# Adaptive Acquisition for DS-SS Systems with Antenna Diversity

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*Abstract*— **We propose an adaptive acquisition system that adjusts the degree of diversity combining without explicit knowledge of the SIR. This system starts code acquisition with no diversity and combines corresponding correlations from different antennas at each code phase to increase the degree of diversity combining whenever false alarm is declared. The mean acquisition time for multipath fading environment is evaluated and is compared with other acquisition systems with multiple receiver antennas. The results indicate that the proposed system performs like the parallel search in high SIR and performs like the serial search with full diversity combining in low SIR.**

## I. Introduction

In DS-SS systems, code acquisition is achieved by correlating the received signal with a locally generated PN sequence. When multiple receiver antennas are used for achieving diversity, it is desirable to employ at least one correlator for code acquisition at each antenna, and combine properly the resulting correlations (Fig. 1). The degree of diversity combining may be adjusted depending on the signal to interference ratio (SIR). In one extreme if the SIR is low, then it is recommended to fully combine the correlations from different antennas. When only one correlator is employed for each antenna, full diversity combining yields a serial search. In the other extreme if the SIR is high, then diversity combining may not be necessary; in this case a parallel search can be performed without increasing hardware cost by assigning a distinct code phase offset to each correlator. The advantage of full diversity combining for code acquisition has been analyzed in [1]. It has been shown in [2] that code acquisition methods with full combining can outperform those with no combining for low SIRs, with the opposite being true for high SIRs.

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In this paper, we propose an adaptive scheme that adjusts the degree of combining without explicit knowlege of the SIR. The basic idea of the algorithm is simple: it starts with no diversity combining and increases the degree of combining whenever false alarm is declared either by the tracking loop (single- and double-dwell systems) or by the verification (a double-dwell system). The results will show that the proposed algorithm performs like the parallel search in high SIR and performs like the serial search with full diversity combining in low SIR.

## II. Adaptive Code Acquisition with Antenna Diversity

Consider a code acquisition system in Fig. 1 which employs $M$ antennas for diversity, where $M$ is a power of two. The antennas are sufficiently spaced and the received signals from different antennas are assumed to be independently faded. Each antenna is followed by a correlator that performs frequency-down conversion, integration and dumping, chip rate sampling and matched filtering (Fig. 1 (b)). Assuming a multipath fading channel, the input to the matched filter is given

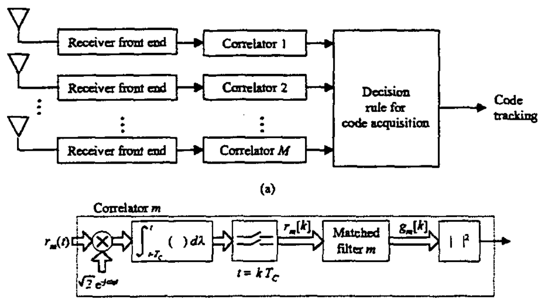


Fig. 1. A code acquisition system with antenna diversity. (a) Structure. (b) Details of the correlator $m$.

by

$$r_m[k] = \sqrt{S}\sum_{q=1}^{Q}\alpha_{q,m}e^{j\theta_{q,m}}\int_{(k-1)T_C}^{kT_C} c(t-(q-1)T_C-\xi)dt + n_m[k],\quad k=1,2,3,\ldots, \tag{1}$$

where $S$ is the signal power, $\alpha_{q,m}$ and $\theta_{q,m}$ represent the amplitude and phase of the channel respectively, $Q$ is the number of resolvable multipaths, $c(t)$ represents the code sequence of length $L$-chips, $\xi$ is the delay with respect to a time reference, and $n_m[k]$ is zero-mean Gaussian noise with variance $\sigma_n^2 = T_C N_0/2$. It is assumed that data are absent and that frequency offset is negligible. In addition, $c(t)$ is assumed to be $\pm1$-valued (rectangular pulse shape). Since all signals arrive at $M$ antennas almost simultaneously, the time delay $\xi$ is constant for all $m$ [3]. The output of the $m$th matched filter is given by

