ORDER STATISTIC LAST OUTPUT REFERENCE FILTERS*

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ABSTRACT

Median filtering can be viewed as operation that selects a sample from each window close to the last output. This observation results in a new edge preserving smoother called the last output reference (LOR) filter. The LOR filter is similar in function to the median or recursive median filters, but has additional advantages particularly in suppressing impulses. It has been shown that repeated applications of LOR filtering produces a sequence that is invariant to subsequent passes through the same filter. Also, it has been proven that any sequence can be converted to a locally monotone sequence by using a combination of "forward" and "backward" LOR filters. When the LOR filter is applied to an actual noisy image, it is shown to perform well.

INTRODUCTION

Median filtering has been recognized as an effective alternative to linear filtering for some digital signal processing applications including speech processing [1] and image enhancement [2]. Recently, some generalizations of median filters [3,4] (often called order statistic filters), and related simple nonlinear filters [5-7] have been introduced. The objectives of these filters are similar to those of the median filter, namely, the suppression of impulsive and/or nonimpulsive noise while preserving edges. While these filters can outperform median filters in some situations, they usually are structurally more complex and/or require more computations. Therefore, median filters are often preferred to these filters for practical applications.

Recursive median filtering [8] is a modification of standard median filtering; recursive median filtering replaces the input at each point with the median of the last N outputs and N + 1 inputs which are the present and the next N inputs. Compared to the median filter, this filter exhibits improved nonimpulsive type noise suppression characteristics due to its recursive nature.

In this paper, we propose a simple nonlinear filter for edge-preserving smoothing that performs like a median filter but has additional advantages. In particular, the filter proposed here is superior to median and recursive median filters in suppression of impulsive noises. The proposed filter can be thought of as an adaptive order statistic filter, but is structurally and computationally almost as simple as the median filter.

THE FILTER

Before describing the proposed filter, let us state an observation for the 1D median filter, which indicates that median filtering can be viewed as an operation that selects a sample from each window close to the last output. The filter that we propose is motivated by this observation and, as an immediate consequence, the proposed filter has characteristics similar to those of a median filter. In stating the observation, the input and the output sequences of the median filter with a window size 2N + 1 are denoted by {X(i)} and {Y(i)}, respectively.

Observation: Suppose that the window is centered at time k. The present output Y(k) of the median filter with N > 1 is equal to either the last output Y(k-1) which is $X_{k-1}^{(N+1)}$ or to one of the three data, $X_{k-1}^{(N)}$, $X_{k-1}^{(N+2)}$, and X(k+N), where $X_{k-1}^{(i)}$ is the i-th smallest sample among the input data within the window centered at k-1. The output Y(k) is equal to X(k+N) only if X(k+N) is in between $X_{k-1}^{(N)}$ and $X_{k-1}^{(N+2)}$.

From the above observation, we can gernerally say that the median filter selects a sample from the window that is close to the last output. This suggests the following filter having properties similar to those of median filtering, but with additional advantages.

The output at each point is a sample with a value closest to the last output among the samples inside the window, which moves over the

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input sequence. We call this filter the last ouput reference (LOR) filter. In LOR filtering we replace the leftmost sample inside each window with the sample value that is the closest to the last output. Thus, unlike the median filter, the window is not centered at a point whose value is to be replaced because the LOR filter replacing the center value inside each window tends to shift the input sequence. If several samples exist whose values are closest to the last output within the window, we select the leftmost sample value. Any positive integer can be the window size of the LOR filter, while the median filter requires an odd window size. Throughout this paper, the window size will be denoted by W. Whenever W represents the window size of median or recursive median filtering, it is an odd integer.

In the sequel, we shall consider the LOR filtering of a sequence of length L. At the beginning of the filtering, we assume that the last output value is the same as the first sample of the sequence. In order to account for the end effect at the right end point of the L-length sequence, W-1 samples are appended to the right end of the sequence. The values of appended samples are equal to the last sample of the original sequence. Under these conditions, the first and the last sample values of the input are invariant to LOR filtering.

The LOR filter can suppress both impulsive and nonimpulsive noises while preserving edges just like median and recursive median filters can. Edge preserving characteristics of these filters are similar. However, LOR filtering is superior to both median and recursive median filtering in suppressing impulsive noise. The LOR filter with a window size W can suppress up to W-1 impluses in a window, while the median and recursive median filter can generally suppress up to (W-1)/2 impluses. Thus, in applications where impulsive noise smoothing is the major concern, the window size of the LOR filter can be smaller by a facter of two than those window sizes of median and recursive median filters. In the paper, the edge preserving smoothing characteristic of these filters will be discussed further through experimental results and compared with each other.

The output of the LOR filter can be easily calculated from the ordered input data inside each window. While the output of median filtering is immediately obtained from the ordered data, LOR filtering requires additional operations which involve a maximum of W+1 comparisons and two subtractions. The number of operations to get the ordered data in a sliding window is, at most, 2W. Thus, the total number of operations for a single window in LOR filtering is, at most, 3W+3. In applications where the window size of the LOR filter can be smaller than that of the median filter by a factor of two, LOR filtering can be computationally more efficient than median filtering.

