A Design and Implementation of Programmable Multiplierless FIR Filters with Powers-of-Two Coefficients

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***Abstract*– An observation which is useful for hardware implementation of FIR filters with powers-of-two coefficients (2PFIR filters) is made. Specifically, it is shown that the exponents of filter coefficients representable by the canonical signed digit(CSD) code with M ternary digits can be chosen from subsets of $\{0, 1, \cdots, M-1\}$. This observation naturally leads to 2PFIR filters with shorter shifters whose length is strictly less than M and, as a consequence, leads to an efficient hardware structure for programmable 2PFIR filtering. In addition, we present some experimental results indicating that the shifters of 2PFIR filters can be shortened further with little degradation of their performance.**

I. Introduction

Due to their simplicity in implementation, FIR filters with powers-of-two coefficients, which are often referred to as 2PFIR filters, have received considerable attention in digital signal processing [1]-[9]. By employing only those coefficients that are sums and differences of signed powers-of-two, each multiplication in 2PFIR filtering can be replaced with simple shift-and-add operations.

Implementation of a 2PFIR filter is particularly efficient when its coefficients are fixed for dedicated applications, since hard-wired shifters can be employed [5]-[7]. On the other hand, realization of a programmable 2PFIR filter [8] is considerably more difficult than that of a fixed filter, because it requires either barrel shifters or shift registers which greatly increase the hardware complexity or slacken the processing speed.

In this paper, we observe that the exponents of 2PFIR filter coefficients representable by the *canonical signed digit* (CSD) code with M ternary digits [4]-[7], [10] can be chosen from subsets of $\{0, 1, \cdots, M-1\}$. This observation naturally leads to 2PFIR filters having shifters of shorter length, and to an efficient hardware structure for programmable 2PFIR filtering.

The organization of this paper is as follows. In section II, we derive a property of the CSD code, and show that the shifter of programmable 2PFIR filter can be shortened by using this property. In section III, we present some experimental results indicating that the shifters of 2PFIR filters can be shortened further.

II. An Efficient Implementation of 2PFIR Filters

The impulse response $h(m)$ of a 2PFIR filter is represented as sums and differences of powers-of-two: specifically, $h(m)$ is given by

$$h(m) = \sum_{k=1}^{L} s_k 2^{-p_k} \tag{1}$$

where $h(m)$ is a fractional number, $s_k \in \{-1, 0, 1\}$, $p_k \in \{0, 1, \cdots, M-1\}$, M is the number of ternary digits and L is the number of nonzero digits. This representation is known as the *radix-2 signed digit* code [10], [12]. In general, there are several signed-digit representations for a given $h(m)$.

A signed-digit code that leads to a unique representation of a number is the CSD code, which is specified as follows: for CSD codes, the number of nonzero digits L should be the minimum and in addition, no two nonzero digits are adjacent, i.e. in (1)

$$\left| p_i - p_j \right| \geq 2 \tag{2}$$

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for any i and j, $i \neq j$. For example, the CSD representation of 0.375 is $2^{-1}-2^{-3}$, and neither $2^{-1}-2^{-2}+2^{-3}$ nor $2^{-2}+2^{-3}$ are CSD codes.

Multiplication of $h(m)$ in (1) with an input value requires L shifters, as illustrated in Fig. 1: Fig. 1(a) and (b), respectively, show an overall structure for realizing a 2PFIR filter and details of a tap. Note that in Fig. 1(b) each product $x(n-m)s_k 2^{-p_k}$ is obtained by using shifters. Of course, in programmable 2PFIR filtering, either barrel shifters or shift registers should be employed. Since each p_k in (1) is conventionally selected from the set $\{0, 1, \cdots, M-1\}$, the shifters should have $N \times M$ bits where N is the wordlength of the input. Next we shall show that the length of the shifters can be shortened by exploiting a property of the CSD codes.

We consider the set of all numbers that can be generated from (1) with fixed M and L. Such a set, which will be denoted by $S_{M,L}$, is the set of all numbers representable by the CSD code with M and L. For example, $S_{2,1}=\{-1, -0.5, 0, 0.5, 1\}$. $S_{M,L}$ is generated from (1) with p_k's which are selected from $\{0, 1, \cdots, M-1\}$. In the following we shall show that $S_{M,L}$ can also be generated with p_k's chosen from some proper subsets of $\{0, 1, \cdots, M-1\}$.

