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# Energy-efficient Power Allocation for OFDM Signaling over a Two-way AF Relay

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***Abstract*—In this paper, the power allocation problem is solved in the context of maximizing the energy efficiency (EE) of orthogonal frequency division multiplexing (OFDM) signaling over a two-way amplify-and-forward (AF) relay network with two sources and one relay. The EE is defined as the ratio of the system throughput over the total power consumption incorporating the transmission power, the fixed circuit power and the dynamic circuit power which is rate- dependent. In the EE maximization (EEM) problem, the efficiencies of the power amplifiers (PAs) of the three terminals can differ from one another. It is shown that EE is a quasi-concave function of the transmission power. Based on this result, we use Dinkelbach's method for fractional programming and propose a two-step allocation (TSA) algorithm to solve the inner loop of the Dinkelbach's method. Simulation results demonstrate that the proposed scheme can efficiently maximize the EE.**

***Index Terms*—Energy efficiency, Two-way relay, Orthogonal frequency-division multiplexing (OFDM), Power allocation, Fractional programming, Dual decomposition.**

## I. Introduction

Two-way relaying (TWR) is an efficient bidirectional relaying scheme where two users exchange information via relay node(s). Due to its bidirectional nature, TWR requires only two time-slots for exchanging the information and can achieve higher spectral efficiency (SE) and energy efficiency (EE), as compared with one-way relaying [1], [2], where EE is defined as the ratio of the system throughput over the total power consumption (bits/Joule).[1] TWR has been considered as an efficient means of improving the SE/EE of recent communication systems. Its applications include cognitive radio [5] and device-to-device communications [6], [7], as well as conventional cellular systems [8].

Energy-efficient TWR has been designed either by minimizing the power consumption or by maximizing the EE. In these design problems, the model for power consumption plays an important role. In general, the power consumption of a communication system is determined by the sum of the transmission power, the fixed circuit power and the dynamic circuit power that varies according to the transmission rate [9], [10]. Here the transmission power depends on the efficiencies of power amplifiers (PAs). The complexity of the optimization can be reduced by ignoring some power consumption terms such as the dynamic circuit power and by assuming that the PA efficiencies of different terminals are identical.

Investigation of the power minimization problem for TWR with amplify-and-forward (AF) relaying includes the power allocation scheme in [11], the relay selection and power allocation for TWR with multiple relays [2], the transmit beamformer and the receive combiner design for TWR with multiple antennas [12], and the hybrid of one-way and two-way relaying for energy-efficient transmission [13]. Among these techniques, [13] takes into account both the transmission power and the fixed circuit power consumption, while the others consider only the transmission power. None of them considers the dynamic power consumption, and all of them assume identical PA efficiencies of the terminals. The hybrid relaying in [13] has been extended to orthogonal frequency division multiplexing (OFDM) signaling [14].

The EE maximization (EEM) problem for the TWR network has been investigated in [15]-[18]. In [15], an optimal power allocation scheme for decode-and-forward (DF) TWR is proposed. For AF TWR networks the research on EEM includes the analysis of EE-SE tradeoff [16], EE maximizing power allocation under individual power constraints [17], and the derivation of the optimal transmission rate minimizing the total energy consumption per information bit, which is the inverse of EE [18]. These EEM techniques take into account both the transmission power and the fixed circuit power, but ignore the dynamic circuit power. [15] and [17] assume identical PA efficiencies, while [16] and [18] consider the general case with different PA efficiencies. All these techniques assume narrowband communications with flat fading channels.

In this paper, we formulate an EEM problem for AF TWR that considers OFDM signaling for wideband communications and the power consumption model in its most general form. The model incorporates the transmission power, the fixed circuit power and the dynamic circuit power; the terminals have different PA efficiencies. We have developed a process for optimally allocating the power to the subcarriers of each node. For a TWR network consisting of two sources and one relay, the optimization is performed to maximize the EE under the power constraints. It is shown that the EE is a *strictly quasi-concave* function of the total transmission power. Based on the quasi-concavity, we adopt the Dinkelbach's method for fractional programming [19] and observe that the inner loop maximization of the Dinkelbach's method reduces to a modified version of the SE maximization (SEM) problem



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[1]Comprehensive reviews on the results of investigations regarding various analysis and design issues for EE may be found in [3], [4].

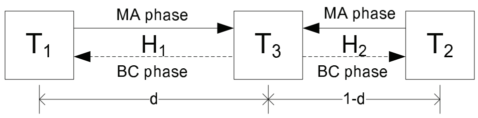


Fig. 1. The two-way relay channel where the distance between $T_1$ and $T_2$ is normalized to 1 and the distance between $T_1$ and $T_3$ is $d$.

considered in [20], [21]. In fact, the inner loop problem can be solved by the dual decomposition method (DDM) in [20]; however, the computational load of this method can be excessive. On the other hand, the two-step allocation (TSA) method in [21] is computationally efficient, but its use is limited to the case where the efficiencies of the PAs of the three nodes are the same. For general cases with different PA efficiencies, we propose the modified TSA by incorporating the iterative optimization algorithm in [22] that maximizes the lower bound of the objective function of the inner loop problem. Simulation results demonstrate that the proposed method, which is based on Dinkelbach's method and the modified TSA, maximizes the EE.

The organization of the paper is as follows. Section II presents the system model and the EEM problem formulation. In Section III, we prove the quasi-concavity of the EE with respect to (w.r.t.) the total transmission power and develop the optimization process to solve the EEM problem. The simulation results are presented in Section IV. Finally, Section V presents the conclusion.

