Multiplierless FIR Filters Based on Cyclotomic and Interpolated Second-Order Polynomials with Powers-of-Two Coefficients

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Abstract - We propose an optimal method for designing multiplierless FIR filters with cascaded prefilter-equalizer structures. In particular, simultaneous design of multiplierless prefilters and equalizers based on cyclotomic polynomials (CPs) and interpolated second-order polynomials (ISOPs) with powers-of-two coefficients is introduced. After employing CPs for prefiltering and ISOPs for equalization, a cascade of the CP prefilter and the ISOP equalizer is simultaneously optimized by mixed integer linear programming (MILP) [1]. Design examples demonstrate that this method leads to more efficient cascaded FIR prefilter-equalizers than existing methods.

I. INTRODUCTION

One approach to the design of efficient FIR filters requiring fewer arithmetic operations than conventional one is based on a cascade structure composed of a prefilter, which is often multiplierlss, followed by an FIR equalizer [1]-[5]. The prefilter provides reasonable stopband attenuation, and the equalizer makes the overall filter meet passband and stopband specifications. Most prefilters introduced so far are based on the use of either recursive running sum (RRS) [2], [3] or CPs [4], [5] which include the RRS as a special case. An optimal procedure for designing CP prefilters with minimal complexity is introduced in [1]. The equalizer is usually designed via a modified Parks-McCllean algorithm.

Although the prefilter and the equalizer have been individually designed under some optimality criterion, the optimality of the overall filter which is the cascade of the two can hardly be guaranteed. In practice, several cascaded FIR prefilter-equalizers that meet given specifications are derived and the one with minimal complexity is selected. The objective of this paper is to develop optimization techniques for designing the overall filter with minimal complexity. We shall introduce a class of ISOPs which powers-of-two coefficients and show that the ISOP can serve as a basic building block of equalizers. Following the approach in [1], the overall filter design problem will be formulated as an optimization problem with linear objective and constraint functions by applying the logarithm to the transfer function of the CP prefilter cascaded with the ISOP equalizer. The design problem is then solved by MILP. This approach guarantees the optimality of the overall filter through simultaneous design of the prefilter and equalizer.

II. PROBLEM FORMULATIONS

A. CP Prefilter

Fig. 1 illustrates the CP prefilter and ISOP equalizer. The system function of the prefilter $P(z)$ is represented as

$$P(z) = \prod_{q=1}^{Q} F_q(z)^{m_q} \tag{1}$$

where m_q are nonnegative integers and $F_q(z)$ s are eligible CPs in z^{-1}. As in [5], CPs are selected based on the following criterion: Roots of a $F_q(z)$ should lie in stopband plus intrusion of f_{in}, typically 5 to 20 percent into the transition band. In order to obtain multiplierless CP prefilter, only the first 104 CPs which contain only the coefficients {-1, 0, 1} are considered. In addition, to further reduce the hardware complexity, some CPs are combined [5]. For example, let $C_n(z)$ be an n-th cyclotomic polynomial, then

$$\begin{aligned} C_{21}(z) &= 1 - z^{-1} + z^{-3} - z^{-4} + z^{-6} - z^{-8} + z^{-9} - z^{-11} + z^{-12} \\ &= C_3(z^7)/C_3(z) \\ &= \frac{1 + z^{-7} + z^{-14}}{1 + z^{-1} + z^{-2}} \end{aligned} \tag{2}$$

Note that by realizing $C_3(z^7)/C_3(z)$ instead of $C_{21}(z)$, we can save 4 adders at the expense of 2 delays. After selecting eligible CPs from the set of first 104 CPs, Some efficient combination of CPs are added.