Property: Let $Z_k \subset \{0, 1, \cdots, M-1\}$, $1 \leq k \leq L$, be a set of successive integers given by

$$Z_k=\{2(k-1), 2(k-1)+1, \cdots, (M-1)-2(L-k)\} \quad (3)$$

Then $S_{M,L}$ can be generated from (1) with $p_k \in Z_k$

Proof: Consider $Z_1=\{0, 1, \cdots, M-1\}$. Since $|p_i - p_j| \geq 2$ and $0 \in Z_1$, then $p_k \geq 2$ for all $k \geq 2$. Thus it is not necessary to include 0 and 1 in all Z_k's, $k \geq 2$. Note that z_2 should include 2 because $0 \in Z_1$, and so we can eliminate 3 and 4 from all Z_k's, $k \geq 3$. In this manner, it can be shown that $\{0, 1, \cdots, 2(k-1)-1\}$ can be excluded from Z_k. Finally, consider $Z_L=\{2(L-1)+1, \cdots, M-1\}$. Due to (2) and $(M-1) \in Z_L$, $(M-1)$ and $(M-2)$ are excluded from all Z_k's, $k \leq L-1$. Similarly, we can show that $\{(M-1)-2(L-k)+1, \cdots, M-1\}$ are not necessarily be elements of Z_k. ∎

The number of elements in Z_k is $M-2L+2$. This property indicates that $N \times (M-2L+2)$ bit shifters can be used in place of $N \times M$ bit shifters, and that the length of each shifter is shortened by $2(L-1)$. Fig. 2 illustrates the structure of a tap employing $N \times (M-2L+2)$ bit shifters. In this structure, the multiplication of $x(n-m)$ with $s_k 2^{-p_k}$ is carried out by using a $2(k-1)$ bit hard-wired shifter followed by a $N \times (M-2L+2)$ bit shifter. Use of shorter shifters will increase the processing speed when shift registers are employed, and decrease the hardware complexity of barrel shifters. For example, when $(M,L)=(12,3)$ the length of shifters is reduced by $2(L-1)=4$, and thus the processing speed for shift registers is increased by $2(L-1)/M=1/3$ (or 33%) and the hardware complexity of barrel shifters are reduced by the same amount.

It should be pointed out that the adders in Fig. 2 can be simpler than those in Fig. 1(b). In contrast with conventional $N+M$ bit full adders in Fig. 1(b), each addition in Fig. 2 requires $N+(M-2L+2)$ bit full adders. This is true because the product $x(n-m)s_k 2^{-p_k}$ is essentially an $N+(M-2L+2)$ digit ternary number, as illustrated in Fig. 3.

III. Further Shortening of Shifters

In this section, we shall show through some experiments that the shifters of 2PFIR filters can be shortened further in many cases with little degradation of their performance.

The length of shifters can be reduced by selecting the values of each p_k from a subset of Z_k in (3). If this happens, the set of all possible numbers generated from (1) with $p_k \in Z'_k$ is a subset of $S_{M,N}$, where Z_k' is a subset of Z_k. We denote such a set by $S'_{M,N}$. Now the following question arises: can as find Z_k' which will lead to a 2PFIR filter whose performance is close to that of an original 2PFIR filter associated with Z_k ? The answer to this question is affirmative, especially when $L \geq 3$. Consider the following example.

Example 1: Let $(M,N)=(12,3)$. The Z_k's in (3) are given by

$$Z_1=\{0, 1, \cdots, 7\}, Z_2=\{2, 3, \cdots, 9\}, \\ Z_3=\{4, 5, \cdots, 11\}, \quad (4)$$

and each shifter has $N \times 8$ bits. To shorten the shifters to $N \times 5$ p_k's may be selected from the sets

$$Z_1'=\{0, 1, \cdots, 4\}, Z_2'=\{3, 4, \cdots, 7\}, \\ Z_3'=\{7, 8, \cdots, 11\}. \quad (5)$$

Now we design 25-tap lowpass 2PFIR filters with coefficients from $S_{12,3}$ and $S'_{12,3}$, which are shown in Fig. 4. The normalized passband and stopband frequencies of the filter to be designed is 0.15 and 0.25, respectively, and the ripple weighting factor is 1. (This filter has been considered in [1], [3] and [4].) The local search method proposed in [3] is used for designing the filter. The magnitude responses of the designed filters are shown in Fig. 5. It is seen that the performance of the 2PFIR filter associated with $S'_{12,3}$ is very close to that of the filter with $S_{12,3}$. The stop band attenuation of the former and the latter is 45.1dB and 45.7dB, respectively.▮

In the following example, it is seen that shortening of shifters causes some degradation of filter performance when $(M,L)=(8,2)$.

