# Adaptive CMMSE Receivers for Space-Time Block Coded MIMO CDMA Systems

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***Abstract*—A technique that can suppress multiple access interference (MAI) in space-time block coded (STBC) multiple-input multiple-output (MIMO) CDMA systems is developed. The proposed scheme, called the constrained minimum mean square error (CMMSE) receiver, is an extension of the method in [4] to MIMO systems. The proposed CMMSE receiver for STBC systems has the identical complexity and rate of convergence with those of CMMSE receiver for SISO system irrespective of the number of transmission antennas. Closed-form expression of signal-to-interference and noise ratio (SINR) of the proposed and conventional maximum likelihood (ML) receivers are derived. As a result, it is shown that the proposed CMMSE receiver can provide significant performance improvement over the existing ML receiver.**

## I. INTRODUCTION

It has been recognized that the capacity of a communication system can be increased by employing a STBC MIMO system, equipped with a maximum likelihood (ML) receiver that optimally combines received signals [1], [2]. In CDMA environment, however, such a system is suffered by MAI, and its performance is degraded rapidly as the number of users increases. This is because the ML receiver treats MAI as additive white Gaussian noise (AWGN). As in the case of conventional CDMA systems, suppression of MAI is important in MIMO CDMA systems.

In this paper we develop an efficient receiver that can suppress MAI for STBC-MIMO CDMA systems. Specifically, the adaptive constrained minimum mean square error (CMMSE) receiver for SISO CDMA [3],[4] is extended to STBC-MIMO systems. A CMMSE optimization problem suitable for STBC is formulated by modifying the cost function for SISO system, and solved by the method of Lagrange multipliers. The proposed receiver determines the transmitted data block after observing all received signals for the corresponding coding block. As the number of transmission antennas increases, the size of coding block also increase. However, the proposed CMMSE receiver for STBC CDMA systems has the identical implemental complexity and rate of convergence with those of CMMSE receiver for SISO system, irrespective of the number of transmission antennas. Computer simulation and statistical analysis indicates that the proposed receiver provides significant performance improvement over the conventional maximum likelihood detector.

## II. SYSTEM MODEL

The communication system considered in the current paper is shown in Fig. 1: it is a MIMO system having $P$ transmission and $M$ receiver antennas operating in frequency selective channels, which is assumed to be quasi-static so that the path gains are constant over a ST coding block and vary from one block to another. The information symbols of the $k$-th user, entering the ST block encoder are grouped into blocks of length $P$, and then each block is converted into a $P$-by-$P$ STBC matrix, denoted by $\mathbf{D}_k(n)$. At each time, $P$ symbols in a column of $\mathbf{D}_k(n)$ are simultaneously spread and then transmitted through $P$ antennas. Specifically, symbols in the $i$th column of $\mathbf{D}_k(n)$, denoted by $\mathbf{d}_k^i(n)$, are spread at time $nP+i-1$ by a normalized spreading code $\mathbf{c}_k = [c_k(1)\ c_k(2)\ \cdots\ c_k(N)]^T$, where $N$ is the numbers of chips per symbol.

The received signal at the $m$th receiver antenna is passed through a chip matched filter and sampled at the chip rate. Under some regulatory conditions in [4] the resulting signal can be expressed as

$$y_m((nP+i-1)N+j) = \sum_{l=1}^{L} \mathbf{h}_{k,ml}(n)\mathbf{d}_k^i(n)c_k(j-l+1) + u_m((nP+i-1)N+j) \qquad (1)$$

where $\mathbf{h}_{k,ml}(n) = [h_{k,1ml}(n)\ \cdots\ h_{k,Pml}(n)]$ and $\{h_{k,pml}|l = 1,\cdots,L\}$ denotes the equivalent impulse response of the quasi-static channel, which is assumed to have $L$ paths, between the $p$th transmission and $m$th receiver antennas and $u_m$ denotes the sum of the intersymbol interference, multiple access interference and background noise. Suppose, without loss of generality, that the 1st user is the user of interest. The CMMSE receiver estimates the transmitted data block $\mathbf{d}_1(n) = [d_{11}(n)\ \cdots\ d_{1P}]^T$ from the received vector $\mathbf{y}_{mi} = [y_m((nP+i-1)N)\ y_m((nP+i-1)N+1)\ \cdots\ y_m((nP+i)N+L-2)]^T$, which can be represented as

$$\mathbf{y}_{mi}(n) = \sum_{l=1}^{L} \mathbf{h}_{1,ml}(n)\mathbf{d}_1^i(n)\cdot\mathbf{c}_{1l} + \mathbf{u}_{mi}(n). \qquad (2)$$

