On the Use of Double Correlation for Frame Synchronization in the Presence of Frequency Offset

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Abstract - A new frame synchronization technique, which is robust to carrier frequency and phase errors, is proposed for M-ary PSK systems. This technique is derived through some modification of the procedure for obtaining the maximum likelyhood(ML) rule in [12]. The proposed rule is based on an operation called the *double* correlation, which evaluates the correlation after properly multiplying the received signal with a sync pattern. It will be shown through computer simulation that the proposed rule outperforms the existing ML rule when the frequency offset is greater than about 0.02/T.

I. Introduction

Almost all digital communication systems require reliable frame synchronization, realizing the start of a data frame at a receiver. Frame synchronization is achieved with the aid of a sync pattern which is either injected periodically into a data stream (continuous transmission) or appended at the beginning of each packet (packet transmission). At the receiver, after recovering timing information, sampled input values[1] are typically correlated with the sync pattern and frame synchronization is accomplished by examing the correlation values[1]-[3]. This type of synchronization method, which are generally referred to as the *correlation rules*, have been popular mainly because of their simplicity in implementation. Frame synchronization may also been done using more optimal rules such as the maximum likelyhood(ML) rules in [8]-[12] and their simplifications. These rules outperform the correlation rules at the expense of additional computation.

Frame synchronization is usually performed before carrier recovery is completed. In particular, popular data-aided methods for estimating carrier frequency and phase [13],[14] require perfect frame sync, and use of these methods needs frame synchronizers which are tolerent of frequency and phase errors. Although robustness to a carrier offset is an important characteristic of frame sync rules, only a few existing rules have such a property. The ML rule in [12] is derived under the assumption that frequency and phase errors are uniformly distributed; and it is tolerent of both frequency and phase offsets. An ad hoc rule in [14], which evaluates the correlation between a differentially encoded input signal and a differentially encoded sync pattern, also has such tolerence. This rule is simpler to implement, but performs worse than the ML rule in [12].

In this paper, we make an attempt to improve the performance of the ML rule in [12], especially for a large frequency offset. Through some modification of the procedure for deriving the ML rule, we shall obtain a new rule that can outperform the existing one when the frequency offset is greater than about $0.02/T$, where T is the symbol period. The proposed rule is based on an operation called the *double* correlation which is an extention of the correlation between differentially encoded input and sync signals in [14].

The organization of this paper is as follows. In section II, we introduce the signal model and briefly review the derivation of the ML rules in [11],[12]. The proposed rule is derived in section III. Simulation results demonstrating the adventage of the proposed rule over the existing rules will be presented in section IV.

[1] Frame synchronization based on the correlation between hard bit decision of input values and sync pattern has been considered in [4]-[7].

II. Signal Model

We consider an M-ary PSK signal which is continuously transmitted over additive white Gaussian noise(AWGN) channel. The frame structure is shown in Fig. 1. Each frame consists of N M-ary symbols: the first L symbols form a fixed frame synchronization pattern denoted as $\{s_k | k = 0, 1, ..., L-1\}$ and the remaining $N-L$ symbols are random data. We shall assume that each data symbol is chosen equally likely from the M-ary signal constellation $\{e^{j(2\pi m/M)} | 0 \leq m < M\}$. The received baseband signal is written as

$$r_n = e^{j(\theta_n + n\omega_o T + \phi_o)} + G_n \tag{1}$$

where $e^{j\theta_n}$ is the M-ary phase-modulated symbol; T is the symbol period; $\omega_o = 2\pi f_o$; f_o and ϕ_o are the frequency and phase offsets, respectively; G_n is complex white Gaussian noise with variance $\sigma_G^2 = N_o/E_s$; E_s denotes symbol energy; and n is the time index. In [11], the received signal r_n is modeled as $r_n = e^{j(\theta_n + \phi_n)} + G_n$, where ϕ_n is i.i.d. and uniformly distributed over $[-\pi, \pi]$. This model leads to a frame synchronization rule which is vulnerable to a frequency offset. In [12] the signal model is given by (1) with the following assumptions: ϕ_o is uniformly distributed over $[-\pi, \pi]$, and the normalized frequency offset $f_o T$ is uniformly distributed over $[-f_m, f_m]$ where f_m is a known constant. In this work, we also start with (1) but assume that both ϕ_o and $\omega_o T(=2\pi f_o T)$ are uniformly distributed over $[-\pi, \pi]$. This assumption will lead to a rule that is simpler to implement and more robust to a frequency offset than the ML rule in [12].