$$g_m[k] = \sum_{l=0}^{L_P-1} r_m[k-l]c[w_m + L_P - l],\quad 1 \le m \le M \tag{2}$$

where $L_P$ is the span of the matched filter in the units of chips, $c[l]$ can be obtained by sampling $c(t)$ with period $T_C$, i.e. $c[l] = c(t)|_{t=lT_C}$, and $w_m$ denotes the code phase offset associated with the matched filter. Assuming perfect chip synchronization, $g_m[k]$ is rewritten as

$$g_m[k]=\begin{cases}\sqrt{S}\alpha_{q,m}e^{j\theta_{q,m}}L_PT_C + n'_m, & \text{when } [k-(q-1)-\frac{\xi}{T_C}]_{\text{mod } L}=w_m+L_P \quad (3a)\\ n'_m, & \text{otherwise} \quad (3b)\end{cases}$$

where $[x]_{\text{mod } L}$ denotes the remainder obtained from dividing $x$ by $L$ and $n'_m$ is zero-mean Gaussian noise with variance $\sigma_n^2 = L_P T_C N_0/2$. The expression in (3a) is obtained when the $q$-th signal component of $r_m[k]$ matches with the matched filter coefficients.

The decision rule block in Fig. 1(b) properly combines $|g_m[k]|^2$ and declares acquisition at the code phase associated with the maximum correlation. Referring to Fig. 2, the proposed algorithm for adaptive combining is described as follows:

*A. Single – Dwell System*

*Initial value.* Set $J = 1/2$.

*Step* 1. $J = J \times 2$. For each $m$, $1 \le m \le M$, allocate the code sequence with phase offset $w_m = \lceil (m-1)/J \rceil J \lceil L/M \rceil$ to the $m$th matched filter where $\lceil x \rceil$ stands for the largest integer equal to or less than $x$. The matched filter coefficients are given by $\{c[w_m], c[w_m+1], \ldots, c[w_m+L_P-1]\}$ which are the first $L_P$ values of the assigned code sequence.

*Step* 2. Evaluate correlation values in the search region of each matched filter. The search region of the $m$th matched filter is defined as $[w_m, w_m+J\lceil L/M \rceil - 1]$ for $1 \le m \le M-1$ and as $[w_M, L-1]$ for $m = M$. Note that $J$ correlation values are obtained at every code phase, and $J$ is a power of two.

*Step* 3. Add the $J$ corresponding correlation values at each code phase and obtain the maximum of added (combined) correlations. Acquisition is declared at the code phase associated with the maximum.

*Step* 4. Start code tracking. If false alarm is declared, then go to Step 1 when $J < M$ and go to Step 2 when $J = M$. Code synchronization is completed unless false alarm is declared.

When $J = 1$, this algorithm is equivalent to the parallel acquisition with no diversity combining [2]; it reduces to the serial acquisition with full diversity combining when $J = M$.

*B. Double – Dwell System*

The proposed algorithm for double-dwell systems is identical to the above algorithm with the exception that verification is performed between Steps 3 and 4. (In a double-dwell system, Steps 1 to 3 constitute a *search* mode and Step 3 declares only tentative acquisition.) The procedure for verification mode is described as follows.

*Initial value.* Set $v = 0$.

*Step* V1. $v = v+1$. Evaluate correlation values in the

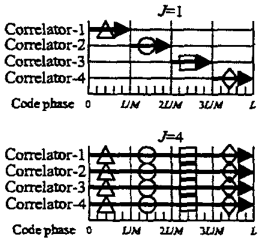


Fig. 2. Span of correlator search regions (bold arrows) for various $J$ and corresponding correlation values at each code phase when $M = 4$ and $L = 20$.

verification region of each matched filter. The verification region of the $m$-th filter is $[w_m + vJ\lceil L/M\rceil, w_m + (v+1)J\lceil L/M\rceil - 1]$.

*Step V*2. Add $J$ corresponding correlation values at each code phase and obtain the maximum of combined correlations.

