H, \tag{2}$$

where $N_{\rm ray}$ is the number of rays per cluster, $\alpha_{l,i}^{(p)}$ is the complex gain of the i-th ray, and $\phi_{l,i}^{{\rm r},(p)}$ $\left(\theta_{l,i}^{{\rm r},(p)}\right)$ and $\phi_{l,i}^{{\rm t},(p)}$ $\left(\theta_{l,i}^{{\rm t},(p)}\right)$ are the azimuth (elevation) angle of arrival (AoA) and angle of departure (AoD), respectively. The vectors $\mathbf{a}_{\rm r}\left(\phi_{l,i}^{{\rm r},(p)}, \theta_{l,i}^{{\rm r},(p)}\right)$ and $\mathbf{a}_{\rm t}\left(\phi_{l,i}^{{\rm t},(p)}, \theta_{l,i}^{{\rm t},(p)}\right)$ are the array response vectors at the receiver and at the transmitter, respectively. If we assume an N element uniform linear array (ULA), its array response $\mathbf{a}(\phi_{l,i})$ is given by

$$\mathbf{a}(\phi_{l,i}) = \left[1,\ e^{j\frac{2\pi}{\lambda}d\sin(\phi_{l,i})},\ \dots\ ,\ e^{j\frac{2\pi}{\lambda}(N-1)d\sin(\phi_{l,i})}\right]^T, \tag{3}$$

where λ is the carrier wavelength, and d is the inter-element spacing.

We assume OFDM communication in which the frequency-selective channel is divided into K frequency-flat subchannels. The frequency domain channel matrix for the k-th subchannel is given by [15]

$$\mathbf{H}_p\left(e^{j2\pi(k/K)}\right) = \sum_{l=0}^{L-1} \bar{\mathbf{H}}_{p,l}e^{-j2\pi l(k/K)}, k = 0,1,\dots,K-1. \tag{4}$$

To simplify notations, we express $\mathbf{H}_p\left(e^{j2\pi(k/K)}\right)$ by $\mathbf{H}_{p,k}$ where $\mathbf{H}_{1,k}$ and $\mathbf{H}_{2,k}$ are $N_{\rm rel} \times N_{\rm T}$ and $N_{\rm R} \times N_{\rm rel}$ matrices, respectively. In addition, we denote by $\mathbf{H}_{p,k} = \mathbf{U}_{p,k}\mathbf{S}_{p,k}\mathbf{V}_{p,k}^H$ the singular value decomposition (SVD) of $\mathbf{H}_{p,k}$, where the entries of the diagonal matrix $\mathbf{S}_{p,k}$ (singular values) are arranged in decreasing order. All nodes are assumed to have perfect knowledge of the channels of both hops.

Referring to Fig. 1, the transmitted signal at the source in the k-th subchannel is given by

$$\mathbf{x}_k = \mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\mathbf{s}_k, \tag{5}$$

where $\mathbf{s}_k$ is the $N_{\rm S} \times 1$ vector of complex information symbols to be transmitted, and $\mathbf{F}_{\rm RF}$ and $\mathbf{F}_{{\rm BB},k}$, respectively, are the $N_{\rm T} \times N_{\rm RF}$ RF beamforming matrix and the $N_{\rm RF} \times N_{\rm S}$ baseband precoding matrix. It is assumed that $\mathcal{E}\left[\mathbf{s}_k\mathbf{s}_k^H\right] = \mathbf{I}_{N_{\rm S}}$ and $\left\|\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\right\|_F^2 = N_{\rm S}$. Since the RF beamformers are implemented using the analog phase shifters, the entries of $\mathbf{F}_{\rm RF}$ are constrained to have equal magnitudes. No hardware constraints are considered on $\mathbf{F}_{\rm BB}$ except for the total power constraint. Note that the RF beamforming matrix $\mathbf{F}_{\rm RF}$ should be the same for all subchannels, because the RF beamformer cannot be implemented separately for each subchannel. During the first transmission phase, the transmitted signal is received through the $N_{\rm RF} \times N_{\rm R}$ RF beamforming matrix $\mathbf{G}_{\rm Rx-RF}$ at the relay. In the second transmission phase, the relay multiplies the beamformer output by the $N_{\rm RF} \times N_{\rm RF}$ AF relay gain matrix $\mathbf{G}_{{\rm BB},k}$ and forwards the resulting signal to the destination through the $N_{\rm R} \times N_{\rm RF}$ RF beamforming matrix $\mathbf{G}_{\rm Tx-RF}$. The received signal at the destination is given by

