SELECTIVE MEDIAN FILTERS

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ABSTRACT

New edge enhancing filters called the selective average (SA) and the selective median (SM) filters are proposed. It is shown theoretically and experimentally that these filters enhance blurred edges while attenuating noise. Root structures of the SA and SM filters are studied. An efficient implementation of the SA filter that requires only one multiplier, four additions, and one comparison irrespective of the filter length is introduced. Comparisons of the SA and SM filters with other edge enhancing techniques indicate that the proposed filters are simpler to implement and perform better than the others.

I. INTRODUCTION

Most edge enhancing or image sharpening techniques that counteract blur without knowledge about the blur are essentially linear high freqency emphasis filters, and thus they are very sensitive to noise [1],[2]. To enhance edges while reducing noise, some nonlinear techniques have been proposed. In [3], an effective edge enhancing smoother often called the Hachimura-Kuwahara (H-K) filter is introduced. In H-K filtering, several windows are set up around each pixel. The average and the sample variance of the values inside each window are calculated, and the pixel value is replaced by the average from the window having the smallest variance. The H-K filter has been applied to enhance edges of radionuclide images [4], [5], and to extract contour primitives from single-photon emission computed tomography images [6]. (The edge enhancing property of H-K type filters are also discussed in [7].) Although the H-K filter is an effective edge enhancing and noise suppressing technique, it is computationally expensive. Recently, another nonlinear edge enhancing technique called the comparison and selection (CS) filter has been introduced [8]. This filter can also enhance edges while reducing noise components, and is usually simpler to implement than the H-K filter. The CS filter, however, enhances edges at the expense of noise suppression.

It should be important to point out that a repeated usage of the H-K or CS filter tends to convert blurred edges into ideal step edges in contrast to linear edge enhancing techniques such as unsharp masking [1]. Therefore a piecewise constant image is usally obtained by repeated H-K or CS filtering. This property is particularly useful when these filters are applied for contour or edge detection [6], [8].

In this paper, we introduce and analyze a new class of edge enhancing filters which are modifications of H-K filters. These filters have subfilters as in H-K filtering, but unlike H-K filtering the input pixel value is replaced by the output of one of the subfilters which is closest in value to the input value. Two types of subfilters which are average and median filters are employed in the proposed filters. The filters with average and median subfilters are called the selective average (SA) and selective median (SM) filters, respectively. The SA and SM filters greatly alleviate the computational burden since sample variances are not evaluated. We shall show that these filters are simpler to implement and perform better than H-K and CS filters. For the sake of simplicity, only one-dimensional (1-D) SA and SM filtering will be studied, and their properties will be compared with those of 1-D H-K and CS filters.

II. THE SELECTIVE AVERAGE AND SELECTIVE MEDIAN FILTERS

The output of the 1-D SA filter with window size 2N+1 is given by

$$Y(n)=\begin{cases} A_1(n), & \text{if } |A_1(n)-X(n)|\le|A_2(n)-X(n)| \\ A_2(n), & \text{otherwise} \end{cases} \tag{1}$$

where A_1 = Average $\{X(n-N), \ldots, X(n-1)\}$, A_2 = Average $\{X(n+1), \ldots, X(n+N)\}$, and $\{X(i)\}$ is an input sequence. In (1), replacing $A_1(n)$ and $A_2(n)$ with $M_1(n)$ and $M_2(n)$, repectively, gives the output representation of the 1-D SM filter. Here M_1 = Median $\{X(n-N), \ldots, X(n-1)\}$, M_2 = Median $\{X(n+1), \ldots, X(n+N)\}$, and N is assumed to be an odd integer. These filter types are computationally efficient and very effective in enhancing edges while suppressing noise.

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III. EDGE ENHANCING PROPERTIES

The convex/concave (C/C) edge is a useful blurred edge model introduced in [8]. In this section, we first show that C/C edges become ideal step edges after multiple passages through SA or SM filters. The C/C edge is defined as follows: Consider a sequence {X(i)} that contains a blurred edge, defined by

$$X(n) = \begin{cases} 0, & i \leq 0 \\ f(i), & 0 < i \leq D \\ H, & D < i \end{cases} \qquad (2)$$

where $\{f(i) : -\infty < i < \infty\}$ is a sequence that determines the blurred edge shape, H is a constant and D is a positive integer. The edge in (2) is called the C/C edge if {f(i)} is convex [i.e., $d(i-1) \leq d(i)$] for $1 \leq i \leq T$, and concave [i.e., $d(i-1) > d(i)$] for $T < i \leq D+1$ where T is an integer, $1 \leq T \leq D+1$, and $d(i) = f(i) - f(i-1)$.