## II. System Model and Problem Formulation

Consider an OFDM two-way relay system in which terminals $T_1$ and $T_2$ exchange data through terminal $T_3$, which acts as an AF relay (Fig. 1). We assume that there is no direct path between $T_1$ and $T_2$ and that the system works in half-duplex mode. Let $\mathbf{x}_k \triangleq [x_k(1), \cdots, x_k(N)]$, $k \in \{1,2\}$, denote the OFDM symbol to be transmitted by terminal $T_k$, where $\mathbf{x}_k \in \mathcal{C}^N$, $N$ is the number of subcarriers, and $E[|x_k(n)|^2] = 1$. In a time slot for the multiple access (MA) phase, $T_1$ and $T_2$ transmit $\sqrt{\mathbf{P}_1}\mathbf{x}_1$ and $\sqrt{\mathbf{P}_2}\mathbf{x}_2$, respectively, to relay $T_3$ where $\mathbf{P}_k \triangleq diag\{p_k(1), \cdots, p_k(N)\}$ is an $N$-by-$N$ diagonal matrix that controls the transmission power. The relay receives

$$\mathbf{y}_3 = \mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \mathbf{w}_3 \tag{1}$$

where $\mathbf{H}_k = diag\{h_k(1), \cdot, h_k(N)\}$ is an $N$-by-$N$ diagonal matrix representing the channel between terminal $T_k$ and relay $T_3$ and $\mathbf{w}_3 \in \mathcal{CN}(0, \mathbf{I}_N)$ is additive white Gaussian noise at the relay. The relay amplifies each subcarrier of the received signal by multiplying $\mathbf{y}_3$ with $\Gamma \triangleq diag\{\gamma(1), \cdots, \gamma(N)\}$ where $\gamma(n) = \sqrt{p_3(n)/(p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + 1)}$ and $p_3(n)$ is the transmission power of the relay for the $n^{th}$ subcarrier. Then, in the broadcast (BC) phase the relay broadcasts the signal to $T_1$ and $T_2$. The received signals at $T_1$ and $T_2$ after self-interference suppression, which is assumed to be perfect, are written as $\mathbf{y}_1 = \Gamma\mathbf{H}_1\mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \Gamma\mathbf{H}_1\mathbf{w}_3 + \mathbf{w}_1$, and $\mathbf{y}_2 = \Gamma\mathbf{H}_2\mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \Gamma\mathbf{H}_2\mathbf{w}_3 + \mathbf{w}_2$, where $\mathbf{w}_k \in \mathcal{CN}(0, \mathbf{I}_N)$ is additive white Gaussian noise at the terminal $T_k$.[2] We assume that the channels are reciprocal and that all terminals have the perfect knowledge of both channels $\mathbf{H}_1$ and $\mathbf{H}_2$.

The throughput of the system is given by

$$R_T(\underline{\mathbf{p}}) = \sum_{n=1}^{N} r_n(\mathbf{p}_n), \tag{2}$$

where $\mathbf{p}_n = [p_1(n), p_2(n), p_3(n)]$, $\underline{\mathbf{p}} = [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_N]$, and $r_n(\mathbf{p}_n) = \frac{W}{2}\sum_{i=1}^{2}\log_2(1+\gamma_i(n))$, with SNR values $\gamma_1(n) = p_2(n)p_3(n)|h_1(n)|^2|h_2(n)|^2/[p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + p_3(n)|h_1(n)|^2 + 1]$, $\gamma_2(n) = p_1(n)p_3(n)|h_1(n)|^2|h_2(n)|^2/[p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + p_3(n)|h_2(n)|^2 + 1]$. and W is the per-subcarrier bandwidth. The SE is defined as $R_T(\underline{\mathbf{p}})/NW$.

The total power consumption of the system is given by

$$P_T(\underline{\mathbf{p}}) = P_c + \sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n) + \beta R_T(\underline{\mathbf{p}}), \tag{3}$$

where the first and third terms in the right-hand-side (RHS) of (3) denote the fixed and dynamic (rate-dependent) circuit power consumption of the three terminals with $\beta$ being a constant representing the power consumption per unit data rate. The second term is the transmission power consumed by the transmitters' power amplifiers with $\alpha_k > 1$ denoting the reciprocal of the efficiency of power amplifiers [9], [10].[3]

The EEM problem, which will be referred to as problem ($\text{P}_{EE}$), is formulated as follows:

$$\text{maximize}_{\underline{\mathbf{p}}} \quad \eta_{EE}(\underline{\mathbf{p}}) = \frac{R_T(\underline{\mathbf{p}})}{P_T(\underline{\mathbf{p}})} \tag{4a}$$

$$subject\ to\ \sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n) \le P_{max}\ and\ p_k(n) \ge 0, \forall\, k,\ n, \tag{4b}$$

where $\eta_{EE}(\underline{\mathbf{p}}) = \frac{R_T(\underline{\mathbf{p}})}{P_T(\underline{\mathbf{p}})}$ is the EE and $P_{max}$ is the maximum power budget of the system.[4] If the objective function in (4) is replaced with $R_T(\underline{\mathbf{p}})$ and $\alpha_k = 1$ for all $k$, then the EEM problem becomes the conventional SEM problem in [20], [21].

## III. Proposed Energy Efficient Power Allocation Method

Problem ($\text{P}_{EE}$) in (4) is a complex fractional problem that is not convex in $\underline{\mathbf{p}}$. Fortunately, however, this problem can be converted into a single variable optimization problem which is strictly quasiconcave given a sufficiently large number of

[2]The self-interferences at T1 and T2 are written as $\Gamma\mathbf{H}_1\mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1$ and $\Gamma\mathbf{H}_2\mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2$, respectively. Since terminals T1 and T2 know their own transmitted symbols they can subtract the self-interferences, assuming perfect knowledge of the corresponding channels.

[3]Devices in radio frequency (RF) chains such as filters, low-noise-amplifiers and mixers contribute to fixed power consumption, while arithmetic operations of the baseband processing unit vary depending on the data rate and contribute to dynamic power consumption (see [23] for a more detailed discussion on power consumption of wireless terminals).

[4]In TWR systems, it is desirable to assign more power to the relay that transmits the sum of two signals from the sources [21]. By using the joint maximum transmit power constraint in (4b), we can optimally allocate the transmission power to the three nodes of the TWR system.

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subcarriers. In this section, we first derive the single variable optimization form of problem (P$_{EE}$) and show its quasiconcavity. Then the problem is solved by Dinkelbach's method for fractional programming [19].