B. ISOP Equalizer

The ISOP is an interpolated version of the following second order polynomial with symmetric coefficients.

$$S(z) = \left(a + bz^{-1} + az^{-2}\right) \tag{4}$$

where a and b are real and $a \neq 0$. When $b^2 - 4a^2 > 0$, the roots of this polynomial are real and cannot be located on the unit circle. Since polynomials whose roots are located on the unit circle are not suitable for equalizers, we assume that $b^2 - 4a^2 > 0$. The ISOP is defined as

$$S^I(z) = \left(a + bz^{-I} + az^{-2I}\right) \quad (5)$$

where I is an interpolation factor. Fig. 2 illustrates the magnitude responses of some ISOPs for several (a,b,I) values. The fluctuations observed from these responses are useful for equalization. The ISOP equalizer is a cascade of ISOPs. Its system function $E(z)$ is defined as

$$E(z) = \prod_{r=1}^{R}(a_r + b_r z^{-I} + a_r z^{-2I})^{n_r} = \prod_{r=1}^{R}(S_r^I)^{n_r} \quad (6)$$

where n_r is a nonnegative integer; a_r and b_r are assumed to be powers-of-two coefficients so that there are finite number of ISOPs. In our design, we select eligible ISOPs under the following criteria described below.

i) As shown in Fig. 3, typical magnitude responses of equalizers are convex in passbands. Based on this observation, for given stopbands and passbands, we select ISOPs which have only one local minimum and do not have any local maximum in the passbands.

ii) A large number of eligible polynomials $S^I(z)$ s have their roots densely clustered each other in passbands. These polynomials exhibit very similar magnitude responses, therefore we select only one polynomial having minimal complexity among them.

C. Optimization Problem

The system function of the overall filter $H(z)$ is given by

$$H(z) = KP(z)E(z) \quad (7)$$

where K is a positive constant. Now $H(z)$ can be designed following the method in [1]. We define

$$\begin{aligned} H_{dB}(\omega) &= 20log\left|H(\omega)\right| \\ &= K_{dB} + \sum_{q=1}^{Q} m_q F_{qdB}(\omega) + \sum_{r=1}^{R} n_r S_{rdB}^I(\omega) \end{aligned} \quad (8)$$

where, $K_{dB} = 20log\left|K\right|$, $F_{pdB}(\omega) = 20log\left|F_p(\omega)\right|$, $S_{rdB}^I(\omega) = 20log\left|S_r^I(\omega)\right|$. The overall filter $H(z)$ with minimal complexity is designed by solving the following optimization problem. For given δ_p and δ_s which are passband and stopband ripples, respectively,

$$\begin{aligned} &\textit{Minimize} \quad \sum_{q=1}^{Q} m_q \cdot (a_{F,q} + c_d \cdot d_{F,q}) + \sum_{r=1}^{R} n_r \cdot (a_{S,r} + c_d \cdot d_{S,r}) \\ &\qquad (\textit{measure of complexity}) \\ &\textit{Subject to} \quad 20log(H_d - \delta_p) < H_{dB}(\omega) < 20log(H_d + \delta_p) \\ &\qquad (\textit{in passbands}) \\ &\qquad H_{dB}(\omega) < 20log\delta_s \\ &\qquad (\textit{in stopbands}) \end{aligned} \quad (9)$$

where $a_{F,q}$ ($d_{F,q}$) and $a_{S,r}$ ($d_{S,r}$) denote the number of adders(delays) required for implementing $F_q(z)$ and $S_r^I(z)$, respectively; c_d is a constant which is determined by comparing the hardware complexity of an adder with that of a delay element. (We set $c_d = 0.5$ in our design examples.) This optimization can be solved by MILP, treating K_{dB}, m_q, and n_r as variables.

III. Examples

To compare the proposed design method with the previous ones, our method is applied to the filter design problems considered in [5]. In our design, we assume that coefficients of $S_r^I(z)$ are 8 bit (including sign bit) power-of-two coefficients. Using the commercial MILP package in [8], we were able to solve the MILP problem within an hour in a Sparc 20.

Example 1 (Low pass filter): The specifications in normalized frequency are:

passband: $F \in [0,\ 0.021]$,
stopband: $F \in [0.07,\ 0.5]$
$\delta_p \le 0.1dB$ in passband, $\delta_s \ge 60dB$ in stopband.