III. Derivation of the Proposed Frame Synchronization

The frame synchronization problem is to estimate the frame boundary position in an arbitrarily selected segment of the AWGN channel output

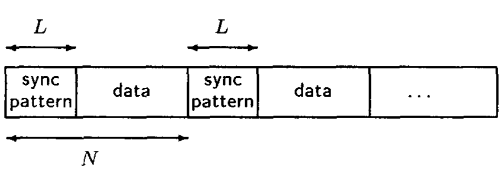

Fig.1. Frame Structure.

observations corresponding to N transmitted symbols. If the sync pattern starts at the μ-th position, $\mu \in [0, N-1]$, of the N observations, then the ML estimate $\hat{\mu}$ is the integer that maximizes the conditional probability density $f(\vec{\mathbf{r}}|\mu)$ of the received signal $\vec{\mathbf{r}} = (r_0, r_1, ... r_{N-1})$. To derive $f(\vec{\mathbf{r}}|\mu)$, we first consider

$$f(\vec{\mathbf{r}}|\mu, \vec{\mathbf{d}}, \omega_o T, \phi_o) = \prod_{k=0}^{N-1} \frac{E_s}{\pi N_o} e^{-|r_k - \exp\{j(\theta_k + k\omega_o T + \phi_o)\}|^2 E_s/N_o} \tag{2}$$

where $\vec{\mathbf{d}} = (d_0, d_1, ..., d_{N-L-1})$ is the random data. Taking the expectation of (2) with respect to ϕ_o yields

$$\begin{aligned} f(\vec{\mathbf{r}}|\mu, \vec{\mathbf{d}}, \omega_o T) &= \int_{-\pi}^{\pi} f(\vec{\mathbf{r}}|\mu, \vec{\mathbf{d}}, \omega_o T, \phi_o) \frac{1}{2\pi} d\phi_o \\ &= \left(\frac{E_s}{\pi N_o}\right)^N \prod_{i=0}^{N-1} e^{-(|r_i|^2+1)E_s/N_o} \\ &\cdot \; I_o\left(\frac{2E_s}{N_o}\left|\sum_{k=0}^{N-1} r_k^* e^{j\theta_k} e^{jk\omega_o T}\right|\right) \end{aligned} \tag{3}$$

where $I_o(x) = (1/2\pi)\int_{-\pi}^{\pi} e^{x\cos\theta} d\theta$ is the zero*th*-order modified Bessel function of first kind. If we take the expectation of (3) with respect to $\omega_o T$ and all possible data symbol, then after some calculation we get

$$\begin{aligned} f(\vec{\mathbf{r}}|\mu) = \left(\frac{E_s}{\pi N_o}\right)^N \prod_{i=0}^{N-1} e^{-(|r_i|^2+1)E_s/N_o} \sum_{all \vec{\mathbf{d}}} \frac{1}{M^{N-L}} \\ \cdot \int_{-\pi}^{\pi} I_o\left(\frac{2E_s}{N_o}\left|\sum_{k=0}^{N-1} r_k^* e^{j\theta_k} e^{jk\omega_o T}\right|\right) \frac{1}{2\pi} d(\omega_o T) \end{aligned} \tag{4}$$

Maximizing (4) with respect to μ requires extremely heavy computation. To obtain a test with much less complexity, we approximate[2] $I_0(x)$ by $(1 + x^2/4 + x^4/64)$ for small x. Then

$$\begin{aligned} f(\vec{\mathbf{r}}|\mu) \approx C(\vec{\mathbf{r}}) \sum_{all \vec{\mathbf{d}}} \frac{1}{M^{N-L}} \int_{-\pi}^{\pi} \{1 \\ + \left(\frac{E_s}{N_o}\right)^2 \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} r_k^* e^{j(\theta_k + k\omega_o T)} r_l e^{-j(\theta_l + l\omega_o T)} \\ + \left(\frac{E_s}{\sqrt{2} N_o}\right)^4 \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} r_k^* r_l r_m^* r_n \\ e^{j\{\theta_k - \theta_l + \theta_m - \theta_n + (k-l+m-n)\omega_o T\}} \} \frac{1}{2\pi} d(\omega_o T) \end{aligned} \tag{5}$$

[2]In [12], $I_0(x)$ is approximated by x^2.