$$\mathbf{y}_k = \mathbf{H}_{2,k}\mathbf{G}_{\rm Tx-RF}\mathbf{G}_{{\rm BB},k}\mathbf{G}_{\rm Rx-RF} \times (\mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{z}_k) + \mathbf{w}_k, \tag{6}$$

where $\mathbf{z}_k$ and $\mathbf{w}_k$ are $N_{\rm rel}\times 1$ and $N_{\rm R}\times 1$ noise vectors at the relay and the destination, respectively. They are assumed to be mutually independent circularly-symmetric complex Gaussian vectors with $\mathcal{E}\left[\mathbf{z}_k\mathbf{z}_k^H\right]=\sigma_z^2\mathbf{I}_{N_{\rm rel}}$ and $\mathcal{E}\left[\mathbf{w}_k\mathbf{w}_k^H\right]=\sigma_w^2\mathbf{I}_{N_{\rm R}}$. After passing through the hybrid MIMO processor, the received signal becomes

$$\tilde{\mathbf{y}}_k=\mathbf{W}_{{\rm BB},k}^H\mathbf{W}_{\rm RF}^H\mathbf{H}_{2,k}\mathbf{G}_{\rm Tx-RF}\mathbf{G}_{\rm BB,k}\mathbf{G}_{\rm Rx-RF} \times\left(\mathbf{H}_{1,k}\mathbf{x}_k+\mathbf{z}_k\right)+\mathbf{W}_{{\rm BB},k}^H\mathbf{W}_{\rm RF}^H\mathbf{w}_k, \quad (7)$$

where $\mathbf{W}_{\rm RF}$ and $\mathbf{W}_{{\rm BB},k}$ are the $N_{\rm R}\times N_{\rm RF}$ RF beamforming matrix and the $N_{\rm RF}\times N_{\rm S}$ baseband combining matrix at the destination, respectively.

III. Hybrid MIMO Design for the Mm-wave AF Relay Link

In this section we first present the OMP-based algorithms for precoding and combining of OFDM signals at the source and the destination, respectively. Then the algorithms are extended for jointly designing the Rx/Tx hybrid MIMO processors of the AF relay.

A. Precoding/Combining for OFDM Signals in Hybrid MIMO Systems

Let $\mathbf{F}_{{\rm opt},k}$ denote the $N_{\rm T}\times N_{\rm S}$ optimal precoder for the k-th subchannel when each antenna is connected to a separate RF chain (full complexity RF chains). The optimal precoder is given by $\mathbf{F}_{{\rm opt},k}=\tilde{\mathbf{V}}_{1,k}\boldsymbol{\Sigma}_{{\rm F},k}^{1/2}$ where $\tilde{\mathbf{V}}_{1,k}$ consists of the $N_{\rm S}$ columns of $\mathbf{V}_{1,k}$ corresponding to the $N_{\rm S}$ largest singular values of $\mathbf{H}_{1,k}$, and $\boldsymbol{\Sigma}_{{\rm F},k}$ is the $N_{\rm S}\times N_{\rm S}$ diagonal matrix for power allocation.

Following the approach in [11]-[13], we minimize the sum of the Frobenius norms of the differences between $\mathbf{F}_{{\rm opt},k}$ and $\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}$:

$$\left(\mathbf{F}_{\rm RF}^{\rm opt},\mathbf{F}_{{\rm BB},k}^{\rm opt}\right)=\mathop{\arg\min}_{\mathbf{F}_{\rm RF},\mathbf{F}_{{\rm BB},k}}\sum_{k=1}^{K}\left\|\mathbf{F}_{{\rm opt},k}-\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\right\|_F \quad \text{s.t. } \mathbf{F}_{\rm RF}^{(j)}\in\mathcal{S}_{\rm S,A},\ \left\|\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\right\|_F^2=N_{\rm S}, \quad (8)$$

where $\mathbf{F}_{\rm RF}^{(j)}$ is the j-th column of $\mathbf{F}_{\rm RF}$ which is selected from the set of array response vectors, $\mathcal{S}_{\rm S,A}=\left\{\mathbf{a}_{\rm t}\left(\phi_{l,i}^{{\rm t},(1)},\theta_{l,i}^{{\rm t},(1)}\right)|1\le l\le L,1\le i\le N_{\rm ray}\right\}$, and $\left\|\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\right\|_F^2=N_{\rm S}$ is the power constraint at the source. Here $\mathcal{S}_{\rm S,A}$ is the set of all $(LN_{\rm ray})$ array response vectors pointing the AoDs of the $LN_{\rm ray}$ rays of the channel $\{\mathbf{H}_{1,k}\}^{\dagger}$.

