Constrained MMSE-RAKE Receivers for DS/CDMA Systems in Multipath Fading Channels

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Abstract—**The constrained minimum mean square error (CMMSE)-RAKE receivers for multipath fading channels are analyzed. In particular, it is shown that for known channels the output signal-to-interference and noise ratio (SINR) of the CMMSE-RAKE receiver with a single constraint(Type 1) is greater than or equal to that of the CMMSE-RAKE with multiple constraints(Type 2). For Type 1 CMMSE-RAKE receiver, a maximum likelihood (ML) channel estimate is derived. Computer simulation results indicate that CMMSE receivers improve the bit error rate (BER) as compared with the conventional RAKE and currently available adaptive MMSE receivers, and that Type 1 performs the best.**

I. INTRODUCTION

Various adaptive MMSE receivers have been proposed for detection of DS-CDMA systems. For AWGN channels, adaptive MMSE receivers were developed based on the standard MSE cost function [1]-[2]. In the case of flat fading channels, a channel estimator was employed and, in an attempt to improve the tracking capability, the MSE cost was modified [3]-[4]. Furthermore, in [4] a constraint regarding filter coefficients was imposed on the MMSE problem. It was shown that the resulting receiver, called the constrained MMSE (CMMSE) receiver, can outperform the other MMSE receivers. For frequency selective fading channels, use of an adaptive filter for each resolvable transmission path has been suggested, and the receivers in [3]-[4] may be applied to each path. In this case, the receiver performance would be degraded due to interpath interference (IPI), and in [5] a CMMSE receiver with multiple constraints was proposed in an attempt to reduce the effect of IPI.

In this paper, we first analyze two kinds of CMMSE receivers for multipath fading channels. These receivers are RAKE receivers that employ a CMMSE receiver at each finger: the one employs the receiver in [4] which is developed for flat fading channels and the other employs the receiver proposed in [5]. The former, equipped with an ML channel estimator, and the latter are called Type 1 and Type 2 CMMSE receivers, respectively. It is shown that CMMSE receivers perform better than existing RAKE receivers and that Type 1 outperforms the others.

II. SYSTEM MODEL

An asynchronous DS-CDMA system with K users and M propagation paths will be considered. Phase-shift keying (PSK) modulated data of the kth user is spread by the waveform $c_k(t) = \sum_{j=0}^{N-1} c_k(j)p(t - jT_c)$, where N is the number of chips per symbol, T_c is the chip duration, $p(t)$ is the chip pulse shape, and $c_k(j)$ is the jth chip of the kth user's pseudonoise(PN) sequence. Here $c_k(j) \in \{-1/\sqrt{N}, 1/\sqrt{N}\}$ for $j = 0, \cdots, N-1$ and $c_k(j) = 0$, otherwise. The complex envelop of the received signal can be expressed as

$$y(t) = \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} d_k(n)c_k(t - nT) * h_k(t) + \eta(t) \quad (1)$$

where $d_k(n)$ is the nth transmitted data symbol, $*$ denotes convolution, $h_k(t)$ is the impulse response of the kth user's radio channel, and $\eta(t)$ denotes complex zero-mean additive white Gaussian noise. $h_k(t)$ is represented as $h_k(t) = \sum_{m=1}^{M} h_{k,m}(t)\delta(t - \tau_{k,m})$ where $h_{k,m}(t)$ is the complex attenuation factor (or channel coefficient) of the kth user's mth path and $\tau_{k,m}$ is the propagation delay. It is assumed that the attenuation factors are not changed for a symbol duration and are uncorrelated each other. The attenuation factor over the nth symbol duration is denoted by $h_{k,m}(n)$. For simplicity, we also assume that $|\tau_{k,m} - \tau_{k,m'}| = |m - m'|T_c$ for all k and $m \neq m'$. The received signal is rewritten as

$$y(t) = \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} \sum_{m=1}^{M} d_k(n)h_{k,m}(n)c_k(t - nT - \tau_{k,m}) + \eta(t) \quad (2)$$