Example 2: Let $(M,N)=(8,2)$. The Z_k's in (3) are

$$Z_1=\{0,1,\cdots,5\}, Z_2=\{2,4,\cdots,7\}. \quad (6)$$

Consider the sets

$$Z'_1=\{0,1,\cdots,4\}, Z'_2=\{3,4,\cdots,7\} \quad (7)$$

and design again the 25-tap lowpass 2PFIR filters of Example 1. This time the filter coefficients are from $S_{8,3}$ and $S'_{8,3}$. The filters with $S_{8,3}$ and $S'_{8,3}$ have the stopband attenuation of 38.1dB and 35.4dB, respectively; the difference is about 2.7dB. ▮

In summary, the shifters of 2PFIR filters can be shortened further by using proper $Z'_k \in Z_k$. The degradation caused by the shortening is generally minor, and often negligible when $L\geq 3$. The set Z'_k is obtained by discarding successive integers from Z_k. As a rule of thumb, largest integers (least significant bits) are discarded when $k\leq L-1$, and smallest integers (most significant bits) are excluded when $k=L$.

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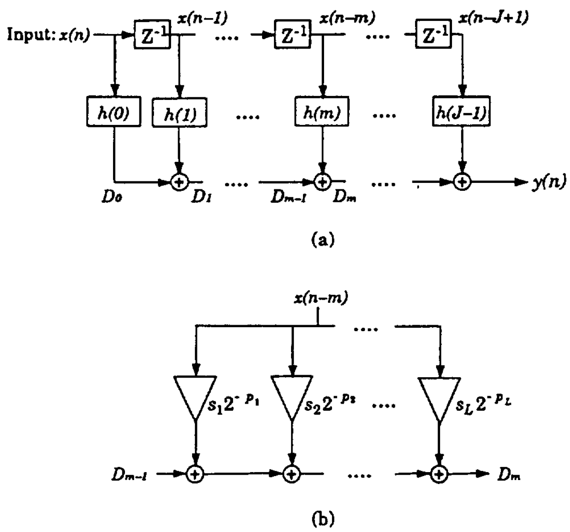

Fig. 1. (a) A structure for realizing 2PFIR filters with J taps
(b) Details of a tab evaluating $D_{m-1}+h(m)x(n-m)$
where $D_{m-1}=\sum_{j=0}^{m-1}h(j)x(n-j)$.

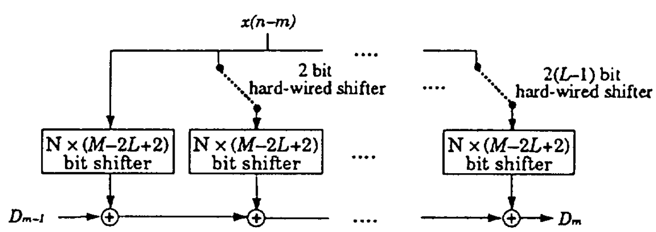

Fig. 2. Evaluating $D_{m-1}+h(m)x(n-m)$ using N×(M−2L+2) bit shifters and adders where N is the wordlength of the input.

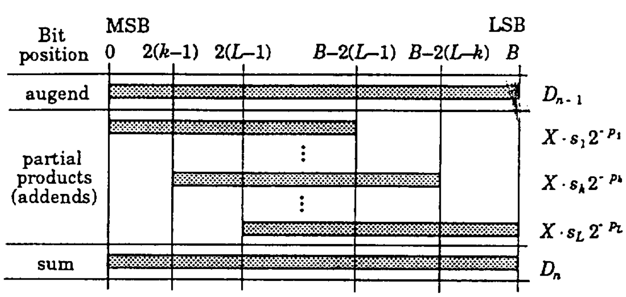

Fig. 3. Addition pattern of 2PFIR filters. Shaded areas represent possible bit positions of augend, addends and the resulting sum.
Here B : N+M

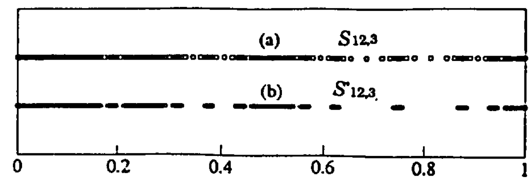

Fig. 4. Distribution of $S_{12,3}$ and $S'_{12,3}$

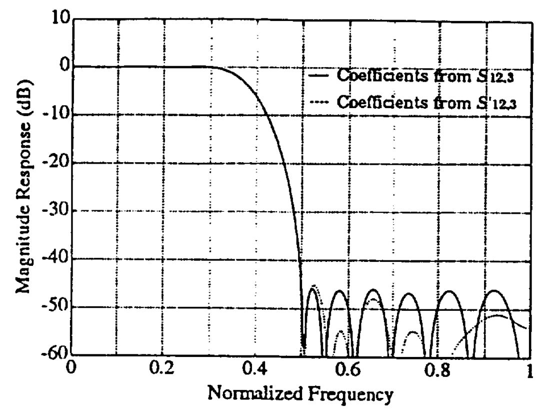

Fig. 5. Magnitude responses for 2PFIR filters of Example 1.