where $C_1(\vec{\mathbf{r}}) = (E_s/\pi N_o)^N \prod_{i=0}^{N-1} e^{-(|r_i|^2+1)E_s/N_o}$ is a constant and independent of μ. Since $\int_{-\pi}^{\pi} e^{jk\theta}d\theta = 2\pi\delta(k)$, terms which satisfy $l+n-k-m=0$ are remained after the above integration. Let $i = n-k = m-l$, where $i \in [-N+1, N-1]$. Then

$$f(\vec{\mathbf{r}}|\mu) \approx C(\vec{\mathbf{r}}) \sum_{all\vec{\mathbf{d}}} \frac{1}{M^{N-L}} \int_{-\pi}^{\pi} \Bigg\{ 1 + \left(\frac{E_s}{N_o}\right)^2 \sum_{p=0}^{N-1} |r_p|^2 + \left(\frac{E_s}{\sqrt{2}N_o}\right)^4 \left(\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} |r_k|^2|r_l|^2 + 2\sum_{i=0}^{N-1}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} r_k r_l^* r_{k+i}^* r_{l+i} e^{j(\theta_{k+i}+\theta_l-\theta_k+\theta_{l+i})} \right) \Bigg\} \quad (6)$$

Dropping the terms independent of μ and calculating expectation with respect to all possible data symbol yields following likelyhood function:

$$\begin{aligned} L(\mu) &= \sum_{all\vec{\mathbf{d}}} \sum_{i=1}^{N-1}\sum_{k=i}^{N-1}\sum_{l=i}^{N-1} r_k r_l^* r_{k-i}^* r_{l-i} e^{j(\theta_{k-i}+\theta_l-\theta_k-\theta_{l-i})} \\ &= \sum_{i=1}^{L-1} \Bigg\{ \sum_{m=0}^{\mu-1} |r_m|^2 |r_{m+i}|^2 + \sum_{k=1}^{L-1}\sum_{l=1}^{L-1} r_{\mu+k} r_{\mu+l}^* r_{\mu+k-i}^* r_{\mu+l-i} s_{k-i} s_l s_k^* s_{l-i}^* \\ &\quad + \sum_{m=\mu+L}^{N-1} |r_m|^2 |r_{m-i}|^2 \Bigg\} + \sum_{i=L}^{N-1}\sum_{m=0}^{N-1-i} |r_m|^2|r_{m+i}|^2 \quad (7) \end{aligned}$$

where the second equality comes from $e^{j\theta_{\mu+L+k}} = d_k$, $k = 0, 1, ..., N-L-1$ and $\sum_{all\ \vec{\mathbf{d}}} d_k = 0$, since a symmetric constellation is assumed in M-ary PSK signal. Subtracting a constant term $\sum_{i=1}^{N-1}\sum_{m=0}^{N-1-i} |r_m|^2|r_{m+i}|^2$ from (7) yields a simpler form

$$L_1(\mu) = \sum_{i=1}^{L-1} \Bigg\{ \left| \sum_{k=i}^{L-1} r_{\mu+k} s_k^* r_{\mu+k-i}^* s_{k-i} \right|^2 - \sum_{k=\mu+i}^{\mu+L-1} |r_k|^2 |r_{k-i}|^2 \Bigg\}. \quad (8)$$

This is our proposed test function. The first term inside the bracket of (8) is the magnitude square of the correlation between $r_{\mu+k}s_k^*$ and $r_{\mu+k-i}s_{k-i}^*$. This term $\sum_{k=i}^{L-1} r_{\mu+k}s_k^*(r_{\mu+k-i}s_{k-i}^*)^*$ will be referred to as the *double* correlation with lag i.

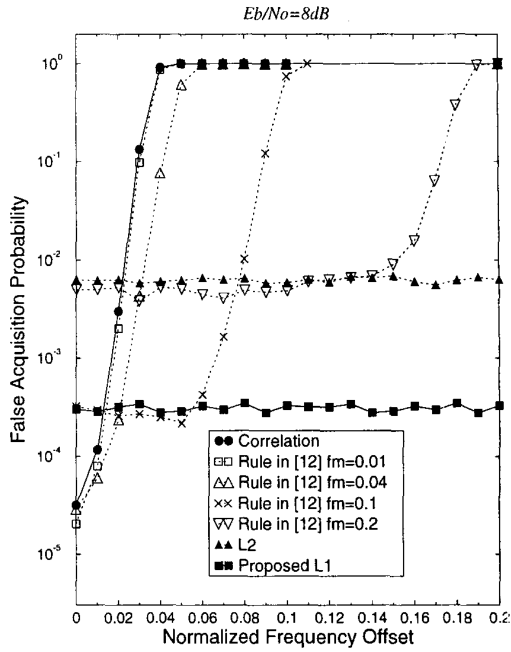

Fig.2. False acquisition probability vs. frequency offset when SNR is 8dB.