Here $\mathbf{c}_{1l} = [\mathbf{0}_{l-1}^T\ \mathbf{c}_1^T\ \mathbf{0}_{L-l}^T]^T$, $\mathbf{c}_1 = [c_1(0)\cdots c_1(N-1)]^T$, $\mathbf{0}_l$ is a $l$-by-1 vector with all zero elements and $\mathbf{u}_{mi}(n) = [u_m((nP+i-1)N)\ u_m((nP+i-1)N+1)\ \cdots\ u_m((nP+i)N+L-2)]^T$. Note that $\{d_{1i}(n)|i = 1,\cdots,P\}$ are estimated from $\{\mathbf{y}_{mi}(n)|i = 1,\cdots,P\}$. In matrix form $\mathbf{Y}_m(n) =$

 

$[\mathbf{y}_{m1}(n) \cdots \mathbf{y}_{mP}(n)]$ is written as

$$\mathbf{Y}_m(n) = \mathbf{C}_1\mathbf{H}_{1,m}(n)\mathbf{D}_1(n) + \mathbf{U}_m(n). \quad (3)$$

where $\mathbf{C}_1 = [\mathbf{c}_{11} \ \cdots \ \mathbf{c}_{1L}]$, $\mathbf{U}_m(n) = [\mathbf{u}_{m1}(n) \ \cdots \ \mathbf{u}_{mP}(n)]$ and $\mathbf{H}_{1,m}(n)$ is an $L$-by-$P$ matrix whose $(i,j)$-th entry is given by $h_{1,jmi}(n)$. Dropping the time index $n$ and the subscript indicating the desired user for notational simplicity, (2) and (3) are rewritten as

$$\mathbf{y}_{mi} = \sum_{l=1}^{L} \mathbf{h}_{ml}\mathbf{d}^i \cdot \mathbf{c}_l + \mathbf{u}_{mi}, \quad (4)$$

$$\mathbf{Y}_m = \mathbf{C}\mathbf{H}_m\mathbf{D} + \mathbf{U}_m. \quad (5)$$

Next we shall show that the correlation matrices of $\{\mathbf{y}_{mi}\}$ are identical for all $i$. This property is derived under the following assumptions:

- $\{\mathbf{h}_{ml}\}$, $\{\mathbf{d}^i\}$ and $\{\mathbf{u}_{mi}\}$ are statistically independent.
- Channel parameters $\{h_{k,pml}\}$ and the original data before ST coding $\{d_{k,p}\}$ are independent for different values of $k$, $p$, $m$ and $l$. They have zero mean.

Under these assumptions, the correlation matrix of $\mathbf{u}_{mi}$, $\mathbf{R}_{\mathbf{u}_{mi}} = E[\mathbf{u}_{mi}\mathbf{u}_{mi}^H]$, can be written as

$$\mathbf{R}_{\mathbf{u}_{mi}} = \sum_{k=2}^{K}\sum_{l=1}^{L} E\left[\sum_{p=1}^{P} |h_{k,pml}|^2 \mathbf{c}_{kl}\mathbf{c}_{kl}^H\right] + \sigma^2, \quad (6)$$

where $\sigma^2$ is the variance of the background noise. Since the right hand side (RHS) of (6) is not dependent upon $i$, $\mathbf{R}_{\mathbf{u}_{m1}} = \mathbf{R}_{\mathbf{u}_{m2}} = \cdots = \mathbf{R}_{\mathbf{u}_{mP}} \triangleq \mathbf{R}_{\mathbf{u}_m}$. Similarly, the correlation matrix of $\mathbf{y}_{mi}$, $\mathbf{R}_{\mathbf{y}_{mi}} = E[\mathbf{y}_{mi}\mathbf{y}_{mi}^H]$, can be expressed as

$$\mathbf{R}_{\mathbf{y}_{mi}} = \sum_{l=1}^{L} E\left[\sum_{p=1}^{P} |h_{1,pml}|^2 \mathbf{c}_{1l}\mathbf{c}_{1l}^H\right] + \mathbf{R}_{\mathbf{u}_m}, \quad (7)$$

and thus $\mathbf{R}_{\mathbf{y}_{m1}} = \mathbf{R}_{\mathbf{y}_{m2}} = \cdots = \mathbf{R}_{\mathbf{y}_{mP}} \triangleq \mathbf{R}_{\mathbf{y}_m}$.