*Step V*3. If $v < A$, where $A$ is a positive integer, then store the code phase at which the maximum combined correlation occurs and go to Step V2. If $v = A$, go to Step V4.

*Step V*4. Examine the stored code phases. If at least B out of A codes phases are identical to the code phase obtained in Step 3 of the search mode in which tentative code acquisition is declared, then initiate tracking in Step 4 above. Otherwise, declare false alarm; go to Steps 1 and 2 of the search mode when $J < M$ and $J = M$, respectively.

## III. Performance Analysis

In this section, the performance of the proposed system is analyzed, under the assumption that, for a given $q$, $1 \le q \le Q$, the channel parameters $\alpha_{q,m}$ and $\theta_{q,m}$, $1 \le m \le M$, are i.i.d., and that $\alpha_{q,m}\, e^{j\theta_{q,m}}$ is complex Gaussian which is invariant over the acquisition period. When $g_m[k]$ is expressed as (3a), $|g_m[k]|^2$ has a chi-square distribution with 2 degrees of freedom, and thus the test statistic $X_J[k] \triangleq \sum_{m=1}^{J} |g_m[k]|^2$ has a chi-square distribution with $2J$ degrees of freedom. In this case the conditional density of $X_J[k]$ given $\underline{\alpha}_{\mathbf{q},\mathbf{J}}$

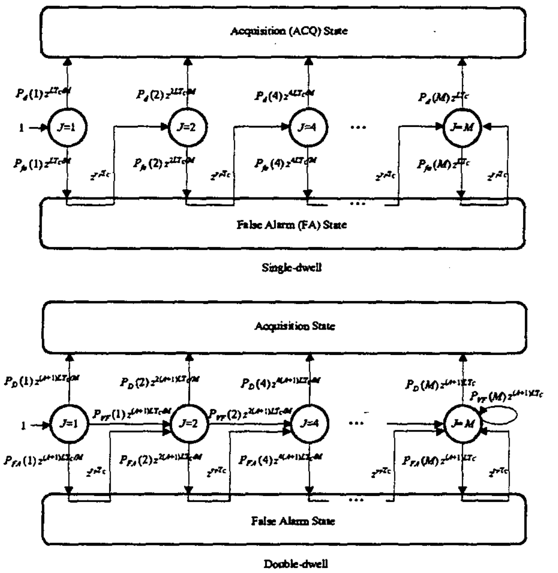


Fig. 3. Flow graph diagrams of the proposed adaptive acquisition with antenna diversity.

$\triangleq [\alpha_{q,1}, \alpha_{q,2}, \dots, \alpha_{q,J}]$ is represented by

$$f_{qJ}(x|\underline{\alpha}_{\mathbf{q},\mathbf{J}}) = \frac{1}{2\sigma_0^2}\left(\frac{x}{Ja_q^2}\right)^{(J-1)/2} \exp\left(-\frac{x + Ja_q^2}{2\sigma_0^2}\right) \cdot I_{J-1}\left(\frac{\sqrt{xJa_q^2}}{\sigma_0^2}\right) \quad (4)$$

where $\sigma_0^2 = L_P T_C N_0/2$, $a_q^2 = E_C L_P^2 T_C \sum_{m=1}^{J} \alpha_{q,m}^2 / J$. This is a non-central chi-square density function [5]. When $g_m[k]$ is expressed as (3b), $X_J[k]$ has a central chi-square density given by

$$f_{0J}(x) = \frac{1}{(2\sigma_0^2)^J (J-1)!} x^{(J-1)} \exp\left(-\frac{x}{2\sigma_0^2}\right). \quad (5)$$