This design problem is also considered in [2], [6], and [7]. Conventional linear phase equiripple filter with infinite precision coefficients requires 59 taps (30 multipliers, 58 adders, and 58 delays) to meet the specifications. In [5], a CP prefilter and an equalizer with infinite precision coefficients are designed under these specifications. In our design, the number of eligible polynomials are: Q=33, R=72. Fig. 4(a) illustrates the magnitude response of the resulting prefilter, equalizer and the overall filter. Table I compares the results of the proposed method with those of [2], [5]-[7]. It is seen that the filter designed by the proposed method is considerably simpler to implement than the others are.

Example 2 (Bandpass Filter): The specifications in normalized frequency are:

passband: $F \in [0.189,\ 0.211]$,
stopband: $F \in [0,\ 0.168] \cup [0.232, 0.5]$

$\delta_p \leq 0.25dB$ in passband, $\delta_s \geq 60dB$ in stopband.

The above specifications for bandpass example were chosen by other researchers [2]-[4]. Conventional linear phase equiripple filter with infinite precision coefficients requires 121 taps (61 multipliers, 120 adders, and 120 delays) to meet the specifications. Using a 10 percent transition band intrusion criterion, we obtained 15 eligible CPs. By examining all possible combinations of these, 25 combined polynomials were obtained and add to the set of eligible CP polynomials ($Q = 40$). The number of eligible ISOPs was 38 (R=38). The frequency response of the designed cascade form FIR digital filter is shown in Fig. 4(b). Table II compares the results of the proposed method with those in [2]-[5]. Again, the filter designed by the proposed simultaneous design method is remarkably simpler than others are.

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TABLE I
COMPARISON AND SUMMARY OF EXAMPLE 1.

	Modified K-H and Remez Equalizer [2]	Tapped Cascaded Identical Subfilters [6]	Interpolated FIR Filter [7]	CP Prefilter/ Remez Equalizer [5]	CP Prefilter/ Equalizer by Subset Selection [5]	CP Prefilter and ISOP Equalizer By MILP
multiplications	17	4	5	10	4	-
additions	37	17	15	28	16	16
delays	73	100	78	71	85	82

TABLE II
COMPARISON AND SUMMARY OF EXAMPLE 2.

	Modified K-H and Remez Equalizer [2]	Cabezas-Diniz Design 2 [3]	CP Prefilter/ Remez Equalizer [4]	CP Prefilter/ Remez Equalizer [5]	CP Prefilter/ Equalizer by Subset Selection [5]	CP Prefilter and ISOP Equalizer By MILP
multiplications	43	7	31	17	13	-
additions	163	48	82	58	49	24
delays	162	217	163	161	180	172

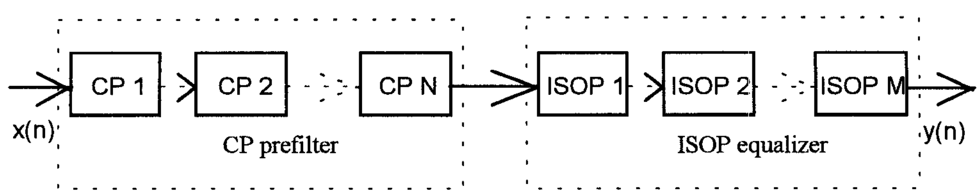


Fig. 1. The CP prefilter cascaded with the ISOP equalizer.

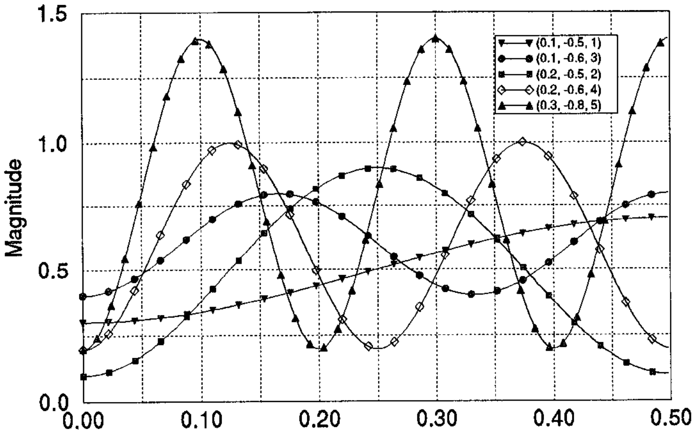


Fig. 2. Magnitude responses of some ISOPs.