## III. CMMSE RECEIVER FOR STBC CDMA SYSTEMS

At the receiver, due to ST block coding, it is necessary to determine $\{d_1, \cdots, d_P\}$ simultaneously after observing $\{\mathbf{y}_{mi} | 1 \le m \le M, \ 1 \le i \le P\}$ where $\{d_1, \cdots, d_P\}$ are the original data block before ST block coding. An efficient receiver structure suitable for this simultaneous detection is shown in Fig. 2. Signals entering each antenna are passed through $L$ adaptive CMMSE blocks for filtering MAI of each transmission path, and then filtered signals from all antennas are combined for detecting $\{d_1, \cdots, d_P\}$. In CMMSE blocks, MAIs in $\{\mathbf{y}_{mi}\}$ are suppressed by linear filtering. To be specific, let $\mathbf{w}_{mli}$ be the filter weight for $\mathbf{y}_{mi}$ at the $l$-th resolvable path. Then the filter output is expressed as $\mathbf{w}_{mli}^H\mathbf{y}_{mi}$, and the weight is obtained based on the following constrained optimization:

$$\text{minimize}_{\mathbf{w}_{mli}} E\left[\left|\mathbf{h}_{ml}\mathbf{d}^i - \mathbf{w}_{mli}^H\mathbf{y}_{mi}\right|^2\right] \ \text{subject to } \mathbf{w}_{mli}^H\mathbf{c}_l = 1. \quad (8)$$

This optimization problem is a direct extension of the SISO CMMSE optimization (type 1) in [4] to the MIMO systems. As in the case of SISO, the constraint in (8) prevents $\mathbf{w}_{mli}$ from being a zero vector and allows channel estimation from the outputs of CMMSE filters. Note that the proposed CMMSE block needs $P$ filter weights $\{\mathbf{w}_{ml1}, \cdots, \mathbf{w}_{mlP}\}$ to suppress MAIs of $\{\mathbf{y}_{m1}, \cdots, \mathbf{y}_{mP}\}$ at the $l$-th path of the $m$-th antenna (Fig. 3). However, in practice, it is not necessary to maintain $P$ weights because the optimal weights $\{\mathbf{w}_{mli}^o\}$ are identical for all $i$. Specifically, using the method of Lagrange multipliers, it is possible to show that $\{\mathbf{w}_{mli}^o\}$ minimizing the cost in (8) is given by

$$\begin{aligned}\mathbf{w}_{mli}^o &= \mathbf{R}_{\mathbf{y}_m}^{-1}\mathbf{c}_l(\mathbf{c}_l^H\mathbf{R}_{\mathbf{y}_m}^{-1}\mathbf{c}_l)^{-1} \\ &\triangleq \mathbf{w}_{ml}^o, \end{aligned} \quad (9)$$

where $\mathbf{R}_{\mathbf{y}_m}$ and $\mathbf{R}_{\mathbf{u}_m}$ are given by (7) and (6), respectively. This result and the fact that $\{\mathbf{y}_{mi}, i = 1, \cdots, P\}$ successively[1] enter, not in parallel, the CMMSE block considerably simplify the implementation: as shown in Fig. 4, MAIs of $\{\mathbf{y}_{mi}, i = 1, \cdots, P\}$ can be suppressed by a single filter irrespective of the number of transmission antennas $P$.

The structure in Fig. 1 suggests the following optimization that minimizes the sum of mean square errors (MSEs) associated with $P$ filter outputs $\{\mathbf{w}_{ml}^H\mathbf{y}_{ml}\}$:

$$\text{minimize}_{\mathbf{w}_{ml}} E\left[\sum_{i=1}^{P}\left|\mathbf{h}_{ml}\mathbf{d}^i - \mathbf{w}_{ml}^H\mathbf{y}_{mi}\right|^2\right] \quad \text{subject to } \mathbf{w}_{ml}^H\mathbf{c}_l = 1. \quad (10)$$

The optimal weight for this problem is also given by (9). The adaptation rule can be derived by applying the method of Lagrange multipliers to (10) [4]. The rule for updating the filter weight is given by

$$\mathbf{w}_{ml}(n+1) = \mathbf{w}_{ml}(n) + \mu\sum_{i=1}^{P} e_{mli}^*(n)\mathbf{P}_{\mathbf{c}_l}^{\perp}\mathbf{y}_{mi}(n), \quad (11)$$

where the error signals are given by $e_{mli}(n) = \mathbf{h}_{ml}(n)\mathbf{d}^i(n) - \mathbf{w}_{ml}^H(n)\mathbf{y}_{mi}(n)$, and $\mathbf{P}_{\mathbf{c}_l}^{\perp} = \mathbf{I} - \mathbf{c}_l(\mathbf{c}_l^H\mathbf{c}_l)^{-1}\mathbf{c}_l^H$. In (11) $\mathbf{c}_l$ is served as the initial weight vector. Therefore the initial state of the proposed CMMSE receiver is identical to the conventional ML receiver. This rule updates the weight at every $P$ symbols, yet its rate of convergence is identical to its SISO counterpart that updates the weight at symbol rate. This is true because of the summation of $P$ errors in the RHS of (11). In practice, the error signal in (11) is evaluated using the channel estimates $\hat{\mathbf{h}}_{ml}(n)$ and previous decisions $\hat{\mathbf{d}}^i(n)$. Estimation of channel parameters and derivation of the decision rule are described below.