This signal is passed through a filter matched to the chip pulse shape and sampled at the chip rate to yield a received sequence denoted by $\{y_j\}$. Suppose, without loss of generality, that the 1st user is the user of interest and that $\tau_{1,m} = (m-1)T_c$. The MMSE receivers that we shall consider estimate the nth symbol $d_1(n)$ from $\{y_{nN}, y_{nN+1}, \cdots, y_{(n+1)N+M-2}\}$, where y_{nN+l} is given by

$$y_{nN+l} = \int_{(nN+l)T_c}^{(nN+l+1)T_c} y(t)p(t - nNT_c - lT_c)dt \quad (3)$$

$$= \sum_{m=1}^{M} d_1(n)h_{1,m}(n)c_1(l - (m-1)) + u(nN+l) \quad (4)$$

where $u(n)$ denotes the sum of intersymbol interference, multiple access interference and background noise. In vec-

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tor form,

$$\mathbf{y}(n) = [y_{nN}\ y_{nN+1}\ \cdots\ y_{(n+1)N+M-2}]^T \quad (5)$$

$$= \sum_{m=1}^{M} d_1(n)h_{1,m}(n)\mathbf{c}_{1,m} + \mathbf{u}(n), \quad (6)$$

where $\mathbf{c}_{1,m} = [\mathbf{0}_{m-1}^T\ \mathbf{c}_1^T\ \mathbf{0}_{M-m}^T]^T$, $\mathbf{c}_1 = [c_1(0)\cdots c_1(N-1)]^T$, $\mathbf{0}_m$ is a m-by-1 vector with all zero elements and $\mathbf{u}(n) = [u(nN)\cdots u((n+1)N+M-2)]^T$. Dropping the subscript indicating the desired user for notational simplicity, $\mathbf{y}(n)$ is rewritten as

$$\mathbf{y}(n) = \sum_{m=1}^{M} d(n)h_m(n)\mathbf{c}_m + \mathbf{u}(n) \quad (7)$$

$$= \mathbf{C}\mathbf{h}(n)d(n) + \mathbf{u}(n) \quad (8)$$

where $\mathbf{C} = [\mathbf{c}_1\ \mathbf{c}_2\ \cdots\ \mathbf{c}_M]$ and $\mathbf{h}(n) = [h_1(n)\ h_2(n)\ \cdots\ h_M(n)]^T$.

III. MMSE Receivers in Frequency-Selective Fading Channels

An MMSE-type receiver, proposed in [3], employs an adaptive filter for each resolvable path and combines the resulting outputs. In this receiver, $d(n)$ is estimated by $\hat{d}(n) = \sum_{m=1}^{M} \hat{h}_m^*(n)\mathbf{w}_m^H(n)\mathbf{y}(n)$ where $\hat{h}_m(n)$ denotes channel estimates and the weight for the mth filter $\mathbf{w}_m(n)$ is obtained using an adaptive normalized least-mean-square (NLMS) algorithm. In this architecture, channel estimation needs some caution: it was observed that estimation of $h_m(n)$ from the filter outputs $\{\mathbf{w}_m^H(n)\mathbf{y}(n)\}$ tends to force the weight $\mathbf{w}_m(n)$ to a zero vector [4], and $\hat{h}_m(n)$ are obtained from the input $\mathbf{y}(n)$ in [3]. To overcome the difficulty in channel estimation, in [4] it was suggested to add a constraint to the MMSE problem. Specifically, the filter weights $\{\mathbf{w}_m(n)\}$ are obtained based on the following optimization:

$$\text{minimize}_{\mathbf{w}_m} E[|h_m(n)d(n) - \mathbf{w}_m^H\mathbf{y}(n)|^2] \quad (9)$$
$$\text{subject to } \mathbf{c}_m^H\mathbf{w}_m = 1.$$