To adopt the sparse approximation framework [14], the error matrix $\left(\mathbf{F}_{{\rm opt},k}-\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\right)$ in (8) is rewritten as $\left(\mathbf{F}_{{\rm opt},k}-\mathbf{A}_{\rm T}\tilde{\mathbf{F}}_{{\rm BB},k}\right)$ where $\mathbf{A}_{\rm T}$ is the $N_{\rm T}\times LN_{\rm ray}$ measurement matrix containing all elements of $\mathcal{S}_{\rm S,A}$ as its columns and $\tilde{\mathbf{F}}_{{\rm BB},k}$ is an $LN_{\rm ray}\times N_{\rm S}$ matrix having $N_{\rm RF}$ non-zero rows which constitute $\mathbf{F}_{{\rm BB},k}$ (columns of $\tilde{\mathbf{F}}_{{\rm BB},k}$ are $N_{\rm RF}$-sparse vectors). The positions of the non-zero rows correspond to the selected array response vectors: the i-th row of $\tilde{\mathbf{F}}_{{\rm BB},k}$ is non-zero if the i-th column of $\mathbf{A}_{\rm T}$ is chosen for RF beamforming. Under this framework, an OMP algorithm for jointly designing $\mathbf{F}_{\rm RF}$ and $\left\{\mathbf{F}_{{\rm BB},k}|1\le k\le K\right\}$ can be derived by modifying the original OMP algorithm, as shown in Algorithm 1. Referring to steps 4 and 5 of Algorithm 1, at each iteration this algorithm chooses the column of $\mathbf{A}_{\rm T}$ that is most strongly correlated with the residual errors $\left\{\mathbf{F}_{{\rm res},k}\right\}$. Then the selected column is appended to $\mathbf{F}_{\rm RF}$ (step 6); $\mathbf{F}_{{\rm BB},k}$ is obtained by evaluating the least squares solution of $\mathbf{F}_{{\rm opt},k}=\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}$ (step 7); and the contribution of the chosen array response vector to $\mathbf{F}_{{\rm opt},k}$ is subtracted to update $\mathbf{F}_{{\rm res},k}$ (step 8).

In a similar manner, we can design the hybrid combining matrices at the destination receiving OFDM signals. In this case the optimal combiner assuming full complexity RF chains at the destination can be set to $\mathbf{W}_{{\rm opt},k}=\tilde{\mathbf{U}}_{2,k}^H$ where $\tilde{\mathbf{U}}_{2,k}$ consists of the $N_{\rm S}$ columns of $\mathbf{U}_{2,k}$ corresponding to the $N_{\rm S}$ largest singular values of $\mathbf{H}_{2,k}$. The procedure for sparsely approximating $\mathbf{W}_{{\rm opt},k}$ at the destination can be developed by modifying Algorithm 1, and the details will not be presented.