Suppose that a pilot block is inserted at every $Q_p$-coding blocks. In the receiver, the outputs of the CMMSE filters are multiplied with pilot symbols to remove data dependency. Specifically, at time $jQ_p$, we evaluate

$$b_{pml}(j) = \sum_{i=1}^{P} d_{(p,i)}^*(jQ_p)\mathbf{w}_{ml}^H(jQ_p)\mathbf{y}_{mi}(jQ), \quad (12)$$

[1]There are overlap of $L-1$ chips between $\mathbf{y}_{mi}$ and $\mathbf{y}_{m(i+1)}$.

where $d_{(p,i)}(jQ_p)$ is the $(p,i)$th element of the pilot coding matrix $\mathbf{D}(jQ)$. After some calculation, $b_{pml}(j)$ can be expressed as

$$b_{pml}(j) = \sum_{l'=1}^{L} h_{pml'}(jQ)\mathbf{w}_{ml}^H(jQ_p)\mathbf{c}_{l'} + \eta, \tag{13}$$

where $\eta$ denotes the background noise. Note that the RHS of (13) is independent of the transmitted data. The channel parameters $\{h_{pm1}, \cdots, h_{pmL}\}$ are estimated simultaneously from $\{b_{pml}(j), l = 1, \cdots, L, j = 1, \cdots, N_p\}$, where $N_p$ is a positive integer, in a maximum likelihood (ML) sense under the following assumptions:

- $\{h_{pml}(n)\}$ are fixed during $N_pQ_p$ coding-block period.
- $\{\mathbf{w}_{ml}(n)\}$ are given.
- $\eta$ is Gaussian.

Let $\mathbf{h}_m^p = [h_{pm1} \ \cdots \ h_{pmL}]^T$ and $\mathbf{b}_{pm}(j) = [b_{pm1}(j) \ \cdots \ b_{pmL}(j)]^T$. Then the joint probability density function (pdf) of $\mathbf{b}_{pm}$, $p = 1, \cdots, P$, conditioned on $\mathbf{h}_m^p$ can be written as

$$f(\mathbf{b}_{pm}|\mathbf{h}_m^p) = \prod_{j=1}^{N_p-1} \frac{e^{-(\mathbf{b}_{pm}(n-j)-\mathbf{m}_{pm})\mathbf{C}_{pm}^{-1}(\mathbf{b}_{pm}(n-j)-\mathbf{m}_{pm})}}{\pi^L \cdot \det(\mathbf{C}_{pm})} \tag{14}$$

where $\det(\mathbf{C}_{pm})$ denotes the determinant of a matrix $\mathbf{C}_{pm}$; $\mathbf{m}_{pm}$ and $\mathbf{C}_{pm}$ are the conditional mean and covariance matrix of $\mathbf{b}_{pm}$, respectively. The ML estimator that maximizes the pdf in (14) with respect to $\mathbf{h}_m^p$ is expressed as

$$\hat{\mathbf{h}}_m^p = (\mathbf{W}_m^H\mathbf{C})^{-1} \cdot \frac{1}{N_p}\sum_{j=1}^{N_p} \mathbf{b}_{pm}(j). \tag{15}$$

where $\mathbf{W}_m = [\mathbf{w}_{m1} \ \mathbf{w}_{m2} \ \cdots \ \mathbf{w}_{mL}]$. This estimator is unbiased because $\mathbf{m}_{pm} = E[\mathbf{b}_{pm}|\mathbf{h}_m^p] = \mathbf{W}_m^H\mathbf{C}\mathbf{h}_m^p$.

Finally, in this section, the decision rule is derived. Define a $P$-by-1 vector $\bar{\mathbf{r}}_{ml}$ collecting all CMMSE outputs at the $l$-th path of the $m$-th antenna:

$$\bar{\mathbf{r}}_{ml} = \begin{cases} [\mathbf{w}_{ml}^H\mathbf{y}_{m1} \ (\mathbf{w}_{ml}^H\mathbf{y}_{m2})^*]^T & , P = 2 \\ [\mathbf{w}_{ml}^H\mathbf{y}_{m1} \ \cdots \ \mathbf{w}_{ml}^H\mathbf{y}_{mP}]^T & , P > 2 \end{cases} \tag{16}$$

