Differential Detection Based Timing Recovery for Continuous Phase Modulation

Inhyoung Kim, Yong Hoon Lee

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu, Taejon, 305-701, Korea
PH : +82-42-866-0800 FAX: +82-42-866-0804
E-mail: yohlee@eekaist.kaist.ac.kr

Abstract - **A data-aided (DA) timing recovery method for general continuous phase modulation (CPM) signaling is introduced. This method requires only symbol rate sampling and is independent of the carrier phase offset, owing to the use a differential operator. By examining the phase of the data after the differential operation, we derive an expression of the phase error caused by the timing offset and obtain an efficient timing error function. Then a feedback timing recovery algorithm is developed by exploiting this timing error function. The characteristics of the algorithm are examined through computer simulation.**

I. Introduction

It has been recognized that the timing recovery of CPM signals with partial response formats and multilevel alphabets is a difficult task [1]. The non-data-aided (NDA) timing recovery methods in [2]-[5] perform well for minimum shift keying (MSK) and Gaussian-MSK (GMSK) signaling, which have full-response type format, but tend to exhibit poor performance with partial response formats, especially with long frequency pulses. The DA method in [6] has excellent tracking performance but its acquisition is prone to spurious locks which occur, in particular, with multilevel partial response formats. Our objective in this paper is to develop a new timing algorithm that works well for general CPM signaling.

The proposed method starts with symbol rate sampling of the received signal. To remove the effect of the carrier phase offset, the sampled data is passed through a differential operator. Then the phase of the resulting signal is decomposed into two terms: a reference phase, which is the phase of the transmitted signal and a phase error caused by the timing offset. We derive an efficient timing error function from the phase error term, and develop a feedback timing recovery algorithm by exploiting the timing error function. This algorithm is simple to implement. It is shown through computer simulation that the algorithm performs well for general CPM signals encompassing both full and partial response formats.

II. The Proposed Timing Recovery Scheme

The baseband equivalent form of a received CPM signal passed through a distortionless channel may be expressed as

$$s(t,\boldsymbol{\alpha}_n) = \exp(j\phi(t,\boldsymbol{\alpha}_n) + j\phi_0) \tag{1}$$

where $nT \le t < (N+1)T$; T is the symbol period; $\boldsymbol{\alpha}_n=[\alpha_0, \alpha_1, \ldots, \alpha_n]$ is an M-ary information sequence; ϕ_0 is the carrier phase offset;

$$\phi(t,\boldsymbol{\alpha}_n) = 2\pi h \sum_{i=0}^{n} \alpha_i q(t-iT) \tag{2}$$

and $q(t)$ is a continuous, non-decreasing function of t satisfying

$$q(t) = \begin{cases} 0, & t<0 \\ 1/2, & t \ge LT. \end{cases} \tag{3}$$

The constant h in (2) and L in (3) are the modulation index and the correlative length, respectively. Suppose that $s(t,\boldsymbol{\alpha}_n)$ is sampled at nT-τ where τ, $-T/2 \le \tau \le T/2$, denotes the symbol timing offset. The phase of the sampled signal is given by

$$\phi(nT-\tau,\boldsymbol{\alpha}_n)+\phi_0 = 2\pi h\sum_{i=0}^{n}\alpha_i q(nT-iT-\tau)+\phi_0 . \quad (4)$$

As a first step for deriving a proper timing error detector, we decompose (4) as follows :

$$\phi(nT-\tau,\boldsymbol{\alpha}_n)+\phi_0 = 2\pi h\sum_{i=0}^{n}\alpha_i q(nT-iT) + 2\pi h\sum_{i=0}^{n}\alpha_i\{q(nT-iT-\tau)-q(nT-iT)\}+\phi_0 \quad (5)$$

$$= \phi(nT,\boldsymbol{\alpha}_n)+2\pi h\sum_{i=n-L}^{n}\alpha_i q_{n-i}(\tau)+\phi_0 \quad (6)$$