### A. Quasicancavity in $P_{tx}$

We introduce a sub-problem of problem (P$_{EE}$), referred to as the *conditional* EE problem (P$_{C-EE}$), which is identical to problem (P$_{EE}$) with the exception that the inequality in (4b) is replaced with the following equality constraint on the total transmission power: $\sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n)=P_{tx}$, for $P_{tx}\in[0,\ P_{max}]$. Let $\underline{\mathbf{p}}^*_{tx}$ be the optimal solution to problem $(P_{C-EE})$. The EE for $\underline{\mathbf{p}}^*_{tx}$, denoted by $\eta^*_{\rm EE}(P_{tx})$, is written as

$$\eta^*_{\rm EE}(P_{tx})=\eta_{EE}(\underline{\mathbf{p}}^*_{tx})=\frac{R_T(\underline{\mathbf{p}}^*_{tx})}{P_c+P_{tx}+\beta R_T(\underline{\mathbf{p}}^*_{tx})} \quad (5)$$

where $R_T(\underline{\mathbf{p}}^*_{tx})$ is the SE for $\underline{\mathbf{p}}^*_{tx}$ satisfying the equality constraint on the total transmission power. In what follows, we denote the numerator and the denominator of $\eta^*_{\rm EE}(P_{tx})$ in (5) by $R^*_T(P_{tx})=R_T(\underline{\mathbf{p}}^*_{tx})$ and $P^*_T(P_{tx})=P_c+P_{tx}+\beta R_T(\underline{\mathbf{p}}^*_{tx})$, respectively. Using these notations, the primary problem (P$_{EE}$) can be rewritten as

$$\text{maximize}_{P_{tx}}\quad \eta^*_{\rm EE}(P_{tx})=\frac{R^*_T(P_{tx})}{P^*_T(P_{tx})} \quad (6a)$$

$$s.\,t.\ \sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n)=P_{tx}\le P_{max}\ and\ p_k(n)\ge 0,\forall\, k,\ n. \quad (6b)$$

This is the single variable optimization form of problem (P$_{EE}$). Next we show the quasi-concavity of $\eta^*_{\rm EE}(P_{tx})$ in $P_{tx}$ by examining the relation between $\eta^*_{\rm EE}(P_{tx})$ and $R^*_T(P_{tx})$.

**Lemma 1**. The optimal power allocation $\underline{\mathbf{p}}^*_{tx}$, which is the solution to problem (P$_{C-EE}$), is also the solution to the following SEM problem, called the *conditional* SEM problem (P$_{C-SE}$):

$$\text{maximize}_{\underline{\mathbf{p}}}\quad R_T(\underline{\mathbf{p}})\quad subject\ to\ (6b) \quad (7)$$

**Proof.** We denote the solution of this sub-problem by $\bar{\underline{\mathbf{p}}}^*_{tx}$ and show that $\underline{\mathbf{p}}^*_{tx}=\bar{\underline{\mathbf{p}}}^*_{tx}$. The EE for $\bar{\underline{\mathbf{p}}}^*$ is written as $\eta_{EE}(\bar{\underline{\mathbf{p}}}^*_{tx})=\frac{R_T(\bar{\underline{\mathbf{p}}}^*_{tx})}{P_c+P_{tx}+\beta R_T(\bar{\underline{\mathbf{p}}}^*_{tx})}$. Note that $\eta_{EE}(\bar{\underline{\mathbf{p}}}^*_{tx})$ is an increasing function of $R_T(\bar{\underline{\mathbf{p}}}^*_{tx})$. This fact indicates that $\bar{\underline{\mathbf{p}}}^*_{tx}$ maximizing $R_T(\bar{\underline{\mathbf{p}}}^*_{tx})$ also maximizes $\eta_{EE}(\underline{\mathbf{p}}_{tx})$. Therefore, $\underline{\mathbf{p}}^*_{tx}=\bar{\underline{\mathbf{p}}}^*_{tx}$, and the EEs in (5) and $\eta_{EE}(\bar{\underline{\mathbf{p}}}^*_{tx})$ are the same. □

The solution to the conditional SEM problem $(P_{C-SE})$ remains the same after replacing the equality constraint in (6b) with the inequality, $\sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n)\le P_{tx}$, because the optimal SE under this inequality constraint is a monotonically increasing function of $P_{tx}$, and when $P_{tx}$ is given, the transmission power corresponding the optimal SE under the inequality constraint is $P_{tx}$. Therefore, problem $(P_{C-SE})$ is in fact a conventional SEM problem and its solution $R^*_T(P_{tx})=R_T(\underline{\mathbf{p}}^*_{tx})$ has the following property.

**Lemma 2**. $R^*_T(P_{tx})$ is a strictly concave function of $P_{tx}$ in the limit as $N\to\infty$.

This lemma follows from Theorem 2 of [24] and indicates that the optimal rate $R_T(\underline{\mathbf{p}}^*_{tx})=R^*_T(P_{tx})$ is asymptotically concave in the transmit power constraint $P_{tx}$ even though $R_T(\underline{\mathbf{p}})$ in (7) is not concave in the power allocation vector $\underline{\mathbf{p}}$. Lemma 2 has been used to solve various resource allocation problems of OFDM systems [9], [25], [26]. In particular, it is shown in [9], [25] that the concavity of the optimal rate holds for practical number of subcarriers N. This is verified by our simulation (see footnote 8 in Section IV). Next we show the quasiconcavity of $\eta^*_{\rm EE}(P_{tx})$.

**Proposition 1**. $\eta^*_{\rm EE}(P_{tx})$ in (5) is a strictly quasiconcave function of $P_{tx}$ in the limit as $N\to\infty$.