Then $\bar{\mathbf{r}}_{ml}$ can be expressed as

$$\bar{\mathbf{r}}_{ml} = \widetilde{\mathbf{H}}_{ml}\mathbf{d}^o + \bar{\mathbf{i}}_{ml} + \bar{\mathbf{u}}_{ml} \tag{17}$$

where $\mathbf{d}^o$ consists of the original data $\{d_1, \cdots, d_p\}$ before ST block coding, $\bar{\mathbf{i}}_{ml}$ denotes the interpath interference (IPI) and $\bar{\mathbf{u}}_{ml}$ contains only AWGN. The $P$-by-$P$ matrix $\widetilde{\mathbf{H}}_{ml}$ consists of channel coefficients: when $P = 2$, 4, and 8, for which rate 1 orthogonal ST block codes exist, $\widetilde{\mathbf{H}}_{ml}$ is given by

$$\widetilde{\mathbf{H}}_{ml} = \begin{bmatrix} h_{1ml} & h_{2ml} \\ h_{2ml}^* & -h_{1ml}^* \end{bmatrix}, \tag{18}$$

$$\widetilde{\mathbf{H}}_{ml} = \begin{bmatrix} h_{1ml} & h_{2ml} & h_{3ml} & h_{4ml} \\ h_{2ml} & -h_{1ml} & -h_{4ml} & h_{3ml} \\ h_{3ml} & h_{4ml} & -h_{1ml} & h_{2ml} \\ h_{4ml} & -h_{3ml} & h_{2ml} & -h_{1ml} \end{bmatrix}, \tag{19}$$

$$\widetilde{\mathbf{H}}_{ml} = \begin{bmatrix} h_{1ml} & h_{2ml} & h_{3ml} & h_{4ml} & h_{5ml} & h_{6ml} & h_{7ml} & h_{8ml} \\ h_{2ml} & -h_{1ml} & -h_{4ml} & h_{3ml} & -h_{6ml} & h_{5ml} & h_{8ml} & -h_{7ml} \\ h_{3ml} & h_{4ml} & -h_{1ml} & -h_{2ml} & -h_{7ml} & -h_{8ml} & h_{5ml} & h_{6ml} \\ h_{4ml} & -h_{3ml} & h_{2ml} & -h_{1ml} & -h_{8ml} & h_{7ml} & -h_{6ml} & h_{5ml} \\ h_{5ml} & h_{6ml} & h_{7ml} & h_{8ml} & -h_{1ml} & -h_{2ml} & -h_{3ml} & -h_{4ml} \\ h_{6ml} & -h_{5ml} & h_{8ml} & -h_{7ml} & h_{2ml} & -h_{1ml} & h_{4ml} & -h_{3ml} \\ h_{7ml} & -h_{8ml} & -h_{5ml} & h_{6ml} & h_{3ml} & -h_{4ml} & -h_{1ml} & h_{2ml} \\ h_{8ml} & h_{7ml} & -h_{6ml} & -h_{5ml} & h_{4ml} & h_{3ml} & -h_{2ml} & -h_{1ml} \end{bmatrix} \tag{20}$$

respectively. The $P$-by-1 IPI vector $\bar{\mathbf{i}}_{ml}$ is given by

$$\bar{\mathbf{i}}_{ml} = \begin{cases} \sum_{l'=1,l'\neq l}^{L} \begin{bmatrix} \mathbf{h}_{ml'}\mathbf{d}^1\mathbf{w}_{ml}^H\mathbf{c}_{l'} \\ (\mathbf{h}_{ml'}\mathbf{d}^2)^*\mathbf{c}_{l'}^H\mathbf{w}_{ml} \end{bmatrix} & , P = 2 \\ \sum_{l'=1,l'\neq l}^{L} \widetilde{\mathbf{H}}_{ml'}\mathbf{d}^o \cdot \mathbf{w}_{ml}^H\mathbf{c}_{l'} & , P > 2 \end{cases} \tag{21}$$

Under the assumption that the CMMSE filters completely suppress MAI and the IPI is Gaussian, we obtain the following ML decision rule:

$$\hat{\mathbf{d}}_{ML}^o = \arg\min_{\hat{\mathbf{d}}^o \in \mathcal{D}} \sum_{m=1}^{M} |\bar{\mathbf{r}}_m - \widetilde{\mathbf{H}}_m\hat{\mathbf{d}}^o|^2, \tag{22}$$

where $\mathcal{D}$ is the set of all possible symbol pairs $\{d_1, \cdots, d_P\}$. After some calculation, this rule can be decomposed as $P$ decision rules for determining $\{d_1, \cdots, d_P\}$:

$$\begin{aligned} \hat{d}_{i,ML} &= \arg\min_{\hat{d}} \left| \sum_{m=1}^{M}\sum_{l=1}^{L} \left(\widetilde{\mathbf{H}}_{ml}^H\right)_i \bar{\mathbf{r}}_{ml} - \hat{d} \right| + \\ &\quad \left( \sum_{p=1}^{P}\sum_{m=1}^{M}\sum_{l=1}^{L} |h_{pml}|^2 - 1 \right) \cdot |\hat{d}|^2, \end{aligned} \tag{23}$$