The optimal weight, $\mathbf{w}_m^o$, that satisfies (9) is given by

$$\mathbf{w}_m^o = \mathbf{R}_\mathbf{y}^{-1}\mathbf{c}_m(\mathbf{c}_m^H\mathbf{R}_\mathbf{y}^{-1}\mathbf{c}_m)^{-1} \quad (10)$$

where

$$\mathbf{R}_\mathbf{y} = E[\mathbf{y}(n)\mathbf{y}^H(n)]$$
$$= \sum_{m=1}^{M} E[|h_m(n)|^2]\mathbf{c}_m\mathbf{c}_m^H + E[\mathbf{u}(n)\mathbf{u}^H(n)] \quad (11)$$

The adaptive receiver based on (9), which will be referred to as the *basic* CMMSE-RAKE receiver[1], is shown in Fig. 1. The error signal for the mth path is given by

$$e_m(n) = \hat{h}_m(n)d(n) - \mathbf{w}_m^H(n)\mathbf{y}(n) \quad (12)$$

[1] In [4], a CMMSE receiver for flat fading channels was proposed. The *basic* CMMSE-RAKE receiver is a direct extension of the CMMSE receiver to multipath fading channels.

and the filter weight is updated using

$$\mathbf{w}_m(n+1) = \mathbf{c}_m + \mathbf{P}_{\mathbf{c}_m}^{\perp}(\mathbf{w}_m(n) + \mu(n)e_m^*(n)\mathbf{y}(n)) \quad (13)$$

where $\mathbf{P}_{\mathbf{c}_m}^{\perp} = \mathbf{I} - \mathbf{c}_m(\mathbf{c}_m^H\mathbf{c}_m)^{-1}\mathbf{c}_m^H$ and $\mu(n) = \frac{\rho}{\mathbf{y}^H(n)\mathbf{y}(n)}$ $(0 < \rho < 1)$. The channel is estimated by exploiting pilot symbols under the assumption that $\{h_m(n)\}$ are quasi-constant. If a pilot symbol is inserted at every q-symbols, then the channel estimate is given by

$$\hat{h}_m(n) = \frac{1}{N_p}\sum_{i=0}^{N_p-1} d^*((\nu-i)q)\mathbf{w}_m^H((\nu-i)q)\mathbf{y}((\nu-i)q) \quad (14)$$

where $\nu = \lfloor \frac{n}{q} \rfloor$, $d((\nu-i)q)$ is the pilot and N_p is a positive integer. In section VI, it will be shown through computer simulations that the basic CMMSE-RAKE receiver can outperform the receiver in [3]. Next we derive a property of this CMMSE-RAKE receiver.

Property 1: The channel estimate in (14) is biased in multipath fading channels ($M \geq 2$). It is unbiased only for flat fading channels ($M = 1$).

Proof: Let $h_m(n) = h_m(n+1) = \cdots = h_m(n + N_p \cdot q - 1) = h_m$. Then from (14),

$$E[\hat{h}_m|h_m] = \frac{1}{N_p}\sum_{i=0}^{N_p-1} h_m E[|d((\nu-i)q)|^2]\mathbf{w}_m^H((\nu-i)q)\mathbf{c}_m$$
$$+ \frac{1}{N_p}\sum_{i=0}^{N_p-1}\sum_{\substack{m'=1 \\ m'\neq m}}^{M} h_{m'}E[|d((\nu-i)q)|^2]\mathbf{w}_m^H((\nu-i)q)\mathbf{c}_{m'}$$
$$+ \frac{1}{N_p}\sum_{i=0}^{N_p-1} \mathbf{w}_m^H((\nu-i)q)E[d^*((\nu-i)q)\mathbf{u}((\nu-i)q)] \quad (15)$$
$$= h_m + \frac{1}{N_p}\sum_{i=0}^{N_p-1}\sum_{\substack{m'=1 \\ m'\neq m}}^{M} h_{m'}\mathbf{w}_m^H((\nu-i)q)\mathbf{c}_{m'} \quad (16)$$

where (15) reduces to (16) since $|d(i)|^2 = 1, \mathbf{w}_m^H(i)\mathbf{c}_m = 1$ and the data of each user are uncorrelated. In (16), the second term does not vanish unless $M = 1$, because in general $\mathbf{w}_m^H(i)\mathbf{c}_{m'} \neq 0$ for $m \neq m'$. ■

The bias of the channel estimator should degrade the receiver performance. Two techniques that can remove the bias and improve the performance will be presented.