**Proof.** Let $\mathcal{S}_q$ be the superlevel set defined as $\mathcal{S}_q=\{P_{tx}\in[0,\ P_{max}]|\eta^*_{EE}(P_{tx})\ge q\}$ for $q\in\mathbb{R}_+$. Consider $P^i_{tx}\in\mathcal{S}_q$, $i\in 1,2$, $P^1_{tx}\ne P^2_{tx}$ and $P^3_{tx}=\theta P^1_{tx}+(1-\theta)P^2_{tx}$ for $0<\theta<1$. Now, $\eta^*_{EE}(P^3_{tx})=(1-q\beta)R^*_T(P^3_{tx})-qP^3_{tx}-qP_c$ $=(1-q\beta)R^*_T(\theta P^1_{tx}+(1-\theta)P^2_{tx})-q(\theta P^1_{tx}+(1-\theta)P^2_{tx})-q(\theta P_c+(1-\theta)P_c)>\theta\eta^*_{EE}(P^1_{tx})+(1-\theta)\eta^*_{EE}(P^2_{tx})\ge q$, where the strict inequality holds because of the strict concavity of $R^*_T(P_{tx})$ (Lemma 2). Therefore, $P^3_{tx}\in\mathcal{S}_q$ and $\eta^*_{EE}(P_{tx})$ is strictly quasiconcave [27], given a sufficiently large number of subcarriers. □

This proposition indicates that $\max_{p_{tx}}\eta^*_{\rm EE}(P_{tx})$ exists and is unique [27]. Based on the quasi-concavity of $\eta^*_{\rm EE}(P_{tx})$, we can make the following observations regarding the optimization of (6). Let $P^o_{tx}$ be the optimal transmit power maximizing $\eta^*_{\rm EE}(P_{tx})$ in (6) when $P_{max}=\infty$.

**Observation 1**. For a finite $P_{max}$, the solution of (6) becomes $\eta^*_{\rm EE}(P_{max})$ if $P_{max}\le P^o_{tx}$, and $\eta^*_{\rm EE}(P^o_{tx})$, otherwise.

An important characteristic of the quasi-concavity is that it allows the optimization problem (6) to incorporate the data rate constraint, $R^*_T(P_{tx})\ge R_{th}$ for $R_{th}$ being the required rate. Let $P_{th}$ denote the transmit power corresponding to $R_{th}$.

**Observation 2**. Suppose that $P_{max}$, $R_{th}$ and $P_{th}$ are given and that $P_{th}\le P_{max}$. A solution of (6) incorporating the rate constraint then exists and is given by $\eta^*_{\rm EE}(P_{max})$ if $P_{th}<P_{max}<P^o_{tx}$, $\eta^*_{\rm EE}(P^o_{tx})$ if $P_{th}<P^o_{tx}<P_{max}$, and $\eta^*_{\rm EE}(P_{th})$ if $P^o_{tx}<P_{th}<P_{max}$.

### B. Optimization Process

Since $\eta^*_{\rm EE}(P_{tx})$ is strictly quasiconcave, problem (P$_{EE}$) in (6) can be solved by iterative convex optimization methods such as the bisection method [28] and Dinkelbach's method [19], where the former is proposed for general quasiconcave optimization problems and the latter is specified for fractional programming.[5] In the context of fractional programming, both the methods are based on the use of the *subtractive* form of the objective function which can be derived from the definition of the super-level sets: the inequality in $\mathcal{S}_q$, $\eta^*_{\rm EE}(P_{tx})\ge q$, is identical to $R^*_T(P_{tx})-qP^*_T(P_{tx})\ge 0$ and the left-hand-side (LHS) of this inequality is the subtractive form. For iterative optimization, the parameter $q$ is replaced with the $i^{th}$ parameter $q_i$, which is updated at each iteration. Here $i$ is the iteration index and $\{q_i\}$ are determined to

[5] Dinkelbach's method has been adopted to solve various energy efficiency optimization problems. For example, see [29]-[31].

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satisfy $\lim_{i\to\infty} q_i = \max_{P_{tx}} \eta^*_{EE}(P_{tx})$. The bi-section method assumes that the initial interval containing $\max_{P_{tx}} \eta^*_{EE}(P_{tx})$ is known and reduces the interval by half at each iteration by examining $R^*_T(P_{tx}) - qP^*_T(P_{tx}) \geq 0$. This scheme sets $q_i$ at the center of the $i$-th interval and solves a feasibility problem at each iteration. On the other hand, Dinkelbach's method does not need the knowledge on the range of $\max_{P_{tx}} \eta^*_{EE}(P_{tx})$. This method sets $q_i$ to the EE of the $(i-1)^{th}$ iteration (initially, $q_0 = 0$) and directly maximizes the subtractive objective function. As a consequence, the method can be computationally more efficient than the bi-section method [29]. Due to these advantages, we employ Dinkelbach's method, which is summarized in Algorithm 1. At the $i^{th}$ iteration (step 3), this scheme directly solves the following, which is called problem ($\mathrm{P}_{q_i}$):

$$\text{maximize}_{P_{tx}\in[0,\ P_{max}]}\ R^*_T(P_{tx}) - q_i P^*_T(P_{tx}),\ \ subject\ to\ (6b) \tag{8}$$

This step also requires an iterative optimization and is called the *inner* loop. The rest of Algorithm 1, referred to as the *outer* loop, iteratively updates $q_{i+1} = R_T(\underline{\mathbf{p}}^*_i)/P_T(\underline{\mathbf{p}}^*_i)$, where $\underline{\mathbf{p}}^*_i$ is the solution to problem ($\mathrm{P}_{q_i}$) in (8). It has been shown that $q_{i+1} > q_i$ and that $\lim_{i\to\infty} q_i = \max_{P_{tx}} \eta^*_{EE}(P_{tx})$ [19]. When $i = 1$, this algorithm solves the SEM problem since $q_0 = 0$, and $R_T(\underline{\mathbf{p}}^*_i)$ corresponds to the maximum transmit power $P_{max}$. Referring to Observation 1, if $P_{max} \leq P^o_{tx}$, then the algorithm stops here and the optimal EE solving (6) is $q_1 = \eta^*_{EE}(P_{max})$. On the other hand, if $P_{max} > P^o_{tx}$, then the iteration continues until $q_i$ reaches $\eta^*_{EE}(P^o_{tx})$. Here at each iteration, $q_i$ is increased by decreasing the transmit power $P_{tx}$. In a similar manner, the algorithm solves the optimization problem when (6) incorporates the rate constraint (see Observation 2). Next, we describe the procedure for solving the inner loop problem ($\mathrm{P}_{q_i}$).