ROOT PROPERTIES

A root (or fixed points [10]) of a filter is a sequence that is invariant to the filtering. Root properties of median and recursive median filters have been studied extensively [8-10]. We characterize roots of the LOR filter and show that any finite length sequence is converted to a root after finite passages through the LOR filter. The following four theorems have been established regarding the root properties of the LOR filter.

Theorem 1: A sequence {X(i)} is a root of LOR filtering if and only if the leftmost sample inside each window is the closest to the last input among the samples within the window, that is, X(k) is the closest to X(k-1) among {X(k), X(k+1),.... ,X(k+W-1)} for any k.

A sequence {X(i)} is locally monotone of length m, which is denoted by LOMO(m), if {X(k), X(k+1), ... ,X(k+m-1)} is monotone for any k [10]. In the cases of median and recursive median filtering, a sequence is a root if and only if it is LOMO[(W+3)/2] where W is the window size [8,9]. In LOR filtering, it is not difficult to see from Theorem 1 that a LOMO(W+1) is a root of LOR filtering. Some sequences that are not locally monotone can also be roots of this filter unlike cases of median and recursive median filtering. Fig. 1(a) and (b) illustrate some examples of such roots of LOR filtering with W = 3. Later in this paper, we shall show that non-locally monotone roots of the LOR filter can be converted to LOMO(W+1) sequence by "backward" LOR filtering where the window moves over the input sequence from right to left.

In median filtering, Gallagher and Wise [9] proved that any nonroot sequence of length L is converted to a root after a maximum of (L-2)/2 successive window passes. In the case of recursive median filtering, Nodes and Gallagher [8] showed that any finite length sequence becomes a root by a single passage through the recursive median filter. The following theorem shows that any finite length sequence is also converted to a root by successive uses of the LOR filter.

Theorem 2: Any sequence of length L is converted to a root of the LOR filter after a maximum of $\lfloor 2(L-3)/W \rfloor + 1$ passages through this filter, where x is the integer part of x.

The LOR filter that we have described up to this point is the "forward" filtering operation in which a window moves over the input from left to right. We now consider "backward" LOR filtering where a window moves from right to left. The output Y(k) of backward LOR filtering is the sample that is the closest to Y(k+1) among {X(k), X(k-1),..., X(k-W+1)}. In backward LOR filtering of a sequence of length L, we assume that Y(L) = X(L) and append W-1 samples to the left end of the sequence. In the next theorem, we first characterize a root set of forward and backward LOR filtering.

Theorem 3: A sequence is a root of the forward LOR filter and also a root of the backward LOR filter if and only if it is a LOMO(W+1), where W is the window size of the forward and the backward LOR filter.

Next we show that any roots of forward LOR filtering are converted to LOMO(W+1) by a single use of the backward LOR filter.

Theorem 4: If a root of the forward LOR filter is backward LOR filtered, it becomes LOMO(W+1), which is invariant to further forward or backward LOR filtering.

From Theorems 2 and 4, any sequence of length L is converted to a LOMO(W+1) sequence after at most $\lfloor 2(L-3)/W \rfloor + 1$ forward passes followed by a single backward pass of the same window size. Fig. 2 illustrates the results of backward LOR filtering (W = 3) of the roots in Fig. 1. The backward filtered sequences are LOMO(4).

SIMULATION AND EXPERIMENTAL RESULTS

In this section, LOR filters are applied to enhance images corrupted by impulses. The images under consideration consist of 256x256 8-bit pixels. The results of LOR filtering are compared with those of median and recursive median filtering. In order to compare them quantitatively, the empirical mean square errors (EMSE) between the original and the filtered images are computed [3]. All of the filte-ring operations considered are separable filtering in which each row of the input image is passed through a 1D filter, and then each column of the resulting image is passed through the same 1D filter.

The original image is shown in Fig. 3(a). Fig. 3(b) shows the image corrupted by impulses of gray level one that occur at random positions with probability 0.1. This image is filtered by 3x3 separable LOR, median and recursive median filters; the results are shown in Fig. 3(c)-(e), respectively. The LOR filter completely suppressed the impulses while median and recursive median filters fail to remove all of them. It is noted, however, that some fine details of the original image are somewhat blurred by LOR filtering. In order to suppress impulses completely by using a median filter, the 5x5 separable median filter is applied to the noisy image. Fig. 3(f) shows the result which looks similar to the 3x3 LOR filtered image. The 5x5 separable median filter completely suppressed impulses at the expense of some image sharpness. Table I lists the EMSE for the 3x3 LOR filter and the 5x5 median filter. The 3x3 LOR filter performed better than the 5x5 median filter.

In general, we can say that the LOR filter performs like a median filter with a larger window size. In applications where removal of impulses is the major concern, however, the LOR filter is preferred to a median filter with a larger window because the former causes less distortion of the input and is computationally more efficient.