The mean acquisition time of the proposed scheme is derived using the flow graph diagrams in Fig. 3 [4]. In these diagrams, circles indicate states that evaluate the maximum of combined correlations for search/verification modes, z represents the unit delay, and $r_P T_C$ is the penalty time caused by false acquisition. $P_{fa}(J)$, $P_d(J)$, $P_{FA}(J)$, $P_D(J)$ and $P_{VF}(J)$ are probabilities which are defined as follows: $P_{fa}(J)$ is the false alarm probability of a single-dwell system with $J$ degrees of diversity combining. It is the probability that the maximum combined correlation occurs at one of $(L-Q)$ incorrect code phases (there are $Q$ correct code phases out of $L$ phases due to multipath). $P_{fa}(J)$ is represented as

$$P_{fa}(J) = (L-Q)P_0(J) \tag{6}$$

where

$$P_0(J) = \int_0^\infty f_{0J}(x)F_{0J}(x)^{L-Q-1}\prod_{q=1}^{Q}F_{qJ}(x|\underline{\alpha}_{\mathbf{q,J}})dx. \tag{7}$$

$F_{0J}(x)$ and $F_{qJ}(x|\underline{\alpha}_{\mathbf{q,J}})$ are distribution functions that correspond to $f_{0J}(x)$ and $f_{qJ}(x|\underline{\alpha}_{\mathbf{q,J}})$, respectively. The detection (acquisition) probability of a single-dwell system is given by

$$P_d(J) = 1 - P_{fa}(J). \tag{8}$$

$P_{FA}(J)$ denotes the false-alarm probability of a double-dwell system. It is the probability that the tentative acquisition declared by the search mode is incorrect and the verification mode fails to find out the false acquisition. $P_{FA}(J)$ is given by

$$P_{FA}(J) = (L-Q)P_0(J)\sum_{l=B}^{A}\binom{A}{l}P_0(J)^l(1-P_0(J))^{A-l}. \tag{9}$$

The detection probability of a double-dwell system is represented as

$$P_D(J) = \sum_{q=1}^{Q}P_q(J)\sum_{l=B}^{A}\binom{A}{l}P_q(J)^l(1-P_q(J))^{A-l} \tag{10}$$

where

$$P_q(J) = \int_0^\infty f_{qJ}(x|\underline{\alpha}_{\mathbf{q,J}})F_{0J}(x)^{L-Q}\prod_{i=1,i\neq q}^{Q}F_{iJ}(x|\underline{\alpha}_{\mathbf{q,J}})dx. \tag{11}$$

$P_D(J)$ is the probability that the maximum combined correlation occurs at one of Q correct codes phases and the tentative acquisition is confirmed by the verification. The verification declares that the tentative acquisition is false with the probability $P_{VF}(J)$, which is given by

$$P_{VF}(J) = 1 - P_{FA}(J) - P_D(J). \tag{12}$$

The mean acquisition time is obtained following the procedure in [4] under the assumption that $\{\underline{\alpha}_{\mathbf{q,M}}\}$ are given. The gains associated with the branches of the flow graph diagram are defined as follows: $H_{DJ}(z)$ corresponds to the path leading from state $J$ to the ACQ state, and $H_{0J}(z)$ denotes the total path gain between state $J$ and state $J+1$ (when $J=M$, $H_{0M}(z)$ is the gain associated with the path from state $M$ to itself). Then, for a single-dwell system

$$H_{DJ}(z) = P_d(J)z^{(JL/M)T_C} \tag{13}$$

$$H_{0J}(z) = P_{fa}(J)z^{(JL/M+r_P)T_C} \tag{14}$$

and for double-dwell systems

$$H_{DJ}(z) = P_D(J)z^{((A+1)JL/M)T_C} \tag{15}$$

$$\begin{aligned} H_{0J}(z) \quad = \quad & P_{FA}(J)z^{((A+1)JL/M+r_P)T_C} \\ & + P_{VF}(J)z^{((A+1)JL/M)T_C}. \end{aligned} \tag{16}$$

The flow graph generating function $P_{ACQ}(z)$ can be shown to be

$$\begin{aligned} P_{ACQ}(z) = & H_{D1}(z) + \sum_{m=2}^{M'-1}H_{D(2^{m-1})}(z)\prod_{i=1}^{m-1}H_{0(2^{i-1})}(z) \\ & + H_{DM}(z)\prod_{i=1}^{M'-1}H_{0(2^{i-1})}(z)\frac{1}{1-H_{0M}(z)} \end{aligned} \tag{17}$$

where $M' = \log_2 M + 1$. The conditional mean acquisition time $E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\}$ is given by

$$E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\} = dP_{ACQ}(z)/dz\big|_{z=1}. \tag{18}$$