where

$$q_j(\tau) = q(jT-\tau)-q(jT),\ j=0,1,\ldots,L, \quad (7)$$

and (6) follows from the fact that $q(nT\text{-}iT\text{-}\tau) = q(nT\text{-}iT) = 1/2$ for $i \le n-L-1$. The function $q_j(\tau)$ in (7) is non-increasing, as shown in Fig.1, because $q(t)$ in (3) is non-decreasing. In (6), the first term is independent of timing offset τ and can be obtained once $\boldsymbol{\alpha}_n$ is given. The timing error information is contained only in the second term, which is a weighted sum of $q_j(\tau)$ functions. Since $q_j(\tau)$ is non-increasing, it would be possible to estimate τ from this term. Use of (6) for timing recovery, however, is not practical because of the unknown phase offset ϕ_0. To remove the effect of ϕ_0, we multiply $s(nT\text{-}\tau,\boldsymbol{\alpha}_n)$ with $s^*(nT\text{-}T\text{-}\tau,\boldsymbol{\alpha}_n)$. The phase of the resulting signal may be expressed as

$$\phi(nT-\tau,\boldsymbol{\alpha}_n)-\phi(nT-T-\tau,\boldsymbol{\alpha}_n) = \phi_{ref}(n,\boldsymbol{\alpha}_{n,L+1})+\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) \quad (8)$$

where

$$\phi_{ref}(n,\boldsymbol{\alpha}_{n,L+1}) = \phi(nT,\boldsymbol{\alpha}_n)-\phi(nT-T,\boldsymbol{\alpha}_n) = 2\pi h\sum_{j=0}^{L}\alpha_{n-j-1}q_j(T), \quad (9)$$

$$\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) = 2\pi h\sum_{j=0}^{L}(\alpha_{n-j}-\alpha_{n-j-1})q_j(\tau), \quad (10)$$

and $\boldsymbol{\alpha}_{n,L+1} = [\alpha_n, \alpha_{n-1}, \ldots, \alpha_{n-L-1}]$. In (8), the phase error $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ contains the timing information and can be obtained by subtracting $\phi_{ref}(n,\boldsymbol{\alpha}_{n,L+1})$, called the reference phase, from the phase of $s(nT\text{-}\tau,\boldsymbol{\alpha}_n)\cdot s^*(nT\text{-}T\text{-}\tau,\boldsymbol{\alpha}_{n-1})$. To extract the timing information from $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$, we now examine the relation between $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ and τ.

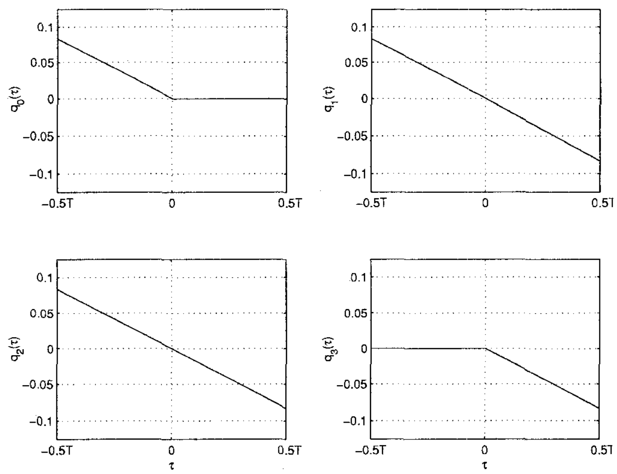


Fig. 1. The function $q_j(\tau)$ when the frequency pulse $dq(t)/dt$ is a rectangular pulse of length 3 (3*REC*, L=3).

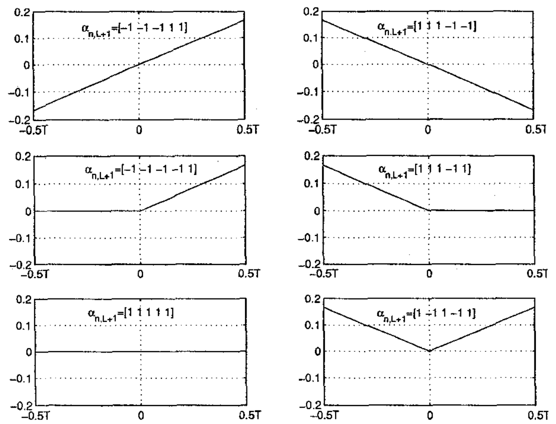
Observation 1: When $\tau = 0$, $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) = 0$ regardless of the data $\boldsymbol{\alpha}_{n,L+1}$.