IV. CMMSE-RAKE Receivers with Unbiased Channel Estimates

Two types of CMMSE-RAKE receivers, which will be referred to as Type 1 and Type 2 receivers, are introduced. It is shown that the channel estimates of these receivers are unbiased.

A. Type 1 Receiver

Type 1 receiver is identical to the basic CMMSE-RAKE receiver with the exception that it employs an unbiased channel estimator instead of the one in (14). In this receiver, all channels $\{h_1(n), \cdots, h_m(n)\}$ are estimated simultaneously from the set of signals $\{d^*(iq)\mathbf{w}_m^H(iq)\mathbf{y}(iq),\ m = 1, \cdots, M\}$(Fig. 2). Here $d(iq)$ are pilot symbols which are multiplied with the filter outputs to remove data dependency—the resulting signal $d^*(iq)\mathbf{w}_m^H(iq)\mathbf{y}(iq)$ can be modeled as a Gaussian process [6]. The channels are estimated under the following assumptions: $h_m(nq) = h_m(nq-1) = \cdots = h_m((n-N_p+1)\cdot q) = h_m$ for $m = 1, \cdots, M$ and the weight vectors are in the steady-state, i.e., $\mathbf{w}_m(nq) = \mathbf{w}_m(nq-1) = \cdots = \mathbf{w}_m((n-N_p+1)\cdot q) = \mathbf{w}_m$ for $m = 1, \cdots, M$. We define an M-by-1 observation vector by $\mathbf{v}(i) = d^*(iq)[\mathbf{w}_1^H\mathbf{y}(iq)\ \mathbf{w}_2^H\mathbf{y}(iq)\ \ldots\ \mathbf{w}_M^H\mathbf{y}(iq)]^T$ and assume that $\{\mathbf{v}(n)\ \mathbf{v}(n-1)\ \cdots\ \mathbf{v}(n-N_p+1)\}$ are given for estimating $\mathbf{h} = [h_1\ h_2\ \cdots h_M]^T$. Then the joint probability density function (pdf) of $\{\mathbf{v}(n)\ \mathbf{v}(n-1)\ \cdots\ \mathbf{v}(n-N_p+1)\}$ conditioned on $\mathbf{h}$ can be written as

$$f(\mathbf{v}|\mathbf{h}) = \prod_{i=0}^{N_p-1} \frac{e^{-(\mathbf{v}(n-i)-\mathbf{m_v})^H \mathbf{C_v}^{-1}(\mathbf{v}(n-i)-\mathbf{m_v})}}{\pi^M \cdot \det(\mathbf{C_v})} \quad (17)$$

where $\det(\mathbf{C_v})$ denotes the determinant of a matrix $\mathbf{C_v}$; $\mathbf{m_v}$ and $\mathbf{C_v}$ are the conditional mean and covariance matrix of $\mathbf{v}$, respectively, which are given by

$$\begin{aligned} \mathbf{m_v} &= E[\mathbf{v}(i)|\mathbf{h}] \\ &= \mathbf{Bh}, \end{aligned} \quad (18)$$

where $\mathbf{B} \triangleq [\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_M]^H\mathbf{C}$, $\mathbf{C_v} = E[(\mathbf{v}(i) - \mathbf{m_v})(\mathbf{v}(i)-\mathbf{m_v})^H|\mathbf{h}] = [\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_M]^H\ \mathbf{R_u}[\mathbf{w}_1\ \mathbf{w}_2 \cdots \mathbf{w}_M]$ and $\mathbf{R_u} = E[\mathbf{u}(n)\mathbf{u}^H(n)]$. The ML estimator that maximizes the pdf in (17) with respect to $\mathbf{h}$ is expressed as

$$\hat{\mathbf{h}} = \mathbf{B}^{-1} \cdot \frac{1}{N_p}\sum_{i=1}^{N_p} \mathbf{v}(i). \quad (19)$$

In general, the matrix $\mathbf{B}$ is nonsingular. This holds because $\{\mathbf{w}_1, \cdots, \mathbf{w}_M\}$, which are determined independently at the fingers, are linearly independent in most cases. Unbiasedness of the ML estimate in (19) follows from (18).