Using (13)-(18), the $E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\}$ can be derived as follows. For a single-dwell system,

$$\begin{aligned} & E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\} \\ & = \Bigg[\frac{L}{M}(1-P_{fa}(1)) + \sum_{m=2}^{M'}\bigg\{\Big((2^m-1)\frac{L}{M}+(m-1)r_P\Big) \\ & \quad \cdot (1-P_{fa}(2^{m-1}))\prod_{i=1}^{m-1}P_{fa}(2^{i-1})\bigg\} + \Big((2M-1)\frac{L}{M} \\ & \quad +(M'-1)r_P+\frac{L+r_P}{1-P_{fa}(M)}\Big)\prod_{i=1}^{M'}P_{fa}(2^{i-1})\Bigg]T_C \end{aligned} \tag{19}$$

and for a double-dwell system,

$$E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\}$$
$$=\Bigg[\frac{(A+1)L}{M}P_D(1)+\sum_{m=2}^{M'}P_D(2^{m-1})\prod_{i=1}^{m-1}(1-P_D(2^{i-1}))$$
$$\cdot\left\{(2^m-1)\frac{(A+1)L}{M}+r_P\sum_{k=1}^{m-1}\frac{P_{FA}(2^{k-1})}{1-P_D(2^{k-1})}\right\}$$
$$+\prod_{i=1}^{M'}(1-P_D(2^{i-1}))\left\{(A+1)L\left(\frac{2M-1}{M}+\frac{1}{P_D(M)}\right)\right.$$
$$\left.+r_P\left(\sum_{k=1}^{M'-1}\frac{P_{FA}(2^{k-1})}{1-P_D(2^{k-1})}T_C+\frac{P_{FA}(M)}{(1-P_D(M))P_D(M)}\right)\right\}\Bigg]T_C. \quad (20)$$

Obviously, the mean acquisition time $E\{T_{ACQ}\}$ is represented as

$$E\{T_{ACQ}\}=\underbrace{\int_0^\infty\cdots\int_0^\infty}_{Q\times M}E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\}f(\underline{\alpha}_{\mathbf{q,M}})d\underline{\alpha}_{\mathbf{q,M}} \quad (21)$$

where $f(\underline{\alpha}_{\mathbf{q,M}})=\prod_{q=1}^{Q}\prod_{m=1}^{M}f(\alpha_{q,m})$ and $d\underline{\alpha}_{\mathbf{q,M}}=\prod_{q=1}^{Q}\prod_{m=1}^{M}d\alpha_{q,m}$. Unfortunately, there is no closed form solution, and (21) must be evaluated numerically.

## IV. Performance Comparison

In this section, the mean acquisition time of the proposed system is evaluated and compared with the mean acquisition time of conventional schemes: serial acquisition with full diversity combining and parallel acquisition with no combining. The conditional mean acquisition time of these schemes are expressed as in Table I. In the computation, $E\{T_{ACQ}\}$ was obtained as follows: after generating $\underline{\alpha}_{\mathbf{q,M}}$ using random number generators, $E\{T_{ACQ}|\underline{\alpha}_{\mathbf{q,M}}\}$ was evaluated and $E\{T_{ACQ}\}$ was estimated by

$$E\{T_{ACQ}\}\approx\frac{1}{N}\sum_{i=1}^{N}E\{T_{ACQ}|(\underline{\alpha}_{\mathbf{q,M}})_i\} \quad (22)$$

(Use of (22) instead of using (21) considerably reduces computational load.) $N$ was set to 100000.