### C. Inner loop maximization

Since $R^*_T(P_{tx}) = R_T(\underline{\mathbf{p}}^*_{tx})$ represents the solution of the SEM that maximizes $R_T(\underline{\mathbf{p}})$ over $\underline{\mathbf{p}}$ under (6b), problem ($\mathrm{P}_{q_i}$) of (8) involves maximization w.r.t. both $P_{tx}$ and $\underline{\mathbf{p}}$. Specifically, the objective of (8) can be rewritten as: $\text{maximize}_{P_{tx}} \left[\text{maximize}_{\underline{\mathbf{p}}} R_T(\underline{\mathbf{p}}) - q_i P_T(\underline{\mathbf{p}})\right]$. Here the maximization over $P_{tx}$ can be dropped, because the maximization can be achieved while maximizing over $\underline{\mathbf{p}}$. Problem ($\mathrm{P}_{q_i}$) in (8) is then equivalent to

$$\text{maximize}_{\underline{\mathbf{p}}}\ (1-\beta q_i)R_T(\underline{\mathbf{p}}) - q_i\left(\sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n)\right), \tag{9a}$$

$$subject\ \ to\ \sum_{k=1}^{3}\alpha_k\sum_{n=1}^{N}p_k(n) \leq P_{max}\ and\ p_k(n)\geq 0, \forall\ k,\ n. \tag{9b}$$

The objective in (9a) is obtained from $R_T(\underline{\mathbf{p}}) - q_i P_T(\underline{\mathbf{p}})$ by dropping the constant $P_c$ from (3). Following the procedure for converting the problem (4) into that of (6), the inner problem (9) can be rewritten as

$$\text{maximize}_{p_{tx}}(1-\beta q_i)R^*_T(P_{tx}) - q_i P_{tx},\ subject\ to\ (9b). \tag{10}$$

Then, due to Lemma 2, the objective function of (10) is a strictly concave function of $P_{tx}$ in the limit as $N\to\infty$. This in turn indicates that (10) can be optimized by solving its Lagrange dual problem; the duality gap approaches zero as $N\to\infty$ [24]-[26]. (There could be convergence problems for a small value of N, but as mentioned below Lemma 2 the concavity of $R^*_T(P_{tx})$ holds for practical values of N and the duality gap is virtually zero for such cases.)

This problem of (9) is a modification of the SEM problem considered in [20], [21]. In what follows, we adopt the two-step allocation (TSA) proposed in [21] which is a computationally-efficient suboptimal method with little degradation in performance as compared with the dual decomposition method (DDM).[6] The TSA consists of two steps called the subcarrier power allocation step and the per-subcarrier optimization step, where in the former the total transmission power for the $n^{th}$ subcarrier, denoted as $p_T(n)$, is determined for each $n$, and in the latter $p_T(n)$ is allocated to each node by solving the per-subcarrier version of problem ($\mathrm{P}_{q_i}$) of (9). In what follows, we first consider the per-subcarrier optimization and then proceed to determining $p_T(n)$.

*1) Per-subcarrier optimization for a given $p_T(n)$:* Using (2) in (10), the objective function of (10) is rewritten as: $\text{maximize}_{p_T(n)\forall n}\sum_{n=1}^{N}\left[(1-\beta q_i)\sum_{i=1}^{2}\log_2(1+\gamma^*_i(n)) - q_i p_T(n)\right]$, and the per-subcarrier version of problem ($\mathrm{P}_{q_i}$), called problem ($\mathrm{P}_{q_i-ps}$), is given by

$$\text{maximize}_{\mathbf{p}_n}(1-\beta q_i)r_n(\mathbf{p}_n) - q_i\left(\sum_{k=1}^{3}\alpha_k p_k(n)\right) \tag{11a}$$

$$subject\ \ to\ \sum_{k=1}^{3}\alpha_k p_k(n) = p_T(n)\ and\ p_k(n)\geq 0, \forall\ k, \tag{11b}$$

where $p_T(n)$ is the total transmit power that is assumed to be allocated to the $n^{th}$ subcarrier of the three terminals. Due to the equality constraint in (11b), this per-subcarrier problem can be reduced to

$$\text{maximize}_{\mathbf{p}_n} r_n(\mathbf{p}_n),\ \ \ subject\ \ to\ (11b). \tag{12}$$

When the parameters for power amplifier efficiency, $\{\alpha_k\}$, are identical to each other, closed-form solutions to problem ($\mathrm{P}_{q_i-ps}$) of (11) can be found following the approach in [21]. They are given by

$$p^*_1(n) = \frac{p'_T(n)(1+|h_2(n)|^2p'_T(n))}{2\left(1+|h_2(n)|^2p'_T(n)+\sqrt{(1+|h_1(n)|^2p'_T(n))(1+|h_2(n)|^2p'_T(n))}\right)}, \tag{13a}$$

**Algorithm 1** Dinkelbach's method for solving problem ($\mathrm{P}_{EE}$).

Input: $I_{outer}$ (maximum number of outer interations)
$\epsilon_{outer}$ (convergence tolerance of outer loop)

1: $i = 0$ , $q_0 = 0$
2: **while** $q_i - q_{i-1} \geq \epsilon$ and $i < I_{outer}$ **do**
3: Solve (8) to obtain its optimal solution $\underline{\mathbf{p}}^*_i$ (inner loop)
4: $q_{i+1} = R_T(\underline{\mathbf{p}}^*_i)/P_T(\underline{\mathbf{p}}^*_i)$
5: $i \leftarrow i+1$
6: **end while**

[6]The DDM solves the Lagrange dual problem and yields the globally optimal solution of the inner problem in the limit as $N\to\infty$ [20]. Use of the TSA for the inner problem can be justified because its performance is almost identical to that of the DDM [21].