Property 1: Any C/C edge with duration D becomes an ideal step edge after at most $\lceil 2D/N+1 \rceil$ and D , where $\lceil x \rceil = x$ if x is an integer, and the integer part of x+1, otherwise .

The proof of this property is rather long, and thus omitted.

Of course, a ramp edge which is a special case of C/C edges [f(i) in (2) is a linear trend with f(0) = 0 and f(D+1) = H], can be converted into an ideal step edge by repeated SA or SM filtering. For this case we can find the position at which the transition from zero to H of the resulting step edge occurs as well as the number of iterations required in producing step edges. We obtain the required number of iterations and the transition position. The results are summarized in Table 1. For comparison, the results associated with 1-D H-K and CS filters are also presented. The above results indicate the following: (i) The edge enhancing behavior of SM filter is the same as or similar to that of the CS filter with J=(N+1)/2. (ii) The SM filter performs better than the SA filter in enhancing edges because the former requires less iterations than the latter.

It should be pointed out that the input and output values considered in obtaining the results in Table 1 have infinite precisions. In practical situations the input and output values are quantized. In order to study the quantization effect, we consider the ramp edge which consists of integers, represented as

$$X(i) = \begin{cases} 0, & i \leq D \\ hi, & 0 < i \leq D \\ h(D+1), & D < i \end{cases}$$

where h and D are positive integers. The ramp edge is passed through the SA filter and decimal fractions of each output value is truncated so that the output sequence is composed of integers. Table 2 presents the number of iterations required in converting the ramp edge into a step edge by using the SA filter. The results indicate that the edge enhancing behavior of the SA filter can be comparable to that of the SM filter when the input and output are quantized.

Filter		Number of iterations	Transition position
SM	$N \leq D$	$\lceil (2D-N+1)/(N+1) \rceil$	D-(n-1)/2
	$N > D$	1	$\lceil D/2 \rceil$
SA	$N \leq D/2$	$\lceil D-N+1 \rceil$	D-N+1
	$N > D/2$	$\lceil D/2 \rceil$	$\lceil D/2 \rceil$
CS	$N \leq D/2$	$\lceil (D-N+1)/J \rceil$	D-N+1
	$N > D/2$	$\lceil D/(2J) \rceil$	$\lceil D/2 \rceil$
H-K	--	∞	--

Table 1 Required number of iterations and transition positions

N	Number of iterations			
	h=1	h=3	h=9	h=12
3	9	10	10	11
5	6	6	7	7
7	6	6	6	6

Table 2. Required number of iterations in SA filtering with quantization (D=20).

IV. NOISE ATTENUATION PROPERTIES

In this section, we compare the noise attenuation properties of edge enhancing filters through computer simulations. We generated a sequence of length 100,000 which is iid with normal desity having mean zero and variance one. The sequence is passed through the edge enhacing filters, and the output sample variances are obtained. Table 3 presents the results. It is observed that the SA filter outperforms the others (the SA filter is almost comparable to the median filter in attenuating this noise), and the noise suppression characteristics of SM and H-K filters are similar to that of the CS filter with J=1.

V. ROOT STRUCTURES

A root of a filter is a sequence that is invariant with respect to the filtering [9]. It is trivial to show that a piecewise constant sequence is a root of the SA filter or SM filter if the duration of each pulse is greater than or equal to 2N in SA filtering, and N+1 in SM filtering. It should be noted that a sequence which is not piecewise constant can also be a root of SA or SM filters. For example, the sequence shown in Fig. 1 is a root of SA or SM filters. This indicates that repeated SA or SM filtering may not yield a piecewise constant sequence. Nontheless, we shall see experimentally in Section VII that a sequence tends to be converted into a piecewise constant one by a repeated usage of the SA or SM filter.

VI. COMPUTATIONAL ASPECTS

The average subfilters of the SA filter can be implemented using recursive structures as in [10]. With the recursive averaging structure shown in Fig. 2, the SA filter can be realized with one scaling multiplier, four additions, and one comparision per output value irrespective of the window length 2N+1. In Fig. 2, replacing the recursive averaging structure in the rectangle depicted by dashed lines with a 1-D fast median filtering structure gives an efficient implementation of the SM filter. With the 1-D fast median filtering algorithm in [11], the SM filter can be realized with two additions and at most 2N+1 compare/swap operations. Thus, the SA filter is usually simpler to implement than the SM filter which is clearly simpler than the CS filter.