This observation is true because $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ is a weighted sum of $q_j(\tau)$ and $q_j(0) = 0$. Fig.2 illustrates $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ for some $\boldsymbol{\alpha}_{n,L+1}$ when frequency pulse $dq(t)/dt$ is a rectangular pulse of length 3 (3REC, L=3) and M∈{2,4}. The phase error $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ for a given $\boldsymbol{\alpha}_{n,L+1}$ is not necessarily a monotonic function of τ, but there always exist monotonic $\{\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})\}$. Through exhaustive search of M^{L+2} elements of the set $\{\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ for all $\boldsymbol{\alpha}_{n,L+1}\}$, we counted the number of monotonic $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ functions. The results are shown in Table 1. It is seen that more than 50% of the $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ functions are monotonic.

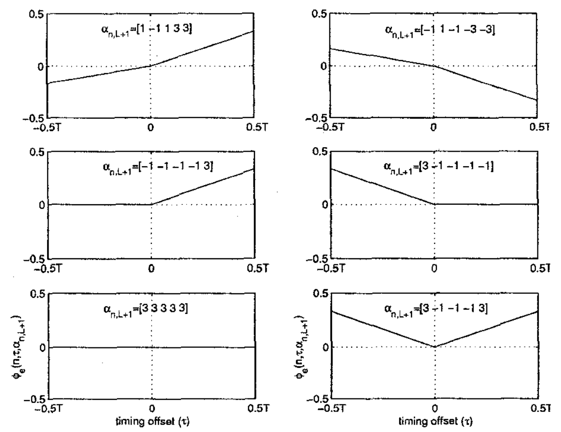
Next we derive a timing error function by using $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$. Consider

$$f(\tau,\boldsymbol{\alpha}_{n,L+1}) = c_\phi(\boldsymbol{\alpha}_{n,L+1})\cdot\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) \quad (17)$$

where



(a) M=2



(b) M=4

Fig. 2. Some representative shapes of $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ for 3REC (L=3, rectangular pulse).

Phase respons	M = 2			M = 4		
	L = 1	L = 2	L = 3	L = 1	L = 2	L = 3
RC	6 (8)	14 (16)	22 (32)	36 (64)	228 (256)	724 (1024)
REC	6 (8)	14 (16)	28 (32)	36 (64)	184 (256)	736 (1024)

Table 1. The number of monotonic $\phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1})$ functions where the numbers in the parentheses are the total number M^{L+2} and RC stands for the raised cosine frequency pulse of duration L.

$$c_\phi(\boldsymbol{\alpha}_{n,L+1}) = \begin{cases} 1, & \text{if } \phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) \text{ given } \boldsymbol{\alpha}_{n,L+1} \text{ is non - decreasing} \\ 0, & \text{if } \phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) \text{ given } \boldsymbol{\alpha}_{n,L+1} \text{ is not monotonic} \\ -1, & \text{if } \phi_e(n,\tau,\boldsymbol{\alpha}_{n,L+1}) \text{ given } \boldsymbol{\alpha}_{n,L+1} \text{ is non - increasing} \end{cases} \tag{18}$$

Owing to Observation 1 and its definition in (17) and (18), the function $f(\tau,\boldsymbol{\alpha}_{n,L+1})$ has the following desirable characteristics:

$$\begin{cases} f(\tau,\boldsymbol{\alpha}_{n,L+1}) \geq 0, & \text{if } \tau > 0 \\ f(\tau,\boldsymbol{\alpha}_{n,L+1}) = 0, & \text{if } \tau = 0 \\ f(\tau,\boldsymbol{\alpha}_{n,L+1}) \leq 0, & \text{if } \tau < 0 \end{cases} \tag{19}$$