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$$p_2^*(n) = \frac{p'_T(n)\sqrt{(1+|h_1(n)|^2 p'_T(n))(1+|h_2(n)|^2 p'_T(n))}}{2\left(1+|h_2(n)|^2 p'_T(n) + \sqrt{(1+|h_1(n)|^2 p'_T(n))(1+|h_2(n)|^2 p'_T(n))}\right)}, \tag{13b}$$

and $p_3^*(n) = p'_T(n)/2$, where $p'_T(n) = p_T(n)/\alpha$. For $\mathbf{p}_n^* = [p_1^*(n), p_2^*(n), p_3^*(n)]$ in (13), the SNR values $\gamma_i$ become identical to each other ($\gamma_1(n) = \gamma_2(n)$).

For $\{\alpha_k\}$ that are different from each other, it is difficult to derive the closed-form solutions of (11), which is non-convex. In this case, to solve problem ($\mathrm{P}_{q_i-ps}$) of (12), we adopt an iterative optimization algorithm in [22] that maximizes a lower bound of the objective function in (12). The lower bound is given by $\overline{r}_n(\mathbf{p}_n) = \frac{1}{2}\sum_{i=1}^{2} a_i \log_2(\gamma_i(n)) + b_i \le r_n(\mathbf{p}_n)$, where $a_i = \gamma_{i,0}(n)/(1+\gamma_{i,0}(n))$, $b_i = \log_2(1+\gamma_{i,0}(n)) - a_i \log_2(\gamma_{i,0}(n))$, and $\gamma_{i,0}$ is a non-negative constant. The equality holds when $\gamma_i(n) = \gamma_{i,0}(n)\ for\ i \in \{1,2\}$. This lower bound can be converted into a convex function by the transformation $p_k(n) = e^{\overline{p}_k(n)}\ for\ k \in \{1,2,3\}$. Using $\overline{p}_k(n)$, the lower bound becomes

$$\overline{r}_n(\overline{\mathbf{p}}_n) = \frac{1}{2}\sum_{i=1}^{2} a_i[\log_2(|h_1(n)|^2|h_2(n)|^2) + \overline{p}_j(n) + \overline{p}_3(n) - \log_2(e^{\overline{p}_1(n)}|h_1(n)|^2 + e^{\overline{p}_2(n)}|h_2(n)|^2 + e^{\overline{p}_3(n)}|h_i(n)|^2 + 1)] + b_i \tag{14}$$

where $j = (i\ mod\ 2) + 1$. $\overline{r}_n(\overline{\mathbf{p}}_n)$ in (14) is concave because the second term in the RHS of (14) is concave (log-sum-exp is convex). By replacing $r_n(\mathbf{p}_n)$ in (12) with $\overline{r}_n(\overline{\mathbf{p}}_n)$ and $\mathbf{p}_n$ by $\overline{\mathbf{p}}_n$, we obtain the following convex optimization problem:

$$\max_{\overline{\mathbf{p}}_n} \overline{r}_n(\overline{\mathbf{p}}_n),\ subject\ \ to\ \sum_{k=1}^{3} \alpha_k e^{\overline{p}_k(n)} = p_T(n). \tag{15}$$

An iterative method for solving problem ($\mathrm{P}_{q_i-ps}$), based on (15), is presented in Algorithm 2. At each iteration, this algorithm solves the convex optimization (15) and updates the parameters $\{a_i, b_i\}$ in (14). It can be shown that Algorithm 2 converges to a solution satisfying the necessary Karush-Kuhn-Tucker (KKT) condition for optimality [22].

*2) Determining $p_T(n)$:* So far, problem ($\mathrm{P}_{q_i-ps}$) is solved under the assumption that the total transmission power for each subcarrier, $p_T(n)$, is given. Next, we describe the process for determining $\{p_T(n)\}$. Using (2) and (3) in (10), the optimal power allocation problem is written as

$$\max_{p_T(n)\forall n} \sum_{n=1}^{N}\left[(1-\beta q_i)\sum_{i=1}^{2}\log_2(1+\gamma_i^*(n)) - (\lambda_0+q_i)p_T(n)\right] \tag{16a}$$

**Algorithm 2** Iterative method ($P_{q_i-ps}$)

Input: $I_{ps}$ (maximum number iteration index)
$\epsilon_{ps}$ (convergence tolerance)
$\overline{\mathbf{p}}_n^0$ and $a_i^0 = 1$, $b_i^0 = 0$ for $i = \{1,2\}$.

$\kappa = 0$
**while** $\|\overline{\mathbf{p}}_n^\kappa - \overline{\mathbf{p}}_n^{\kappa+1}\|^2 \ge \epsilon_{ps}$ and $\kappa < I_{ps}$ **do**
  Solve (15) to obtain its optimal solution $\overline{\mathbf{p}}_n^*$ and objective value $\{\gamma_i^\kappa\}$
  Update $a_i^{\kappa+1} = \gamma_i^\kappa/(1+\gamma_i^\kappa)$
  $b_i^{\kappa+1} = \log_2(1+\gamma_i^\kappa) - a_i^{\kappa+1}\log_2(\gamma_i^\kappa)$
  $\kappa \leftarrow \kappa + 1$
**end while**

$$subject\ \ to\ \sum_{n=1}^{N} p_T(n) \le P_{max}\ and\ p_T(n) \ge 0\ \forall\ n, \tag{16b}$$

where $\gamma_i^*(n)$ is equal to the SNR $\gamma_i(n)$, given the optimal $\mathbf{p}_n^* = [p_1^*(n), p_2^*(n), p_3^*(n)]$. (Recall that $\mathbf{p}_n^*$ is given by (13) when $\{\alpha_k\}$ are equally likely, and obtained by Algorithm 2, otherwise.) This optimization problem can be solved efficiently by the Lagrange dual method [28]. The Lagrange dual problem of (16) is defined as: minimize$_{\boldsymbol{\lambda}\ge\mathbf{0}} g(\boldsymbol{\lambda})$ where $g(\boldsymbol{\lambda})$ is a Lagrange dual function given by

$$g(\boldsymbol{\lambda}) = \max_{p_T(n)\forall n}\sum_{n=1}^{N}\max_{p_T(n)}\left[(1-\beta q_i)\sum_{i=1}^{2}\log_2(1+\gamma_i^*(n)) - (\lambda_0+q_i)p_T(n) + \lambda_n p_T(n)\right] + \lambda_0 P_{max}, \tag{17}$$