Algorithm 1 OMP-based hybrid precoding for OFDM

Require: $\{\mathbf{F}_{{\rm opt},k}\}$

1: $\mathbf{F}_{\rm RF}$=Empty Matrix
2: $\mathbf{F}_{{\rm res},k}=\mathbf{F}_{{\rm opt},k}$, for all $\{k|1\le k\le K\}$
3: **for** $i\le N_{\rm RF}$ **do**
4: $\quad\boldsymbol{\Psi}_k=\mathbf{A}_{\rm T}^H\mathbf{F}_{{\rm res},k}$, for all $\{k|1\le k\le K\}$
5: $\quad q=\arg\max_{m=1,\dots,LN_{\rm ray}}\left(\sum_{k=1}^{K}\boldsymbol{\Psi}_k\boldsymbol{\Psi}_k^H\right)_{m,m}$
6: $\quad\mathbf{F}_{\rm RF}=\left[\mathbf{F}_{\rm RF}|\mathbf{A}_{\rm T}^{(q)}\right]$
7: $\quad\mathbf{F}_{{\rm BB},k}=\left(\mathbf{F}_{\rm RF}^H\mathbf{F}_{\rm RF}\right)^{-1}\mathbf{F}_{\rm RF}^H\mathbf{F}_{{\rm opt},k}$, for all $\{k|1\le k\le K\}$
8: $\quad\mathbf{F}_{{\rm res},k}=\frac{\mathbf{F}_{{\rm opt},k}-\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}}{\|\mathbf{F}_{{\rm opt},k}-\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\|_F}$, for all $\{k|1\le k\le K\}$
9: **end for**
10: $\mathbf{F}_{{\rm BB},k}=\sqrt{N_{\rm S}}\frac{\mathbf{F}_{{\rm BB},k}}{\|\mathbf{F}_{\rm RF}\mathbf{F}_{{\rm BB},k}\|_F}$, for all $\{k|1\le k\le K\}$
11: **return** $\mathbf{F}_{\rm RF}$, $\{\mathbf{F}_{{\rm BB},k}|1\le k\le K\}$

B. AF Relaying for OFDM Signals in Hybrid MIMO Systems

Let $\mathbf{G}_{{\rm opt},k}$ denote the $N_{\rm rel}\times N_{\rm rel}$ optimal relaying matrix for the k-th subchannel when full complexity RF chains are employed. The optimal relaying matrix is given by $\mathbf{G}_{{\rm opt},k}=\tilde{\mathbf{V}}_{2,k}\boldsymbol{\Sigma}_{{\rm G},k}^{1/2}\tilde{\mathbf{U}}_{1,k}^H$ where $\tilde{\mathbf{V}}_{2,k}$ and $\tilde{\mathbf{U}}_{1,k}$ consist of the $N_{\rm S}$ columns of $\mathbf{V}_{2,k}$ and $\mathbf{U}_{1,k}$, respectively, corresponding to the $N_{\rm S}$ largest singular values, and $\boldsymbol{\Sigma}_{{\rm G},k}$ is the $N_{\rm S}\times N_{\rm S}$ diagonal matrix for power allocation.

The problem for minimizing the sum of the Frobenius norms of the differences between $\mathbf{G}_{{\rm opt},k}$ and the cascade of the relay processing matrices, $\mathbf{G}_{\rm Tx-RF}\mathbf{G}_{{\rm BB},k}\mathbf{G}_{\rm Rx-RF}$ is written as (9), shown at the top of the next page, where $\left(\mathbf{G}_{\rm Rx-RF}^T\right)^{(j)}$ and $\mathbf{G}_{\rm Tx-RF}^{(j)}$ are selected from the sets of array response vectors $\mathcal{S}_{\rm Rx-R,A}=\left\{\mathbf{a}_{\rm r}\left(\phi_{l,i}^{{\rm r},(1)},\theta_{l,i}^{{\rm r},(1)}\right)|1\le l\le L,1\le i\le N_{\rm ray}\right\}$ and

†The AoDs are the same for all subchannels.

$$\left(\mathbf{G}_{\mathrm{Tx-RF}}^{\mathrm{opt}}, \mathbf{G}_{\mathrm{BB},k}^{\mathrm{opt}}, \mathbf{G}_{\mathrm{Rx-RF}}^{\mathrm{opt}}\right) = \underset{\mathbf{G}_{\mathrm{Tx-RF}}, \mathbf{G}_{\mathrm{BB},k}, \mathbf{G}_{\mathrm{Rx-RF}}}{\arg\min} \sum_{k=1}^{K} \left\| \mathbf{G}_{\mathrm{opt},k} - \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}} \right\|_F$$
$$\text{s.t. } \mathbf{G}_{\mathrm{Tx-RF}}^{(j)} \in \mathcal{S}_{\mathrm{Tx-R,A}}, \left(\mathbf{G}_{\mathrm{Rx-RF}}^{T}\right)^{(j)} \in \mathcal{S}_{\mathrm{Rx-R,A}}, \text{and}$$
$$\left\| \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}} \left(\mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{z}_k\right) \right\|_F^2 = N_{\mathrm{S}}, \tag{9}$$

