Design of Sparse FIR Filters Based on Branch-and-Bound Algorithm

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Abstract – Branch-and-bound algorithm is applied to the design of sparse FIR filters having intentionally zeroed tap positions. It is shown that this algorithm coupled with a suitable optimization technique for filter design can lead to an optimal sparse FIR filter satisfying given specifications. Design examples demonstrate that the proposed method requires less computation than the conventional optimization such as the subset selection method.

I. Introduction

One approach to efficient FIR filters is to design a filter having intentionally zeroed tap coefficients which are not necessarily equally spaced [1]-[6]. Such a filter, called *sparse* or *thinned* FIR filters, can lead to either reduction of multipliers or additional stopband suppression at the expense of increased delays. In contrast to most of the other efficient FIR filter design techniques [7]-[11], which are mainly useful for narrowband filter design, this approach is effective in designing wide range of filter types including wideband and nonlinear phase FIR filters. As a consequence, sparse filter design can be used in conjunction with the other methods. For example, sparse equalizers in the prefilter-equalizer design [9] and sparse model filters in the interpolated FIR filter design [11] have been shown to be useful for reducing arithmetic complexity [6].

The procedure for designing sparse FIR filters consists of two stages. In the first stage, zeroed tap positions are selected; and in the second the values of non-zero coefficients are determined. In the latter stage, designing optimal filter coefficients given zeroed tap positions is rather straightforward: linear or quadratic programming solving constrained optimization problems can be used. On the other hand, finding best tap positions may require an exhaustive search; several techniques have been proposed to overcome this difficulty. In [1] and [2], a class of FIR filters having zeroed taps at every nth tap position was considered. In an effort to find optimal zeroed tap positions for general FIR filters, the subset selection method that can select a subset of some linearly independent basis functions that best approximate a given process has been applied [9]. This method can indeed find the best tap positions, but it has some chance of evolving into an exhaustive search. As an alternative approach, the problem of finding best tap positions was formulated as linear programming, and solved by using a mixed integer linear programming (MILP) package [6]. This approach also leads to an optimal solution but its use is limited to filter designs with minimax-type error criterion.

In this paper, we propose to use branch-and-bound algorithm for determining optimal zeroed tap positions. It will be shown that a branch-and-bound method coupled with a suitable optimization technique for filter design can lead to a filter satisfying given specifications, with minimal complexity. The proposed method is simpler than the subset selection method, and can design sparse FIR filters under various error criterions.

II. Sparse Filter Design Based on Branch-and-Bound Method

The design method described in this section can be applied to any FIR filter design. In what follows, we shall illustrate our method for the case where the impulse response is symmetric and the filter length is odd.

Let $h(n), n = 0, 1, \cdots, 2N-2$, denote the impulse response of a linear phase, sparse FIR filter. If $h(n) = h(2N-2-n)$ for $n = 0, 1, \cdots, N-2$, its frequency response is given by (omitting the linear phase term $\exp^{-j(N-1)w}$)

$$H(w) = \sum_{k=0}^{N-1} a(k) cos\, kw \qquad (1)$$

where $a(0) = h(N-1)$ and $a(k) = 2h(N-1-k)$ for $1 \le k \le N-1$. The procedure for designing a sparse FIR filter is described as follows.

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Preliminary Stage: A conventional, non-sparse FIR filter that satisfies given filter specifications is designed by using an appropriate optimization technique. The length of the resulting filter is denoted by $2N_c - 1$.

The Branch-and-Bound Algorithm: At the beginning we set N in (1) to an integer greater than N_c. Then a design problem for finding $\{ a(k) : k = 0, 1, \cdots, N-1 \}$ that satisfy the filter specifications is formulated. This problem is designated as P. Next for each $k, 0 \leq k \leq N-1$, a design problem P_k is generated by adding the constraint $a(k) = 0$ to the original problem P. Further branching is performed on each P_k: a subproblem P_{kj} is generated by adding $a(j) = 0$ to P_k. Thus P_{kj} is a filter design problem under the constraints that $a(k) = a(j) = 0$. The branching process may be continued until $N-1$ out of N coefficients are set to zero, as illustrated in Fig. 1 for $N = 4$.