and $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \cdots, \lambda_N]$. Note that the optimal $p_T^*(n)$ for each $n$, maximizing the per-subcarrier objective of (17), can be found by one-dimensional search. The Lagrange dual problem is solved by the *subgradient* method, as summarized in Algorithm 3. At the $j^{th}$ iteration, for a given $\boldsymbol{\lambda}$, the optimal solution of (17), $\{p_T^*(n)\}$, is obtained through the golden section search [32] (step 5). Here during the search, for each candidate $p_T(n)$, $\gamma_i(n)$ is obtained, either by (13) (when $\{\alpha_k\}$ are equally likely), or by solving Algorithm 2 (when $\{\alpha_k\}$ are different). The Lagrange multiplier at the $j^{th}$ iteration is denoted as $\boldsymbol{\lambda}^j = [\lambda_0^j, \lambda_1^j, \cdots, \lambda_N^j]$ and updated by

$$\lambda_0^{j+1} = \left[\lambda_0^j + \xi_0\left(\sum_{n=1}^{N} p_T^*(n) - P_{max}\right)\right]^+, \tag{18}$$

$$\lambda_n^{j+1} = [\lambda_n^j + \xi_n p_T^*(n)]^+,\ for\ n\ \in \{1,\cdots,N\}, \tag{19}$$

where $\{\xi_0, \xi_1, \cdots, \xi_N\}$ represent the step size, $p_T^*(n)$ is the optimal solution at the $j^{th}$ iteration, and $[x]^+ = max(x,0)$ (steps 7-9). The proposed optimization presented in Algorithm 3 is called the *modified* TSA.

The computational load for the modified TSA is in between those of the TSA and the DDM. It can be shown that the TSA and the DDM need $O(IN\log_2(B'))$ and $O(INB^3)$ computations, respectively, where $I$ is the number of iterations required by the subgradient method, and $B$ and $B'$ are the number of power levels that can be taken by $p_k(n)$ and

**Algorithm 3** Lagrange dual method for solving problem ($P_{q_i}$)

Input: $I_{inner}$ (maximum number of inner iterations)
$\epsilon_{inner}$ (convergence tolerance of inner loop)
$\boldsymbol{\lambda}$, and step size $\{\xi_0, \xi_1, \cdots, \xi_N\}$.

1: $j = 0$
2: **while** $\|\boldsymbol{\lambda}^j\|^2 - \|\boldsymbol{\lambda}^{j-1}\|^2 \ge \epsilon_{inner}$ and $j < I_{inner}$ **do**
3:   $j \leftarrow j + 1$
4:   **for** $n : 1 \to N$ **do**
5:     Obtain the optimal solution of (17), $p_T^*(n)$, by the golden search, where for each candidate $p_T(n)$, $\gamma_i(n)$ is obtained either by (13) (when $\{\alpha_k\}$ are the same), or by solving Algorithm 2 (when $\{\alpha_k\}$ are different).
6:   **end for**
7:   Update $\boldsymbol{\lambda}$ with
8:   $\lambda_0 = [\lambda_0 + \xi_0[\sum_{n=1}^{N} p_T^*(n) - P_{max}]]^+$,
9:   $\lambda_n = [\lambda_n + \xi_n p_T^*(n)]^+$, for $n \in \{1,\cdots,N\}$
10: **end while**

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$p_T(n)$, respectively [21]. When $\{\alpha_k\}$ are different from each other, the computational load of the modified TSA is given by $O(IN\log_2(B')I_{ps}I_{(30)})$, where $I_{ps}$ is the maximum number of iterations of Algorithm 2 and $I_{(30)}$ is the number of iterations needed for solving (15).In general, $\log_2(B')I_{ps}I_{(30)}$ can be considerably smaller than $B^3$, and thus, the modified TSA requires less computation than does the DDM.

## IV. Simulation Results

The performance of the proposed optimization is examined by computer simulation. For comparison, two additional schemes are considered: one is the SEM scheme that maximizes the SE and the other is the DDM based scheme which is identical to the proposed method with the exception that the inner loop problem of the Dinkelbach's method is solved by the DDM. The parameters for the simulation are as follows: For the iterative optimization, we set $I_{outer} = 10$, $I_{inner} = 100$, $I_{ps} = 10$, $I_{(30)} = 100$, $\epsilon_{outer} = \epsilon_{inner} = \epsilon_{(30)} = 10^{-3}$, $B = 100$, and $B' = 300$.[7] For the power consumption model, we use $P_c \in \{0.1, 0.5\}$Watt, $\alpha_k \in \{2, 3, 4\}$, $\beta \in \{0.1, 0.01\}$ in the unit of uJ/bit, and $P_{max}$ varying from $0.01$ Watt to $5$ Watt. The bandwidth of the channel is 1MHz, the number of subcarriers $N = 72$, and the noise power spectral density is $-174dBm/Hz$.[8] The channel is modeled by the path-loss, given by $128.1+37.6\log_{10}(D)$ where $D$ is the distance in the unit of $Km$ [33], and uncorrelated Rayleigh fading obeying the complex Gaussian distribution, $CN(0, 1)$. We assume one-dimensional network geometry, as shown in Fig. 1, where the distance between terminals $T_1$ and $T_2$ is $1Km$, and the distance between terminals $T_1$ and $T_3$ is denoted by "d."

Fig. 2 shows the energy efficiency against d. The proposed and the DDM-based schemes perform almost the same and, as expected, outperform the SEM scheme maximizing the SE instead of the EE. These three schemes achieve their maximum EEs when $d = 1 - d = 0.5$, suggesting that the relay be located in the middle to enhance the EE as in the case of SEM in [21]. Comparing the performances of the proposed schemes with different PA efficiencies, the proposed with $[\alpha_1, \alpha_2, \alpha_3] = [4, 4, 2]$ and $[\alpha_1, \alpha_2, \alpha_3] = [2, 2, 4]$ exhibit the best and the worst performances, respectively, and the EE curve for $[\alpha_1, \alpha_2, \alpha_3] = [3, 3, 3]$ lies in between the other two curves. These results indicate that employing a PA with higher efficiency is more important to the relay than to the sources. This is because the proposed scheme assigns more power to the relay that transmits the sum of two signals from the sources. Similar observations can be made for the DDM based and the SEM schemes.