Algorithm 2 OMP-based hybrid relaying for OFDM

Require: $\{\mathbf{G}_{\mathrm{opt},k}\}$

1: $\mathbf{G}_{\mathrm{Rx-RF}} = \mathbf{G}_{\mathrm{Tx-RF}} =$ Empty Matrix
2: $\mathbf{G}_{\mathrm{res},k} = \mathbf{G}_{\mathrm{opt},k}$, for all $\{k|1 \le k \le K\}$
3: **for** $i \le N_{\mathrm{RF}}$ **do**
4: $\quad \mathbf{\Psi}_{\mathrm{Rx},k} = \mathbf{G}_{\mathrm{res},k}\mathbf{A}_{\mathrm{Rx-rel}}^{H}$, for all $\{k|1 \le k \le K\}$
5: $\quad q = \arg\max_{m=1,\ldots,LN_{\mathrm{ray}}} \left(\sum_{k=1}^{K} \mathbf{\Psi}_{\mathrm{Rx},k}^{H}\mathbf{\Psi}_{\mathrm{Rx},k}\right)_{m,m}$
6: $\quad \mathbf{G}_{\mathrm{Rx-RF}} = \left[\mathbf{G}_{\mathrm{Rx-RF}} | \left(\mathbf{A}_{\mathrm{Rx-rel}}^{T}\right)^{(q)}\right]^{T}$
7: $\quad \mathbf{\Psi}_{\mathrm{Tx},k} = \mathbf{A}_{\mathrm{Tx-rel}}^{H}\mathbf{G}_{\mathrm{res},k}\mathbf{G}_{\mathrm{Rx-RF}}^{H}$, for all $\{k|1 \le k \le K\}$
8: $\quad v = \arg\max_{m=1,\ldots,LN_{\mathrm{ray}}} \left(\sum_{k=1}^{K} \mathbf{\Psi}_{\mathrm{Tx},k}\mathbf{\Psi}_{\mathrm{Tx},k}^{H}\right)_{m,m}$
9: $\quad \mathbf{G}_{\mathrm{Tx-RF}} = \left[\mathbf{G}_{\mathrm{Tx-RF}} | \mathbf{A}_{\mathrm{Tx-rel}}^{(v)}\right]$
10: $\quad \mathbf{G}_{\mathrm{BB},k} = \left(\mathbf{G}_{\mathrm{Tx-RF}}^{H}\mathbf{G}_{\mathrm{Tx-RF}}\right)^{-1}\mathbf{G}_{\mathrm{Tx-RF}}^{H}\mathbf{G}_{\mathrm{opt},k}\mathbf{G}_{\mathrm{Rx-RF}}^{H}\left(\mathbf{G}_{\mathrm{Rx-RF}}\mathbf{G}_{\mathrm{Rx-RF}}^{H}\right)^{-1}$, for all $\{k|1 \le k \le K\}$
11: $\quad \mathbf{G}_{\mathrm{res},k} = \frac{\mathbf{G}_{\mathrm{opt},k} - \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}}{\|\mathbf{G}_{\mathrm{opt},k} - \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}\|_F}$, for all $\{k|1 \le k \le K\}$
12: **end for**
13: $\mathbf{G}_{\mathrm{BB},k} = \sqrt{N_{\mathrm{S}}}\frac{\mathbf{G}_{\mathrm{BB},k}}{\|\mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}(\mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{z}_k)\|_F}$, for all $\{k|1 \le k \le K\}$
14: **return** $\mathbf{G}_{\mathrm{Tx-RF}}$, $\mathbf{G}_{\mathrm{Rx-RF}}$, and $\{\mathbf{G}_{\mathrm{BB},k}|1 \le k \le K\}$

$\mathcal{S}_{\mathrm{Tx-R,A}} = \left\{\mathbf{a}_{\mathrm{t}}\left(\phi_{l,i}^{\mathrm{t},(2)}, \theta_{l,i}^{\mathrm{t},(2)}\right) | 1 \le l \le L, 1 \le i \le N_{\mathrm{ray}}\right\}$. Here $\mathcal{S}_{\mathrm{Rx-R,A}}\left(\mathcal{S}_{\mathrm{Tx-R,A}}\right)$ is the set of all array response vectors pointing the AoAs (AoDs) of the LN_{ray} rays of $\mathbf{H}_{1,k}\left(\mathbf{H}_{2,k}\right)$, and $\left\|\mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}\left(\mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{z}_k\right)\right\|_F^2 = N_{\mathrm{S}}$ is the power constraint at the relay.