The number of subproblems to be solved is substantially reduced by fathoming: we say that a subproblem is fathomed when further branching on the subproblem cannot yield any useful information. For a fathomed subproblem branching is not performed. In our case, a subproblem is fathomed if it is infeasible. For example, P_{kj} is fathomed if we cannot design a filter satisfying the given specifications under the constraints that $a(k) = a(j) = 0$.

When designing a sparse FIR filter, it is unnecessary to solve the problem P. Following the depth-first branch-and-bound search [12], the design process is stated as follows: First, the subproblem P_0 is solved. Since $|a(0)|$ tends to be larger than $|a(k)|, k = 1, 2, \cdots, N-1$, P_0 has high probability of being fathomed. If P_0 is feasible, its solution is saved. Then we go down to the lower level and solve P_{01}. This procedure continues until a subproblem under consideration is fathomed. After fathoming a subproblem, we reinitiate the depth-first search process from an unsolved subproblem. This process continues until the feasibility of all the subproblems are checked. The branch-and-bound algorithm implicitly enumerate all subproblems. The final solution is obtained by comparing the solutions of the feasible subproblems. The one exhibiting either the minimal complexity or the best performance is the desired solution.

III. Design Examples

The branch-and-bound algorithm described above can be applied to the design of sparse FIR filters under various error criteria. In this section, we illustrate our method by designing filters having peak-constrained least-squares stopbands [13].

The integral squared error E_s at stopbands is given by

$$E_s = \int_{stopbands} | H(w) |^2 \, dw$$

$$= \boldsymbol{a}^t \boldsymbol{Q} \boldsymbol{a} \quad (2)$$

where $\boldsymbol{a} = [a(0) \; a(1) \; \cdots \; a(N-1)]^t$, $\boldsymbol{Q}$ is an $N \times N$ matrix whose i-th row, j-th column element q_{ij} is given by $q_{ij} = \int_{stopbands} cos\,(iw)\, cos\,(jw)\, dw$, $0 \leq i, j \leq N-1$. In the first example presented below, we design a sparse filter with minimum E_s under the following ripple constraints.

$$\begin{cases} | H(w) - 1 | \leq \delta_p & when\ w \in passband \\ | H(w) | \leq \delta_s & when\ w \in stopband \end{cases} \quad (3)$$

In the second example, sparse filters with minimum complexity are designed under ripple and stopband energy constraints. The designed sparse filters are compared with equiripple passband and peak-constrained least-squares stopband (EPPCLSS) filters in [13]. The filter design problems were solved by using quadratic programming.

Example 1 (Lowpass Filter Design, Minimum E_s): The desired specifications in normalized frequency are as follows:

$$passband : F \in [0,\ 0.15],\ stopband : F \in [0.2,\ 0.5]$$
$$\delta_p = 0.072\,\mathrm{dB},\ \delta_s = -46.05\,\mathrm{dB}$$

The Parks-McClellan algorithm determined that the minimum impulse response length required to meet the specifications was 47 ($N_c = 24$). We designed the EPPCLSS filter of length 47, which minimizes E_s subject to the ripple constraints, and a sparse FIR filter with $N = 29$. The results are summarized in Table 1. The designed sparse filter achieved 5.04 dB gain at the expense of 8 delays, as compared with the EPPCLSS filter. The magnitude responses shown in Fig. 2 demonstrate the advantage of the sparse filter. The number of quadratic problems solved for designing the sparse filter was 65. For comparison, we designed the same filter by the subset selection method. In this case, 85 quadratic problems should have been solved. The branch-and-bound algorithm required less computation than the subset selection method.