B. Type 2 Receiver

Type 2 receiver is proposed in [5]. It is based on the observation that the bias of the channel estimates in (14) is removed if $\mathbf{w}_m(n)$ is chosen to satisfy $\mathbf{w}_m^H(n)\mathbf{c}_{m'} = 0$ for all $m' \neq m$ (see (16)). Type 2 receiver is derived by adding such constraints to the optimization problem in (9). Specifically, this receiver is based on the following optimization:

$$\begin{aligned} &\text{minimize}_{\mathbf{w}_m} E\left[|h_m(n)d(n) - \mathbf{w}_m^H\mathbf{y}(n)|^2\right] \\ &\qquad \text{subject to } \mathbf{C}^H\mathbf{w}_m = \mathbf{g}_m, \end{aligned} \quad (20)$$

where $\mathbf{C}$ is defined in (8) and $\mathbf{g}_m$ is an M-by-1 vector whose mth element is one and the rest are zeros. The optimal weight, $\mathbf{w}_m^o$, that satisfies (20) is given by

$$\mathbf{w}_m^o = \mathbf{R_y}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R_y}^{-1}\mathbf{C})^{-1}\mathbf{g}_m. \quad (21)$$

The structure of Type 2 receiver is identical to the one in Fig 1. Type 2 receiver employs the channel estimate in (14), like the conventional CMMSE-RAKE receiver, but in this case the estimate becomes unbiased owing to the multiple constraints in (20). The weight of Type 2 is updated by

$$\mathbf{w}_m(n+1) = \mathbf{r}_m + \mathbf{P}_\mathbf{C}^\perp(\mathbf{w}_m(n) + \mu(n)e_m^*(n)\mathbf{y}(n)) \quad (22)$$

where $\mathbf{r}_m = \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}_m$, $\mathbf{P}_\mathbf{C}^\perp = \mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$ and $\mu(n) = \frac{\rho}{\mathbf{y}^H(n)\mathbf{y}(n)} (0 < \rho < 1)$.

V. AVERAGE SINR ANALYSIS

In this section, we derive the average SINR under the following assumptions:

- $\{h_m(n), m = 1, \cdots, M\}$, $\{\mathbf{u}(n)\}$ and $\{d(n)\}$ are independent random variables with zero mean. They are mutually independent as well.
- The channel estimation is perfect — in this case, Type 1 receiver becomes identical to the basic CMMSE-RAKE receiver.
- The adaptive filters are in the steady-state, and the filter weights are identical to the optimal weights in (10) and (21).

Referring to Fig. 1 and 2, the decision variable of Type 1 and 2 receivers is given by $\hat{d}(n) = \sum_{m=1}^M h_m^*(n)\mathbf{w}_m^H\mathbf{y}(n)$. Using (7) and $\mathbf{w}_m^H\mathbf{c}_m = 1$, $\hat{d}(n)$ can be rewritten as

$$\begin{aligned} \hat{d}(n) &= \sum_{m=1}^M |h_m(n)|^2 d(n) \\ &+ \sum_{m=1}^M h_m^*(n)\mathbf{w}_m^H\Big(\sum_{\substack{j\neq m\\ j=1}}^M \mathbf{c}_j h_j(n)d(n) + \mathbf{u}(n)\Big) \end{aligned} \quad (23)$$

The first term in the right-hand-side (RHS) of (23) represents the signal of interest and the second term denotes the interference and noise. The average signal power is given by

$$S_p(\mathbf{h}) = E\Big[\Big|\sum_{m=1}^M |h_m(n)|^2\Big|^2\Big]. \quad (24)$$

The interference and noise power averaged over $\mathbf{h}$, $\mathbf{u}(n)$ and $d(n)$ can be written as