The second term in (8) compensates the effect of random data surrounding the sync pattern. It is interesting to note that the magnitude of the double correaltion with lag 1, given by

$$L_2(\mu) = \left| \sum_{k=i}^{L-1} r_{\mu+k} s_k^* r_{\mu+k-1}^* s_{k-1} \right| \quad (9)$$

is equivalent to the ad hoc rule in [14] that evaluates the correlation between the differentially encoded inputs and sync symbols. In the following section, we compare the performances of $L_1(\mu)$ in (8), $L_2(\mu)$ in (9) and the rule in [12] through computer simulation.

IV. Performance Evaluation

To evaluate the performance of the proposed synchronizer, we generate QPSK symbols which are distorted by AWGN noise, constant phase offset and constant frequency offset. It is assumed that the frame length $N = 162$ and the sync pattern length $L = 15$. Ten million independent frames are generated and the false acquisition probability is em-

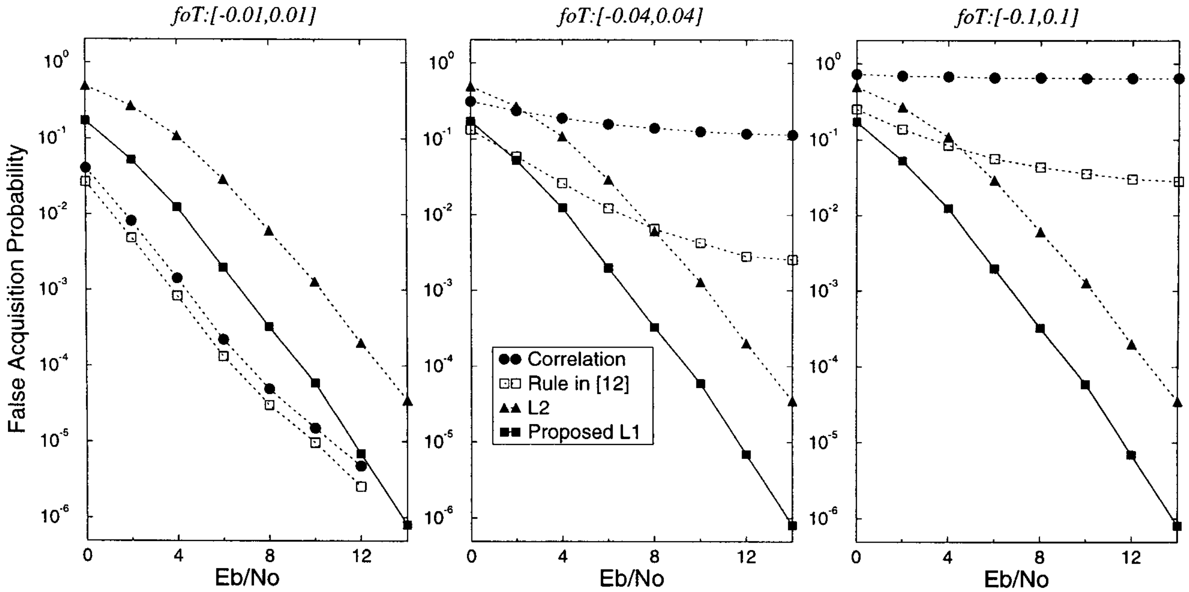

Fig.3. False acquisition probability vs. signal to noise ratio.

pirically estimated by counting the number of frame sync failures.

To obtain the results in Fig. 2, the signal to noise ratio (E_b/N_o) was fixed at 8dB and the normalized frequency offset(f_oT) was varied from 0 to 0.2. For the method in [12] f_m is assumed to be 0.01, 0.04, 0.1 and 0.2. Whereas the conventional correlation and the method in [12] shows severe performance degradation as the frequency offset becomes large, the proposed method and its special case $L_2(\mu)$ are invulnerable to frequency offset.

The results in Fig.3 were obtained under the assumption that the normalized frequency offset is uniformly distributed over [-0.01,0.01], [-0.04,0.04] and [-0.1,0.1]. The method in [12] is assumed to know the exact values of f_m. Except when the distribution of the frequency offset is narrow and the E_b/N_o is small, the proposed synchronizer outperforms the other methods.

V. Conclusion

A new frame synchronizer which is robust to frequency offset was proposed and its performance was examined through computer simulation. The proposed synchronizer performed better than the conventional techniques except when the frequency offset is nearly zero. The proposed scheme is particularly useful in applications where receivers suffer from large frequency offset.

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