Example 2 (Lowpass Filter Design, Minimal Complexity): The desired specifications are as follows:

$$passband : F \in [0,\ 0.165],\ stopband : F \in [0.24,\ 0.5]$$
$$\delta_p = 0.043\,\mathrm{dB},\ \delta_s = -53.17\,\mathrm{dB}$$
$$stopband\ energy\ E_s < -56.51\,\mathrm{dB}$$

The minimum length of the EPPCLSS filter satisfying the given specifications was 37 ($N_c = 19$). We designed a sparse FIR filter with $N = 22$. Table 2 compares the EPPCLSS and the sparse filter. By sparse filtering, we can reduce 3 multiplications and 6 additions at the expense of 2 delays. The magnitude responses are shown in Fig. 3. The number of solved quadratic problems in our branch-and-bound method was 44; the subset selection method required to solve 69 problems.

IV. Conclusions

The branch-and-bound algorithm was applied to the design of sparse FIR filters. This algorithm can efficiently find optimal zeroed tap positions, and enables us to design optimal sparse filters under various error criterions. Through some design examples, we showed that the branch-and-bound algorithm requires less computation than the subset selection method. Further work in this direction will be concentrated on application of the branch-and-bound algorithm to the design of unequally spaced antenna arrays [2].

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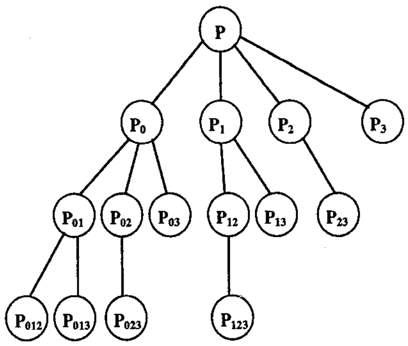

Figure 1: A branch-and-bound tree for $N = 4$ used for the arithmetic complexity minimization.

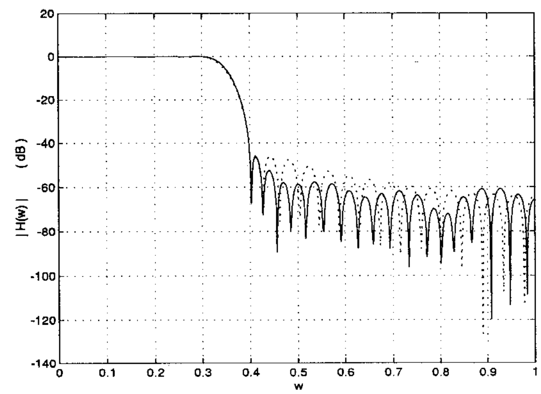

Figure 2: Magnitude responses of the sparse filter (solid line) and the EPPCLSS filter (dotted line) in Example 1.

Table 1: Comparision between the filters in Example 1.

	sparse	EPPCLSS
E_s (dB)	-57.24	-52.76
multiplication	24	24
addition	47	47
delay	55	47

Table 2: Arithmetic complexity comparision between the filters in Example 2.

	sparse	EPPCLSS
multiplication	16	19
addition	31	37
delay	39	37

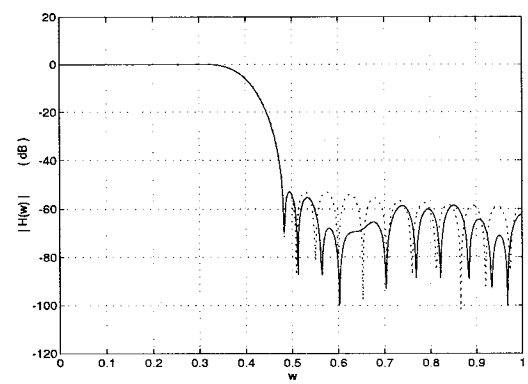

Figure 3: Magnitude responses of the sparse filter (solid line) and the EPPCLSS filter (dotted line) in Example 2.