$$\begin{aligned} I_p(\mathbf{h}) &= E\Big[\Big|\sum_{m=1}^M h_m^*(n)\mathbf{w}_m^H\Big(\sum_{\substack{j\neq m\\ j=1}}^M \mathbf{c}_j h_j(n)d(n) + \mathbf{u}(n)\Big)\Big|^2\Big] \\ &= \sum_{m=1}^M E\Big[|h_m(n)|^2\Big]\mathbf{w}_m^H\Big\{\sum_{\substack{j\neq m\\ j=1}}^M E\Big[|h_j(n)|^2\Big]\mathbf{c}_j\mathbf{c}_j^H + \mathbf{R_u}\Big\}\mathbf{w}_m \end{aligned} \quad (25)$$

where the second equality comes from the independence assumptions. The average SINR obtained from (24) and (25) is

$$\text{SINR} = \frac{E[|\sum_{m=1}^M |h_m(n)|^2|^2]}{\sum_{m=1}^M E[|h_m(n)|^2](\mathbf{w}_m^H\mathbf{R_y}\mathbf{w}_m - E[|h_m(n)|^2])} \quad (26)$$

where $\mathbf{R_y}$ is defined in (11). Equation (26) indicates that SINRs of Type 1 and Type 2 receivers can be compared by examining $\mathbf{w}_m^H \mathbf{R_y} \mathbf{w}_m$: the one with a smaller $\mathbf{w}_m^H \mathbf{R_y} \mathbf{w}_m$ has a large SINR. Let $\xi_{1,m}$ and $\xi_{2,m}$ denote the $\mathbf{w}_m^H \mathbf{R_y} \mathbf{w}_m$ values of Type 1 and Type 2 receivers, respectively. From (10) and (21),

$$\xi_{1,m} = (\mathbf{c}_m^H \mathbf{R_y}^{-1} \mathbf{c}_m)^{-1} \quad (27)$$

$$\xi_{2,m} = \mathbf{g}_m^H (\mathbf{C}^H \mathbf{R_y}^{-1} \mathbf{C})^{-1} \mathbf{g}_m \quad (28)$$

and $\xi_{2,m}$ is the (m, m)-th element of $(\mathbf{C}^H \mathbf{R_y}^{-1} \mathbf{C})^{-1}$. The following lemma is useful for comparing $\xi_{1,m}$ and $\xi_{2,m}$.

Lemma 1: For any positive definite Hermitian matrix $\mathbf{A}$,

$$[\mathbf{A}^{-1}]_{(i,i)} \geq \{[\mathbf{A}]_{(i,i)}\}^{-1}. \quad (29)$$

where $[\cdot]_{(i,i)}$ denotes the (i, i)th element of the matrix. The proof of this lemma is presented in Appendix A.

Property 2: $\xi_{2,m} \geq \xi_{1,m}$ for all m, and thus the average SINR of Type 1 receiver is greater than or equal to that of Type 2 receiver. The average SINRs are the same when M=1.

Proof: Let $\mathbf{A} = \mathbf{C}^H \mathbf{R_y}^{-1} \mathbf{C}$, then $\xi_{1,m} = \{[\mathbf{A}]_{(m,m)}\}^{-1}$, $\xi_{2,m} = [\mathbf{A}^{-1}]_{(m,m)}$, and Lemma 1 indicates that $\xi_{2,m} \geq \xi_{1,m}$. When M=1, (28) becomes identical to (27), and thus the average SINRs are the same. ■

The performance degradation of Type 2 receiver is caused by the reduction of degrees of freedom in choosing the weights. Due to the M constraints in Type 2 receiver, the weights of Type 2 receiver are optimized on a $(N-1)$-dimensional vector space, while those of Type 1 receiver are optimized on a $(N + M - 2)$-dimensional space.

VI. Simulation Results

An asynchronous CDMA system with BPSK modulation was considered. Gold code with length 31 was used for spreading and the data rate was 64kbps. A frequency selective Rayleigh fading channel with an exponentially decaying power delay profile was assumed. The carrier frequency was 2.0GHz (r.m.s. delay spread was assumed to be $1\mu sec$). Transmitted powers of all active users were idectical. The channel parameters $\{h_m(n)\}$ were generated following the approach in [8]. Five CDMA receivers were compared: they were the conventional RAKE, linear MMSE (LMMSE)-RAKE[3], the basic CMMSE-RAKE, Type 1 and 2 receivers. The pilot symbol assisted technique was employed for channel estimation and one pilot symbol was inserted at every ten data symbols (q=10). For channel estimation, the parameter N_p in (14) and (19) was set at three (N_p=3) which was the value exhibiting the best performance.