for $i = 1, \cdots, P$. Here $\left(\widetilde{\mathbf{H}}_{ml}^H\right)_i$ is the $i$-th row of $\left(\widetilde{\mathbf{H}}_{ml}^H\right)$ and decision complexity increase linearly with $P$. For constant envelop modulations like PSK, the decision rule in (23) can be further simplified as

$$\hat{d}_{i,ML} = \arg\min_{\hat{d}} \left| \sum_{m=1}^{M}\sum_{l=1}^{L} \left(\widetilde{\mathbf{H}}_{ml}^H\right)_i \bar{\mathbf{r}}_{ml} - \hat{d} \right|. \tag{24}$$

## IV. Average SINR

In this section, we derive the average SINR for the proposed CMMSE receiver based on SINR analysis for SISO system based on the following assumptions:

- $\{h_{pml}\}$,$\{\mathbf{u}_{mi}\}$ and $\{d_i\}$ for $p = 1, \cdots, P$, $m = 1, \cdots, M$, $l = 1, \cdots, L$, $i = 1, \cdots, P$ are mutually independent random variables with zero mean.
- The channel estimation is perfect.
- The adaptive filters are in the steady-state, and the filter weights are identical to the optimal weights in (9).

The decision variable for the proposed receiver is given by

$$\bar{d}_i = \sum_{m=1}^{M}\sum_{l=1}^{L} \left(\widetilde{\mathbf{H}}_{ml}^H\right)_i \bar{\mathbf{r}}_{ml}, \tag{25}$$

for $i = 1, \cdots, P$. Using (17) and $\mathbf{w}_{ml}^H \mathbf{c}_l = 1$, $\bar{d}_i$ can be rewritten as

$$\bar{d}_i = \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{p=1}^{P} |h_{pml}|^2 \cdot d_i + \sum_{m=1}^{M}\sum_{l=1}^{L} (\widetilde{\mathbf{H}}_{ml}^H)_i \cdot \bar{\mathbf{u}}_{ml} + \sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{p=1}^{P} (\widetilde{\mathbf{H}}_{ml}^H)_{(i,p)} \cdot \sum_{l'=1, l' \neq l}^{L} (\widetilde{\mathbf{H}}_{ml'}^H)_p \mathbf{d}^o \cdot \mathbf{w}_{ml}^H \mathbf{c}_{l'}, \quad (26)$$

where $(\widetilde{\mathbf{H}}_{ml}^H)_{(i,p)}$ denotes the $p$-th element of $(\widetilde{\mathbf{H}}_{ml}^H)_i$. The first term in the right-hand-side(RHS) of (26) represents the signal of interest, the second term denotes the MAI and noise and the third term denotes the IPI. The average signal power is given by

$$S_p(\mathbf{h}) = E\left[\left|\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{p=1}^{P} |h_{pml}|^2\right|^2\right]. \quad (27)$$

The interferences and noise power averaged over $h_{pml}$, $\bar{\mathbf{u}}_m$ and $d_p$ can be written as

$$I_p(\mathbf{h}) = \sum_{m=1}^{M}\sum_{l=1}^{L} E\left[\left|\sum_{p=1}^{P} |h_{pml}|^2\right|^2\right] \times \left(\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml} - E\left[\sum_{p=1}^{P} |h_{pml}|^2\right]\right). \quad (28)$$

The average SINR obtained from (27) and (28) is

$$\text{SINR} = \frac{S_p(\mathbf{h})}{I_p(\mathbf{h})}. \quad (29)$$

The SINR of the proposed CMMSE receiver and the conventional ML receiver can be compared by examining $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml}$ in (28): the one with the smaller $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml}$ has the larger SINR. Let $\xi_{1,ml}$ and $\xi_{2,ml}$ denote the $\mathbf{w}_{ml}^H \mathbf{R}_{\mathbf{y}_m} \mathbf{w}_{ml}$ values of CMMSE receiver and conventional ML receiver, respectively, from (9)

$$\xi_{1,ml} = (\mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m}^{-1} \mathbf{c}_l)^{-1}. \quad (30)$$

Similarly, from $\mathbf{w}_{ml} = \mathbf{c}_l$ for conventional ML receiver

$$\xi_{2,ml} = \mathbf{c}_l^H \mathbf{R}_{\mathbf{y}_m} \mathbf{c}_l \quad (31)$$

**Property 1.** $\xi_{2,ml} \geq \xi_{1,ml}$, for all $m, l$ and thus the average SINR of the proposed CMMSE receiver is greater or equal to that of ML receiver.