VII. SIMULATION RESULTS

In this section, we present one representative set of results illustrating the performance characteristics of the various filters we have discussed so far. Fig. 3 shows a noisy blurred edge which is the test input and the results of median, H-K, CS, SA, and SM filtering with N=5. This test input was obtained by blurring the step edge shown as the dashed lines and adding zero mean white Gaussian noise of variance four. It is seen that the median filter could not enhance the edges, while the others could. Both the CS and SM filters produced roots after six passages, but the SA and H-K filters could not produce roots even after ten passages. It is conspicuous that the SA filter has a superior Gaussian noise attenuation characteristics. In general, among the edge enhancing filters, the SA filter outperformed the others, the performances of H-K and SM filters seem to be similar and the CS filter is the worst in overall performance.

Filter	Variance	
	N = 3	N = 5
SA	0.25	0.16
SM	0.32	0.22
CS (J=1)	0.37	0.22
H-K	0.38	0.21
Median	0.21	0.14

Table 3 Output sample variances when the input is iid, N(0,1).

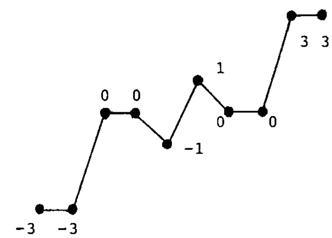

Fig. 1 A root of SA and SM filters.

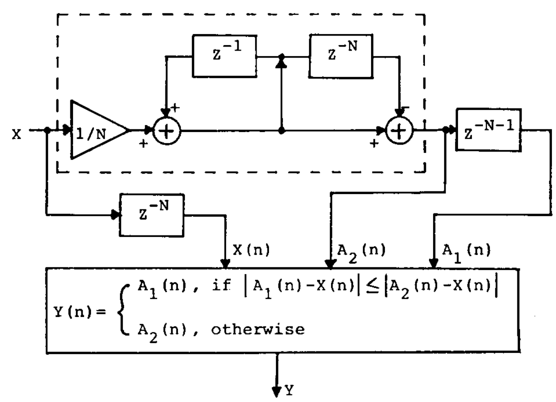

Fig. 2 Efficient implementation of the SA filter.

REFERENCES

[1] W.K. Pratt, Digital Image Proc.. New York: Wiley, 1978.

[2] A. Rosenfeld and A.C. Kak, Digital Picture Proc., vol. 1, 2nd ed. New York: Academic, 1982.

[3] M. Kuwahara, K. Hachimura, S. Eiho, and M. Kinoshita, "Processing of RI-angiocardiographic image," in Digital Proc. of Biomedical Images, K. Preston and M. Onoe, Eds. New York: Plenum, 1976, pp. 187-202.

[4] P.W. Verbeek and D.J. deJong, "Edge preserving texture analysis," IEEE Compt. Soc. Publ. 84CH2046-1, pp. 1030-1032, 1984.

[5] D. Romary, J. Lerallat, and G. Foutenier, "Application of image processing techniques to gamma-angiography," Comput. Biomed. Res., vol. 18, pp. 488-495, 1985.

[6] K. Minato, Y.-N. Tang, G.W. Bennett, and A.B. Brill, "Automatic contour detection using a "fixed-point Hachimura-Kuwahara filter" for SPECT attenuation correction," IEEE Trans. Med. Imaging, vol. MI-6, no 2, pp. 126-133, June 1987.

[7] M. Nagao, and T. Matsuyama, "Edge preserving smoothing," Comput. Graph. Image Proc., vol. 9, pp.394-407, 1979.
[8] Y.H. Lee, and A.T. Fam, "An edge gradient enhancing adaptive order statistic filter," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-35, pp. 680-695, May 1987.
[9] N.C. Gallagher and G.L. Wise, "A theoretical analysis of the properties of median filters," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-29, pp. 1136-1141, Dec. 1981.
[10] P. Heinonen, and Y. Neuvo, "FIR-median hybrid filters," IEEE Trans Acoust., Speech, Signal Processing, vol. ASSP-35, pp. 832-838, June 1987.
[11] J.B. Bednar and T.L. Watt, "Alpha-trimmed means and theirs relationship to median filters," IEEE trans Acoust., Speech, Signal Processing, vol. ASSP-32, pp. 145-153, Feb. 1984.