The system parameters were: $M$=4, $L=2^{15}-1$, $r_PT_C=25000T_C$, $L_P=256$ chips. The channel had exponentially decaying delay power profile [1] and was expressed by

$$E\{\alpha_{q,m}^2\}=\frac{1-e^{-1}}{1-e^{-Q}}e^{-(q-1)}. \quad (23)$$

where the number of resolvable paths $Q=5$. The specular-to-diffuse power ratio of the $q$-th signal path $K_q$ was set at 20dB.

TABLE I

MEAN ACQUISITION TIME OF CONVENTIONAL METHODS

| |
|---|
| **Serial acquisition with full diversity combining** |
| Single-dwell |
| $E\{T_{ACQ}\|\underline{\alpha}_{\mathbf{q,m}}\}=\left(\frac{L+r_PP_{fa}(M)}{1-P_{fa}(M)}\right)T_C$ |
| Double-dwell |
| $E\{T_{ACQ}\|\underline{\alpha}_{\mathbf{q,m}}\}=\left(\frac{(A+1)L+r_PP_{FA}(M)}{P_D(M)}\right)T_C$ |
| **Parallel acquisition with no diversity combining** |
| Single-dwell |
| $E\{T_{ACQ}\|\underline{\alpha}_{\mathbf{q,m}}\}=\left(\frac{L/M+r_PP_{fa}(1)}{1-P_{fa}(1)}\right)T_C$ |
| Double-dwell |
| $E\{T_{ACQ}\|\underline{\alpha}_{\mathbf{q,m}}\}=\left(\frac{(A+1)L/M+r_PP_{FA}(1)}{P_D(1)}\right)T_C$ |

Fig. 4 compares the performances for single-dwell systems. The serial acquisition with full diversity combining performs better than the parallel acquisition with no diversity combining when $E_C/I_0<-12$ dB, but the latter performs better when $E_C/I_0>-12$ dB. As expected, the proposed system performed like the parallel acquisition in high SIR ($E_C/I_0>-10$ dB) and like the serial acquisition with full diversity combining in low SIR. Fig. 5 compares the performances of double-dwell systems with $A=4$ and $B=2$. As in single-dwell systems, the proposed scheme performed like the parallel acquisition in high SIR and like the serial acquisition with full diversity combining in low SIR. Comparing to Fig. 4, the performance gap between the proposed and the serial acquisition with full diversity in low SIR was decreased in the double-dwell case because verification reduced the false alarm probability.

## V. Conclusion

An adaptive acquisition scheme for CDMA systems that employ multiple receiver antennas for diversity

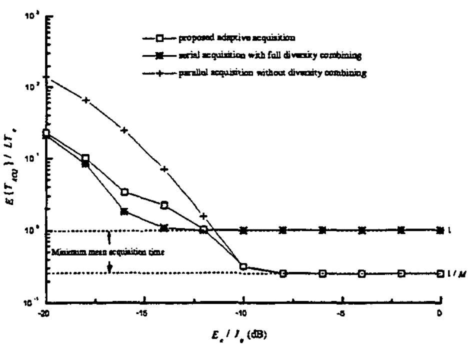


Fig. 4. Performance comparison between the proposed and the conventional methods in an exponentially decaying MIP ($\lambda = 1$) frequency selective multipath channel ($Q = 5$), when single-dwell is considered.

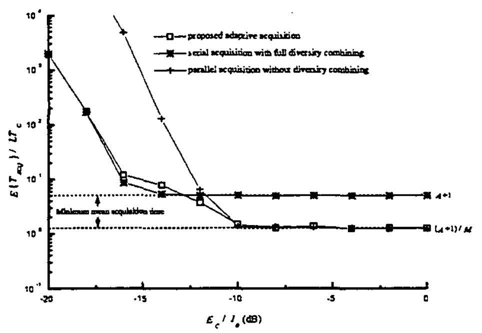


Fig. 5. Performance comparison between the proposed and the conventional methods when double-dwell is considered.

was proposed. It was assumed that each antenna employs a correlator for code acquisition. The proposed scheme adaptively adjusts the degree of diversity combining without explicit knowledge of SIR. Through some performance analysis, it was shown that the proposed method performed like the parallel acquisition in high SIR, and like the serial acquisition with full combining in low SIR.

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