*Proof:* Appendix A.

## V. Simulation Results

The BER performance of the proposed CMMSE receiver was compared with that of the conventional ML receiver. The conventional ML receiver for multipath CDMA systems can be easily obtained by replacing $\mathbf{w}_{ml}$ of CMMSE receiver with $\mathbf{c}_l$. For simulation, an asynchronous CDMA system with BPSK modulation was considered. 3GPP short scrambling code [5] with a length of 256 ($N$=256) was employed. Rayleigh fading channels with an equal power delay profile and $f_D T$ (normalized doppler frequency) =$10^{-3}$ was assumed and there were 50 active users. To assist in the channel estimation, one pilot coding block was inserted every ten coding blocks ($Q_p = 10$). The parameters $N_p$ for the channel estimators were set at two. Figs. 5 and 6 compare the BER performance of the proposed receiver and the conventional ML receiver for two transmit antennas and various values of $M$, $L$, and $E_b/N_o$. Since our proposed receiver removes MAI effectively, it was observed that the CMMSE receiver outperformed the conventional ML detector in both frequency flat fading and frequency selective fading channels. In general, performance gain obtained by increasing receiver diversity gain is less than the performance gain obtained by suppressing the effect of MAI.

## Appendix

### A. Proof of Property 1

By using the unitary similarity transformation, we may write $\mathbf{R}_{\mathbf{y}_m} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H = \sum_{i=1}^{N+L-1} \lambda_i \mathbf{q}_i \mathbf{q}_i^H$ and $\mathbf{R}_{\mathbf{y}_m}^{-1} = \mathbf{Q}\mathbf{\Lambda}^{-1}\mathbf{Q}^H = \sum_{i=1}^{N+L-1} \frac{1}{\lambda_i} \mathbf{q}_i \mathbf{q}_i^H$, where $\mathbf{Q} = [\mathbf{q}_1 \cdots \mathbf{q}_{N+L-1}]$ is the $(N+L-1)$-by-$(N+L-1)$ matrix whose columns are orthonormal set of eigenvectors, and $\mathbf{\Lambda} = diag(\lambda_1, \cdots, \lambda_{N+L-1})$ is the eigenvalue matrix. Then $\xi_{1,ml}$ and $\xi_{2,ml}$ can be rewritten as

$$\xi_{1,ml} = \left(\sum_{i=1}^{N+L-1} {\lambda_i}^{-1} |\mathbf{c}_l^H \mathbf{q}_i|^2\right)^{-1}, \quad (A.1)$$

$$\xi_{2,ml} = \sum_{i=1}^{N+L-1} \lambda_i |\mathbf{c}_l^H \mathbf{q}_i|^2. \quad (A.2)$$

Using Jensen's inequality [8], we can show $\xi_{2,ml} \geq \xi_{1,ml}$. Because $f(x) = 1/x$ is a convex function for $x > 0$, clearly, $\frac{\sum_{i=1}^{N+L-1} \lambda_i^{-1} |\mathbf{c}_l^H \mathbf{q}_i|^2}{\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2}$ is a convex function. From Jensen's inequality,

$$\frac{\sum_{i=1}^{N+L-1} \lambda_i^{-1} |\mathbf{c}_l^H \mathbf{q}_i|^2}{\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2} \geq \frac{1}{\frac{\sum_{i=1}^{N+L-1} \lambda_i |\mathbf{c}_l^H \mathbf{q}_i|^2}{\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2}} \quad (A.3)$$

Since $\mathbf{c}_l$ is a vector in the space spanned by the orthonormal basis $\{\mathbf{q}_i | i = 1, \cdots, N+L-1\}$ and it's norm is one, $\sum_{i=1}^{N+L-1} |\mathbf{c}_l^H \mathbf{q}_i|^2$ is equal to one and finally we have the following result

$$\sum_{i=1}^{N+L-1} \lambda_i |\mathbf{c}_l^H \mathbf{q}_i|^2 \geq \left(\sum_{i=1}^{N+L-1} {\lambda_i}^{-1} |\mathbf{c}_l^H \mathbf{q}_i|^2\right)^{-1}. \quad (A.4)$$

Therefore $\xi_{2,ml} \geq \xi_{1,ml}$. □

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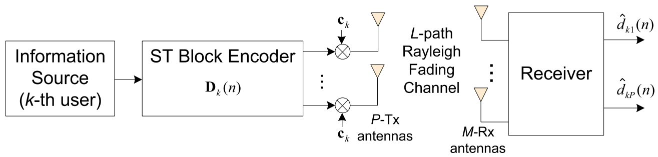


Fig. 1. STBC MIMO system with $P$-Tx and $M$-Rx antennas in multipath fading channels.