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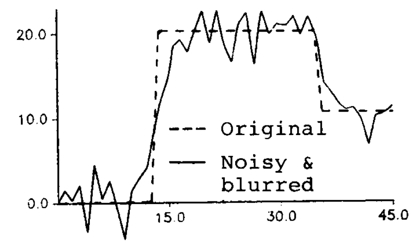

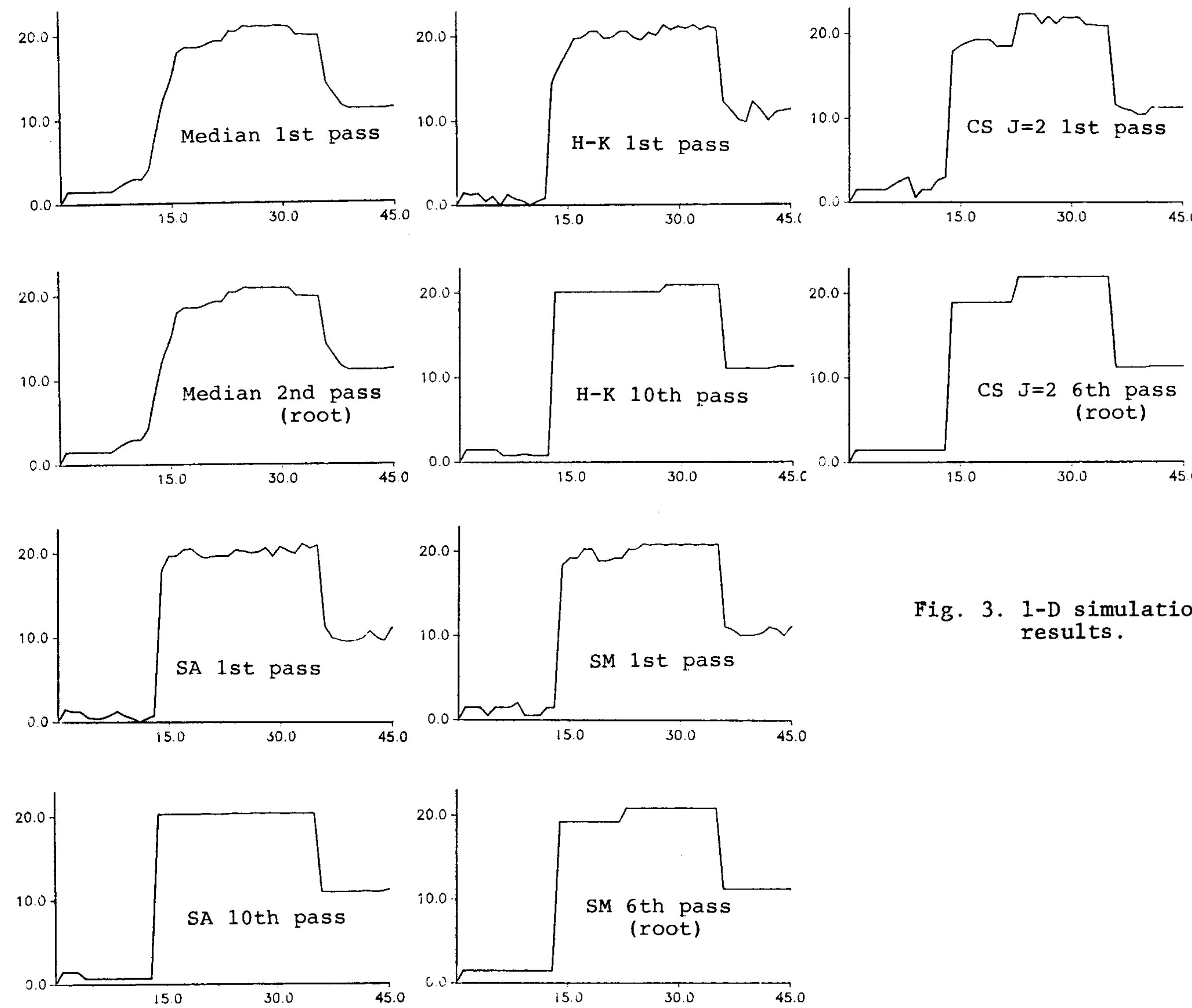

Fig. 3. 1-D simulation results.