In this case we need to jointly choose the Rx/Tx beamforming vectors, while designing the baseband processor. Adopting the sparse approximation approach, the error matrix $\left(\mathbf{G}_{\mathrm{opt},k} - \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}\right)$ in (9) is rewritten as $\left(\mathbf{G}_{\mathrm{opt},k} - \mathbf{A}_{\mathrm{Tx-rel}}\tilde{\mathbf{G}}_{\mathrm{BB},k}\mathbf{A}_{\mathrm{Rx-rel}}\right)$ where $\mathbf{A}_{\mathrm{Rx-rel}} = \left[\mathbf{a}_{\mathrm{r}}\left(\phi_{1,1}^{\mathrm{r},(1)}, \theta_{1,1}^{\mathrm{r},(1)}\right), \ldots, \mathbf{a}_{\mathrm{r}}\left(\phi_{L,N_{\mathrm{ray}}}^{\mathrm{r},(1)}, \theta_{L,N_{\mathrm{ray}}}^{\mathrm{r},(1)}\right)\right]^{T}$ and $\mathbf{A}_{\mathrm{Tx-rel}} = \left[\mathbf{a}_{\mathrm{t}}\left(\phi_{1,1}^{\mathrm{t},(2)}, \theta_{1,1}^{\mathrm{t},(2)}\right), \ldots, \mathbf{a}_{\mathrm{t}}\left(\phi_{L,N_{\mathrm{ray}}}^{\mathrm{t},(2)}, \theta_{L,N_{\mathrm{ray}}}^{\mathrm{t},(2)}\right)\right]$ are $LN_{\mathrm{ray}} \times N_{\mathrm{rel}}$ and $N_{\mathrm{rel}} \times LN_{\mathrm{ray}}$ matrices containing all elements of $\mathcal{S}_{\mathrm{Rx-R,A}}$ and $\mathcal{S}_{\mathrm{Tx-R,A}}$, respectively, as their columns. $\tilde{\mathbf{G}}_{\mathrm{BB},k}$ is an $LN_{\mathrm{ray}} \times LN_{\mathrm{ray}}$ square matrix having N_{RF}^2 non-zero entries that constitute $\mathbf{G}_{\mathrm{BB},k}$. The locations of the non-zero entries are determined according to the selected column and row indices of $\mathbf{A}_{\mathrm{Rx-rel}}$ and $\mathbf{A}_{\mathrm{Tx-rel}}$: the (*i*, *j*)-th entry of $\tilde{\mathbf{G}}_{\mathrm{BB},k}$ is non-zero when the *i*-th column of $\mathbf{A}_{\mathrm{Tx-rel}}$ and the *j*-th row $\mathbf{A}_{\mathrm{Rx-rel}}$ are chosen for RF beamforming.

In contrast to the standard sparse approximation problem having one measurement matrix, the problem under consideration has two measurement matrices, $\mathbf{A}_{\mathrm{Tx-rel}}$ and $\mathbf{A}_{\mathrm{Rx-rel}}$. This fact leads to a modified OMP algorithm, summarized in Algorithm 2, that successively selects the Rx and Tx beams. In steps 4, 5 and 6 of the algorithm, the row of $\mathbf{A}_{\mathrm{Rx-rel}}$ that is most strongly correlated with the residual errors $\{\mathbf{G}_{\mathrm{res},k}\}$ is chosen for Rx beamforming, and the row is appended to $\mathbf{G}_{\mathrm{Rx-RF}}$. Then in steps 7, 8 and 9, the column of $\mathbf{A}_{\mathrm{Tx-rel}}$ that is most strongly correlated with the product of $\mathbf{G}_{\mathrm{res},k}$ and $\mathbf{G}_{\mathrm{Rx-RF}}^{H}$ is chosen for Tx beamforming, and the column is appended to $\mathbf{G}_{\mathrm{Tx-RF}}$. Note that the Rx and Tx beams are jointly selected by considering the product $\mathbf{G}_{\mathrm{res},k}\mathbf{G}_{\mathrm{Rx-RF}}^{H}$, instead of $\mathbf{G}_{\mathrm{res},k}$ in step 7. The baseband processor $\mathbf{G}_{\mathrm{BB},k}$ is obtained by evaluating the least squares solution of $\mathbf{G}_{\mathrm{opt},k} = \mathbf{G}_{\mathrm{Tx-RF}}\mathbf{G}_{\mathrm{BB},k}\mathbf{G}_{\mathrm{Rx-RF}}$ (step 10). Finally, in step 11 of the iteration the contribution of the chosen array vectors to $\mathbf{G}_{\mathrm{opt},k}$ is subtracted to update $\mathbf{G}_{\mathrm{res},k}$.