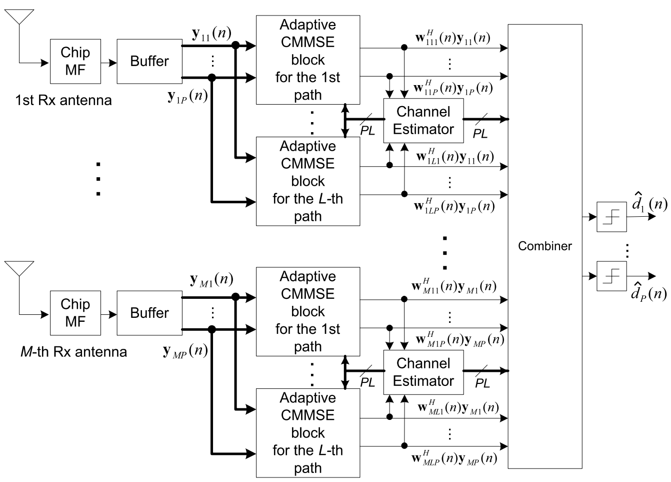


Fig. 2. Adaptive CMMSE receiver for a STBC MIMO CDMA system with $P$ transmitter and $M$ receiver antennas.

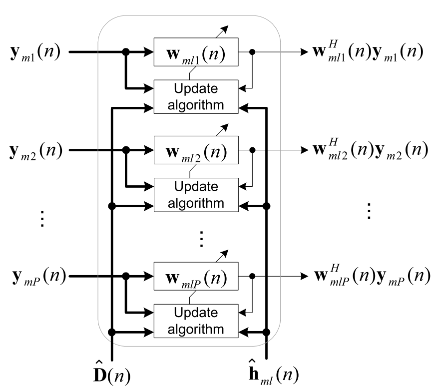


Fig. 3. Conceptual diagram of the adaptive CMMSE block at the $l$-th path of the $m$th antenna.

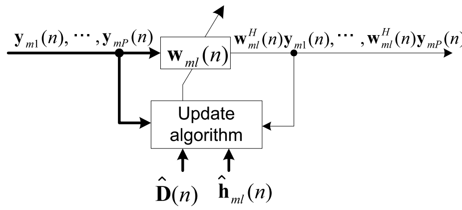


Fig. 4. Practical implementation of adaptive CMMSE block.

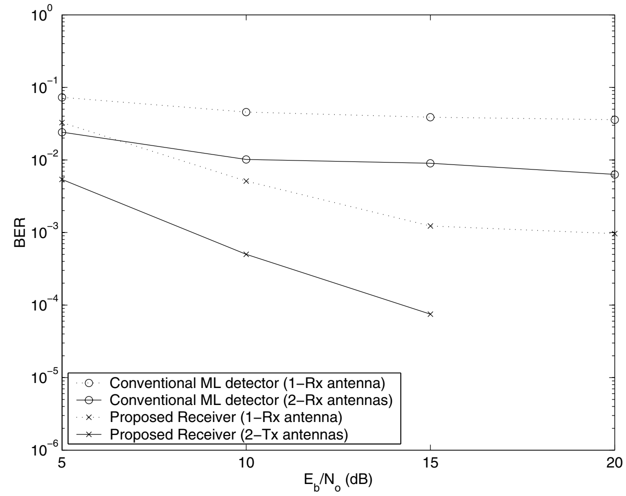


Fig. 5. BER comparison ($N$=256 3GPP short scrambling code, 50 users, $f_D T = 10^{-3}$ frequency flat rayleigh fading, BPSK).

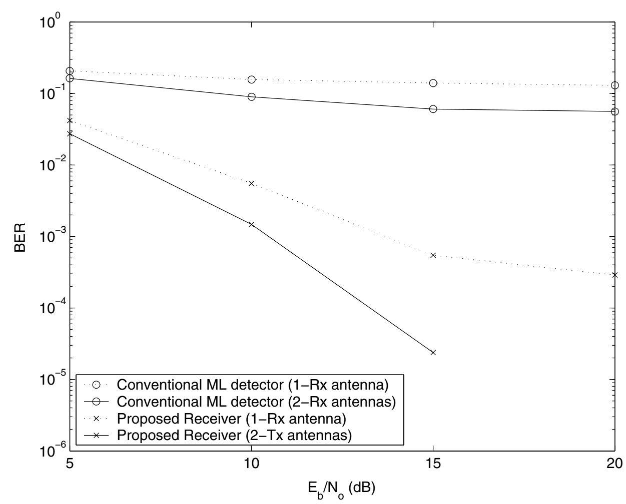


Fig. 6. BER comparison ($N$=256 3GPP short scrambling code, 50 users, $f_D T = 10^{-3}$ 3path frequency selective rayleigh fading, BPSK).