(19) is satisfied irrespective of the data $\boldsymbol{\alpha}_{n,L+1}$ and thus this function can serve as a timing error function. Using (17), we propose a recursive formula for estimating the timing error τ. The proposed formula is written as

$$\begin{aligned} \hat{\tau}_{n+1} &= \hat{\tau}_n - \mu \cdot f(\hat{\tau}_n,\boldsymbol{\alpha}_{n,L+1}) \\ &= \hat{\tau}_n - \mu \cdot c_\phi(\boldsymbol{\alpha}_{n,L+1}) \cdot \phi_e(n,\hat{\tau}_n,\boldsymbol{\alpha}_{n,L+1}) \end{aligned} \tag{20}$$

where $\hat{\tau}_n$ is the n-th estimate of τ and μ is a constant. It is expected that $\hat{\tau}_n$ converges to τ if μ is properly chosen. In practical situations, the symbol vector $\boldsymbol{\alpha}_{n,L+1}$ is unknown and should be detected. When the maximum likelihood sequence estimation (MLSE) receiver is employed for CPM demodulation, decisions are made with some delay D. In a such case, (20) is rewritten as

$$\hat{\tau}_{n+1} = \hat{\tau}_n - \mu \cdot c_\phi(\boldsymbol{\alpha}_{n-D,L+1}) \cdot \phi_e(n-D,\hat{\tau}_n,\boldsymbol{\alpha}_{n-D,L+1}) \tag{21}$$

where $\boldsymbol{\alpha}_{n-D,L+1}$ is the detected symbol vector. This timing offset estimator is illustrated in Fig. 3. Here it is assumed that the set of constants $\{c_\phi(\boldsymbol{\alpha}_{n-D,L+1})\}$ are pre-stored. This timing recovery scheme is simple to implement, yet performs reasonably well for general CPM signals, as demonstrated in the next section through computer simulation.

III. Simulation Results

To examine the behavior of the proposed schemes, timing error variances were estimated empirically for binary

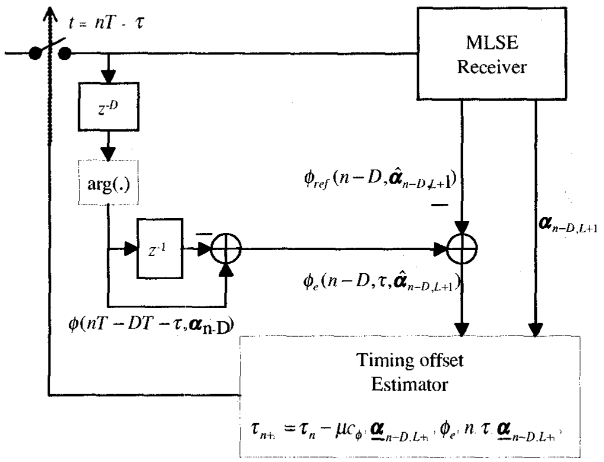


Fig. 3. The block diagram of the proposed timing recovery scheme where z^{-i} represents the delay by I symbol periods

CPM signals (M=2) with various phase responses transmitted over an AWGN channel. In our simulation, each error variance estimate was calculated through ensemble averaging preformed over 100 independent trials. (At each trial, we generated a sequence of length 500.) Fig. 4 illustrates the transient behavior of the proposed algorithm. From this figure, we observe the following:

- As μ is reduced, the rate of convergence of the algorithm is correspondingly decreased.
- A reduction of μ also has the effect of reducing the steady state variance.
- As L is increased, the rate of convergence is decreased.
- An increase in L also has the effect of increasing the steady state variance.

Fig. 5 compares the steady state behavior of the algorithm for various CPM formats. Each steady state error variance in Fig. 5 was estimated by averaging the last 200 values of the corresponding variance estimate sequence of length 500 (Fig. 4 shows such sequences). It is seen that the rectangular pulses cause larger error variances than the raised cosine pulses. As observed above the error variance increase as L is increased.

It is expected that the proposed timing recovery scheme performs well for multilevel CPM signals. Simulation with multilevel signaling remains as a future work.

Reference

[1] U.Mengali, and A.N.D'Andrea, *Synchronization Techniques for Digital Receivers.* New York: Plenum Press, 1997 .

[2] R.de Buda, "Coherent Demodulation of Frequency Shift Keying with Low Deviation Ratio," *IEEE Trans. Commun.*, vol. COM-20, pp.429-235, June 1972.