IV. Simulation Results

The average rates of the proposed OMP-based method are examined through computer simulation. For comparison, we also evaluate the average rates of the optimal AF relaying system with full complexity RF chains, which is designed by the technique in [17], and the beam-steering method that designs the basedband processors as in [17], after steering the RF beams to the N_{RF} strongest rays. The parameters for the simulation are as follows: the number of antennas $N_{\mathrm{T}} = N_{\mathrm{rel}} = N_{\mathrm{R}} = 64$, the number of data streams $N_{\mathrm{S}} = 2$, and the number of subchannels $K = 64$. We assume ULAs with $d = \lambda/2$. The channel has $L = 3$ clusters and exhibits exponential power delay profile: the second and third paths experience 10dB and 20dB more path loss, respectively, than the first path [2]. The number of rays is $N_{\mathrm{ray}} = 10$. The azimuth angle of each ray, $\phi_{l,i}$ in (3), is randomly generated following the extended Saleh-Valenzuela model [18]: the mean angle associated with each cluster is uniform over $[-\pi, \pi)$ and the difference between $\phi_{l,i}$ and its mean is two-sided Laplacian distributed with angular standard deviation 5°. The complex gains $\left\{\alpha_{l,i}^{(p)}\right\}$ in (2) are complex Gaussian with zero mean, and their variances are 1, 0.1 and 0.01 when l =1, 2 and 3, respectively. The noise variance $\sigma_z^2 = \sigma_w^2$, and the signal-to-noise ratio (SNR) is defined as SNR= $N_{\mathrm{S}}/\sigma_z^2$.

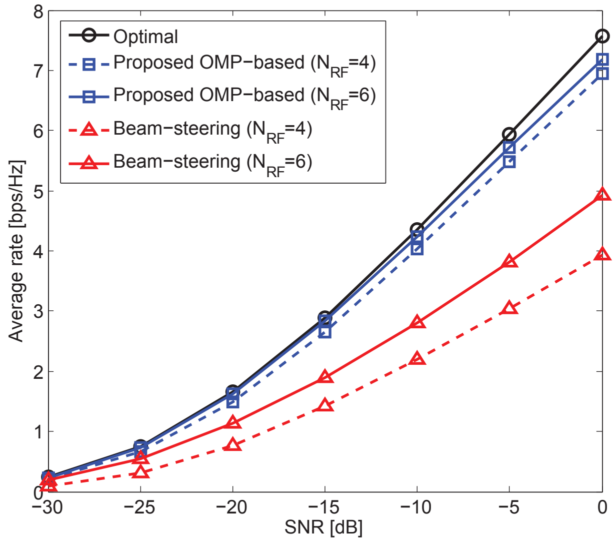

Fig. 2. Average rate comparisons at different SNR levels.

We determine the number of RF chains, N_{RF}, depending on the *effective rank* of the channel which is defined as $r(\kappa) = \arg\max_{r} \sum_{i=1}^{r} \lambda_i \leq \kappa \sum_{i=1}^{64} \lambda_i$ where λ_i is the *i*-th largest eigenvalue of $\mathbf{H}\mathbf{H}^H$ and $0 < \kappa \leq 1$ (here to simplify notation, $\mathbf{H}_{p,k}$ is written as $\mathbf{H}$). In the simulation, we considered two cases with $\kappa \in \{0.95, 0.98\}$ and observed that $r(0.95) \simeq 4$ and $r(0.98) \simeq 6$. Thus we set $N_{\mathrm{RF}} \in \{4, 6\}$.