[7]In the simulation, it was observed that given the convergence tolerances $\epsilon_{outer} = \epsilon_{inner} = \epsilon_{ps} = 10^{-3}$, the outer loop, the inner loop and Algorithm 2 required at most 10, 100 and 10 iterations, respectively. The total number of iterations for the proposed method, $O(IN\log_2(B')I_{ps}I_{(30)})$, is about 1% (0.001%) of that for the DDM, $O(INB^3)$, when $\{\alpha_k\}$ are different (the same).

[8]The duality gap is zero if $R_T^*(P_{tx})$ is a concave function of $P_{tx}$ [24]-[26]. Through an additional simulation not reported here, we examined the concavity of $R_T^*(P_{tx})$ w.r.t. $P_{tx}$ for $N \geq 2$. The results indicate that the concavity holds when $N \geq 8$, coinciding with the result in [25], [26]. Therefore, in our simulation where $N = 72$, the duality gap is virtually zero.

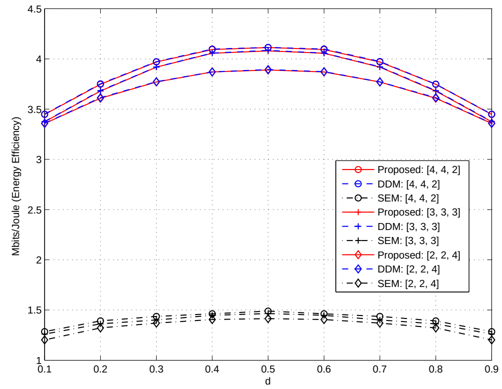


Fig. 2. EE against d where d is the distance between terminal $T_1$ and relay $T_3$. Here $P_{max}$ = 5 Watt, $\beta$ = 0.01, $P_c$ = 0.5, and $[\alpha_1, \alpha_2, \alpha_3] \in \{[4, 4, 2], [3, 3, 3], [2, 2, 4]\}$.

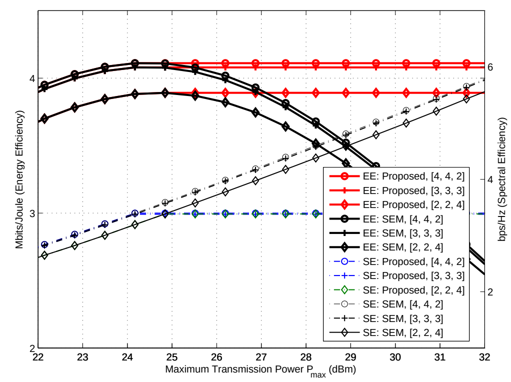


Fig. 3. EE/SE against $P_{max}$ where d = 0.5, $\beta$ = 0.01, and $[\alpha_1, \alpha_2, \alpha_3] \in \{[4, 4, 2], [3, 3, 3], [2, 2, 4]\}$.

Figs. 3 and 4 compare the EE/SE curves of the proposed and the SEM schemes against the total power $P_{max}$, where the former shows the curves for different PA efficiencies and the latter shows those for different $\beta$s and $P_c$s. (As in Fig. 2, the EE/SE curves of the DDM-based schemes overlap those of the corresponding curves of the proposed method, and the DDM curves are not shown.) As $P_{max}$ is increased, the SEM scheme continues to allocate more power to the terminals to increase the SE, and as a result the EE of the SEM scheme decreases after achieving its maximum EE. On the other hand, after attaining the maximum EE, the proposed EEM scheme stops allocating more power to the terminals and its EE/SE curves remain constant. In Fig. 4, it is noted that reducing $P_c$ in the proposed method causes a reduction of the optimal transmit power $P_{tx}^o$ which in turn considerably increases the EE but decreases the SE. The reason why this happens is as follows: referring to the definitions of EE/SE and the total power consumption in (4)-(9), the EE can be increased by simply decreasing the transmit power when both $P_c$ and $\beta$ are close to zero, because the slope of the log function in (5) is steeper when the SNR is small. Therefore, incorporating the data rate constraint with (13) (Observation 2) is particularly

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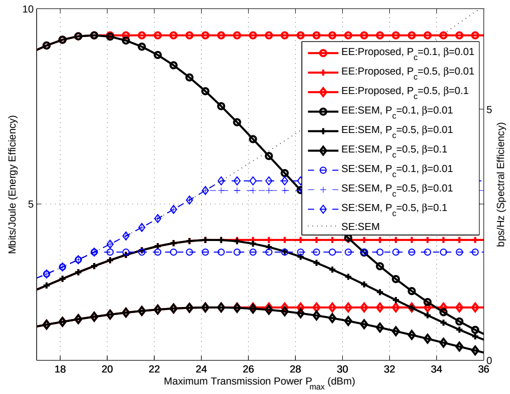


Fig. 4. EE/SE against $P_{max}$ where d = 0.5, $\beta \in \{0.1, 0.01\}$, $P_c \in \{0.1, 0.5\}$, and $[\alpha_1, \alpha_2, \alpha_3] = [3, 3, 3]$.

important when $P_c$ and $\beta$ are close to zero. Comparing the curves in Fig. 4 for different values of $\beta$, the EEs decrease as $\beta$ increases. This happens because the dynamic power consumption increases linearly with $\beta$.

## V. Conclusion

A process for optimally allocating the power to each subcarrier of OFDM signals over a two-way AF relay network was developed. The performance metric for the optimization was the EE defined as the ratio of system throughput over the total power consumption incorporating the transmission power, the fixed circuit power and the rate-dependent dynamic circuit power. The EE was shown to be quasi-concave w.r.t. the transmission power and maximized by Dinkelbach's method in conjunction with the modified TSA algorithm. Simulation results show that the proposed scheme can efficiently maximize the EE. Further work in this area includes the extension of the proposed method to a two-way relay network with multiple relays.

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