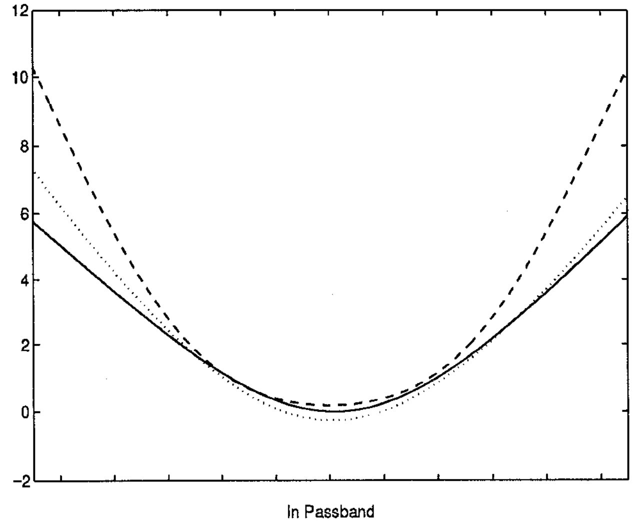
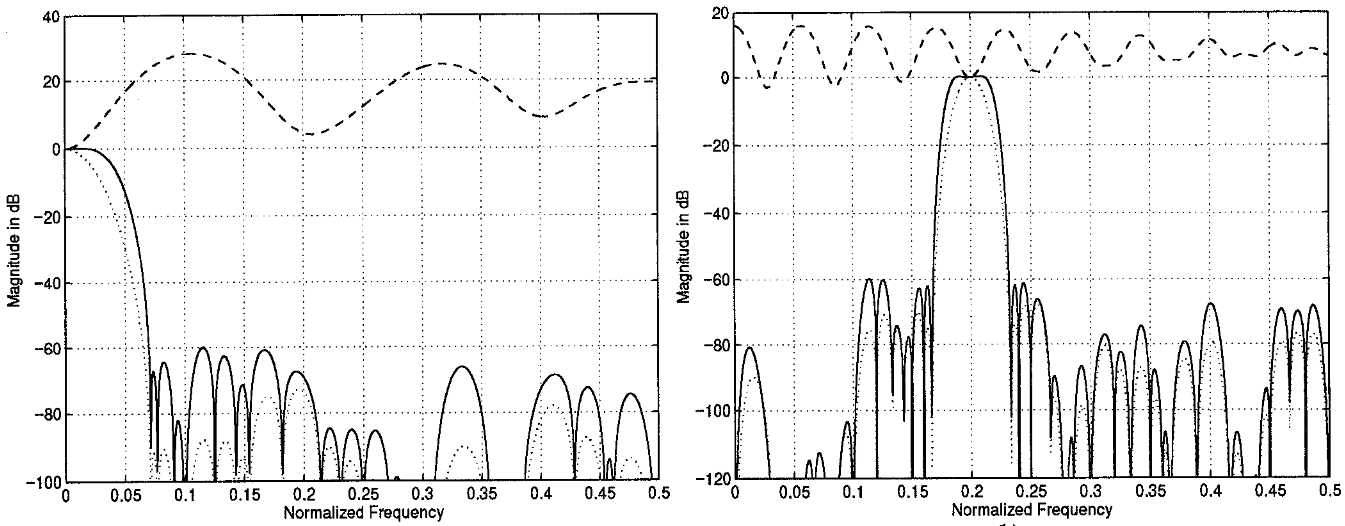


Fig. 3. Typical magnitude responses of equalizers



(a) (b)

Fig. 4. Magnitude responses of the designed CP prefilter (dotted line), the ISOP equalizer (dashed line), and the overall cascaded filter (solid line). (a) Lowpass filter in Ex. 1. (b) Bandpass filter in Ex. 2.