Fig. 2 shows the average rates against the SNR. The proposed method outperforms the beam-steering method, and the performance of the former is close to that of the optimal scheme. The performance gap between the proposed methods with $N_{\mathrm{RF}} = 4$ and 6 is considerably smaller than the corresponding gap for the beam-steering method. This indicates that the proposed scheme can reduce N_{RF} with less degradation compared to the beam-steering method.

V. Conclusion

We considered the design of hybrid MIMO processors for an AF relay-assisted mm-wave communication system with OFDM signaling. The OMP-based algorithms were developed both for OFDM signaling and for jointly designing the Rx/Tx and RF/baseband processors of the relay. Simulation results demonstrate that the proposed method performs better than the beam-steering method that designs the baseband MIMO processor after steering the RF beams.

The proposed method requires the design of optimal precdoer/combiner and relaying matrices for all subchannels, which is computationally expensive. Further work in this area includes the development of an efficient algorithm for reducing the computational burden.

References

[1] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101-107, June 2011.

[2] T. S. Rappaport, S. Sun, R. Mayzus, H. Zhao, Y. Azar, K. Wang, G. N. Wong, J. K. Schulz, M. Samimi, and F. Gutierrez, "Millimeter wave mobile communications for 5G cellular: It will work!," *IEEE Access*, vol. 1, pp. 335-349, May 2013.

[3] IEEE P80211ad, Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications - Amendment 3: Enhancements for Very High Throughput in the 60GHz band, Dec. 2012.

[4] IEEE 802.15 WPAN Millimeter Wave Alternative PHY Task Group 3c. [Online]. Available: www.ieee802.org/15/pub/TG3c.html, Sep. 2011.

[5] X. Zhang, A. F. Molisch, and S. Y. Kung, "Variable phase shift based RF baseband codesign for MIMO antenna selection," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4091-4103, Nov. 2005.

[6] P. D. Karamalis, N. D. Skentos, A. G. Kanatas, "Adaptive antenna subarray formation for MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 11, pp. 2977-2982, Dec. 2006.

[7] A. G. Kanatas, "A receive antenna subarray formation algorithm for MIMO systems," *IEEE Commun. Lett.*, vol. 11, no. 5, pp. 1396-398, May 2007.

[8] V. Venkateswaran and A. van der Veen, "Analog beamforming in MIMO communications with phase shift networks and online channel estimation," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4131-4143, Aug. 2010.

[9] J. Nsenga, A. Bourdoux, and F. Horlin, "Mixed analog/digital beamforming for 60GHz MIMO frequency selective channels," *in Proc. IEEE Int. Conf. Commun.*, June 2010.

[10] J. Nsenga, A. Bourdoux, W. V. Thillo, V. Ramon, and F. Horlin, "Joint Tx/Rx analog linear transformation for maximizing the capacity at 60GHz," *in Proc. IEEE Int. Conf. Commun.*, June 2011.

[11] O. El Ayach, R. W. Heath, Jr., S. Abu-Surra, S. Rajagopal, and Z. Pi, "Low complexity precoding for large millimeter wave MIMO systems," *in Proc. IEEE Int. Conf. Commun.*, June 2012.

[12] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, Jr., "Spatially sparse precoding in millimeter wave MIMO systems," *to appear in IEEE Trans. Wireless Commun.*, 2014.

[13] A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., "Hybrid precoding for millimeter wave cellular systems with partial channel knowledge," *in Proc. Inf. Theory and Appl. Workshop*, Feb. 2013.

[14] J. A. Tropp and S. J. Wright, "Computational methods for sparse solution of linear inverse problems," *Proc. IEEE,* vol. 98, no. 6, pp. 948-958, Jun. 2010.

[15] H. Bolcskei, D. Gesbert, and A. J. Paulraj, "On the capacity of OFDM based spatial multiplexing systems," *IEEE Trans. Commun.*, vol. 50, no.2, pp. 225-234, Feb. 2002.

[16] A. Forenza, D. Love, and R. W. Heath, Jr., "Simplified spatial correlation models for clustered MIMO channels with different array configurations," *IEEE Trans. Veh. Technol.*, vol. 56, no. 4, pp. 1924-1934, July 2007.

[17] I. Hammerstrom and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2798-2802, Aug. 2007.

[18] B. Clerckx and C. Oestges, *MIMO Wireless Networks: Channels, Techniques and Standards for Multi-Antenna, Multi-User and Multi-Cell Systems*, 2nd ed. Academic Press, 2013.