[3] A.N.D'Andrea, U.Mengali, and R.Reggiannini, "A Digital Approach to Clock Recovery in Generalized Minimum Shift Keying," *IEEE Trans. Commun.*, vol.39, no.3, pp.227-234. August 1990.

[4] R.Mehlan, Y.-E. Chen, and H.Meyr, "A Fully Digital Feedforward MSK Demodulator with Joint Frequency Offset and Symbol Timing Estimation for Burst Mode Mobile Radio," *IEEE Trans. Veh. Technol.*, vol.42, no.4, pp.434-443, November 1993.

[5] A.N.D'Andrea, and U.Mengali, "Symbol Timing Estimation with CPM Modulation," *IEEE Trans. Commun.*, vol.44, no.10, pp.1362-1372, October 1996.

[6] J.Huber, and W.Liu, "Data-Aided Synchronization of Coherent CPM-Receivers," *IEEE Trans. Commun.*, vol.40, no.1, pp.178-189, January 1992.

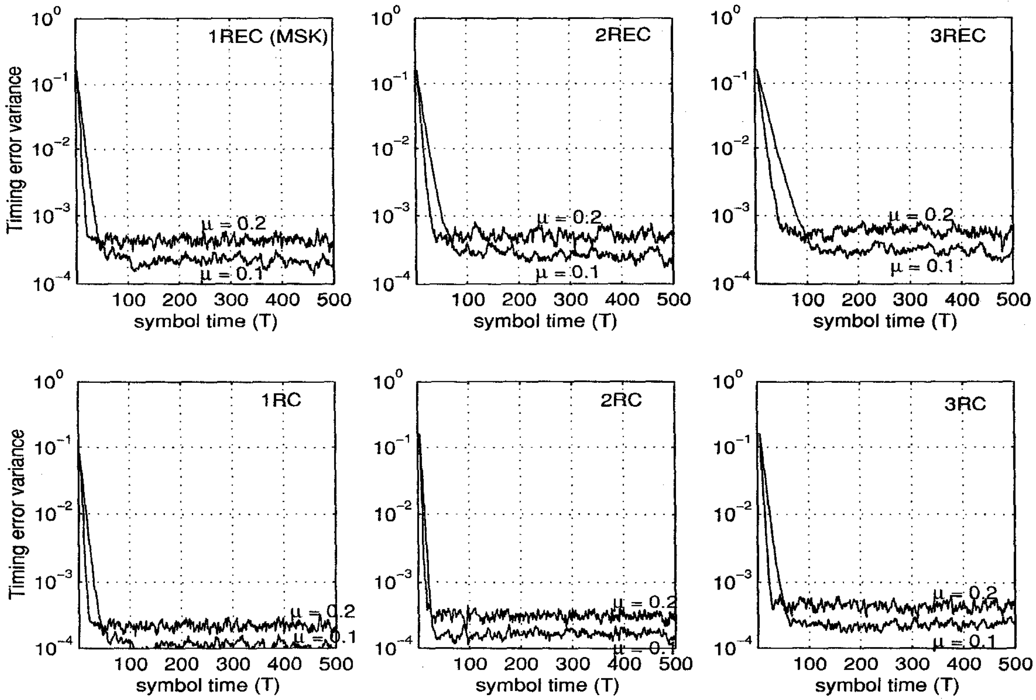


Fig. 4. Timing error variance estimates when E_b/N_o = 20dB.

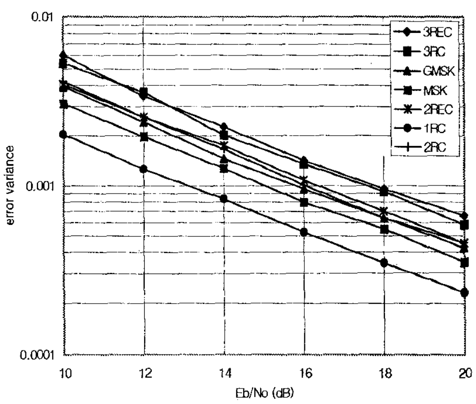


Fig. 5. Steady state error variance for various CPM formats when $\mu = 0.2$.