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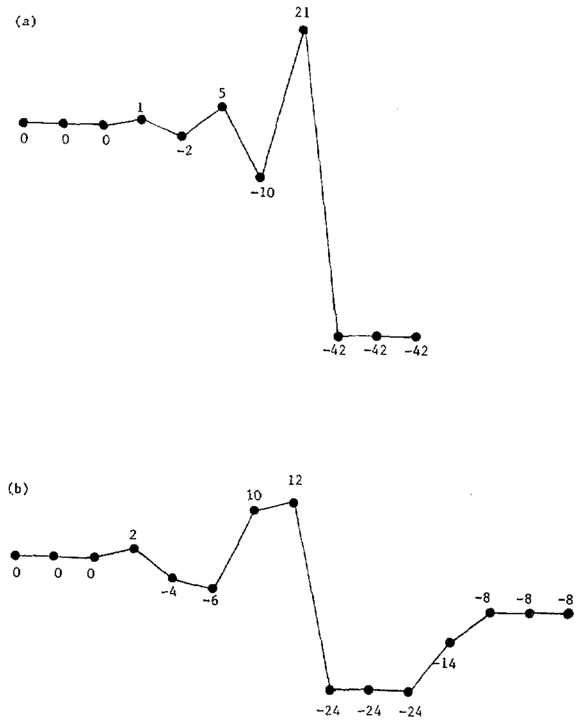

Fig 1. Examples of roots of LOR filtering with W=3 which are not LOMO(W+1).

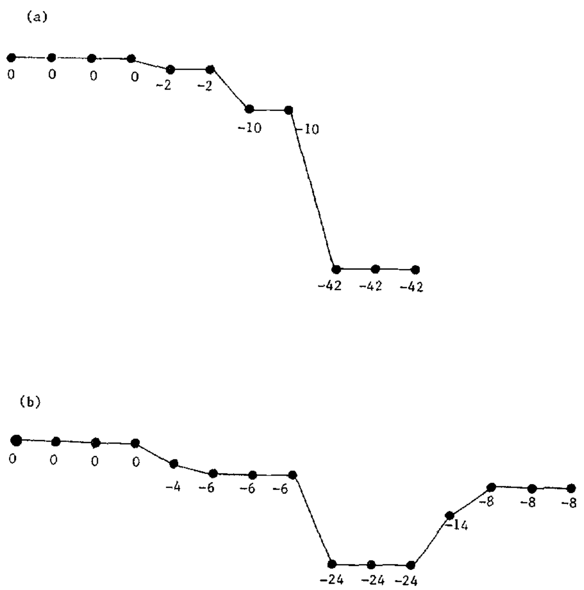

Fig. 2. (a) and (b) illustrate LOMO(W+1) sequences obtained by backward LOR filtering with W=3 of the forward root sequences shown in 1(a) and (b), respectively.

Filter	EMSE
LOR (W = 3X3)	37.84
Median (W = 5X5)	46.55

Table I. EMSE for filtered versions of the image corrupted by impulsive noise.

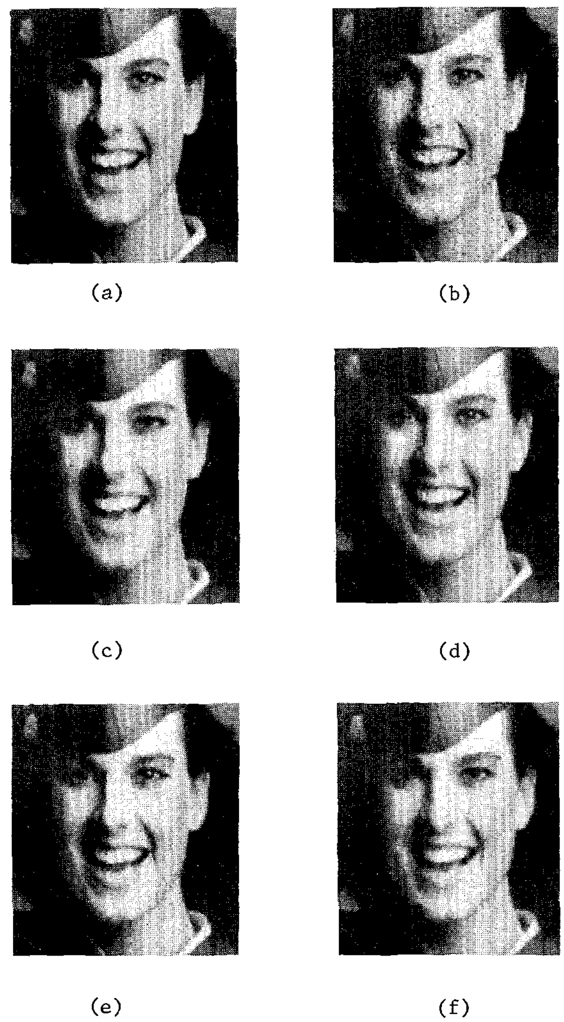

Fig. 3. (a) Original image. (b) Image corrupted by impulsive noise. (c) Result of 3x3 separable LOR filtering. (d) Result of 3x3 separable median filtering. (e) Result of 3x3 separable recursive median filtering. (f) Result of 5x5 separable median filtering.