Fig. 3 shows the BER when K=3, E_b/N_o=15dB, and the number of multipaths (M) increases from 1 to 10. Type 1 receiver offered significant performance improvement over the other receivers. Type 2 outperformed the others when $M \leq 8$, but its performance was almost identical to that of the basic CMMSE receiver when M=9 and 10. The performance degradation of Type 2 for large M was caused by excessive constraints. Similar observations in comparing BER performances can be made from Fig. 4 and 5 which show the BER values when E_b/N_o varies from 2 to 24dB. The CMMSE receivers performed better than the others. The performance gain of Type 1 over the basic CMMSE receiver was about 1dB and 2dB when M=3 (Fig. 4) and M=9 (Fig. 5, BER=10^{-3}), respectively. The behavior of Type 2 was better than the basic CMMSE receiver when M=3, but the performances of the two receivers became almost the same for all values of E_b/N_o when M=9.

Appendix

A. Proof of Lemma 1

To prove Lemma 1, we need the following lemmas.

Lemma A.1: Let $\alpha_1 \leq \alpha_2 \leq \cdots \leq \alpha_N, \beta_1 \leq \beta_2 \leq \cdots \leq \beta_N$ and $\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_N$ be eigenvalues of the Hermitian matrices $\mathbf{Q}, \mathbf{T}$ and $\mathbf{Z} = \mathbf{Q} + \mathbf{T}$. Then, $\alpha_i + \beta_1 \leq \gamma_i \leq \alpha_i + \beta_N$, for $i = 1, 2, \cdots, N$

The proof of this lemma can be found in [9, p 315].

Lemma A.2: Let $\mathbf{A}$ be a positive definite Hermitian matrix, and a denotes the (1,1)-th element of $\mathbf{A}$. Then the (1,1)-th element of $\mathbf{A}^{-1}$, denoted by $[\mathbf{A}^{-1}]_{(1,1)}$, is always greater than or equal to a^{-1}.

Proof: Let $\mathbf{A} = \begin{bmatrix} a & \mathbf{b}^H \\ \mathbf{b} & \mathbf{\Phi} \end{bmatrix}$. Then, $[\mathbf{A}^{-1}]_{(1,1)} - a^{-1} = \det(\mathbf{\Phi})/\det(\mathbf{A}) - 1/a = (a \cdot \det(\mathbf{\Phi}) - \det(\mathbf{A}))/(a \cdot \det(\mathbf{A}))$. Since $a > 0$ and $\mathbf{A}$ is positive definite, $a \cdot \det(\mathbf{A}) > 0$ in the denominator. By Schur complement [7], $\det(\mathbf{A}) = \det(a)\det(\mathbf{\Phi} - \mathbf{b}a^{-1}\mathbf{b}^H)$. Thus it is sufficient to show that $\det(\mathbf{\Phi}) - \det(\mathbf{\Phi} - a^{-1}\mathbf{b}\mathbf{b}^H) \geq 0$. Let the eigenvalues of $\mathbf{\Phi}$ and $\frac{\mathbf{b}\mathbf{b}^H}{a}$, respectively, be $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ and $\zeta_1 \leq \zeta_2 \leq \cdots \leq \zeta_N$ (in fact, $\zeta_N = \frac{\mathbf{b}^H\mathbf{b}}{a}$ and $\zeta_i = 0$ otherwise). Then, $\det(\mathbf{\Phi}) = \prod_{i=1}^N \lambda_i$ and

$$\det(\mathbf{\Phi}) - \det(\mathbf{\Phi} - a^{-1}\mathbf{b}\mathbf{b}^H) \geq \prod_{i=1}^N \lambda_i - \prod_{i=1}^N (\lambda_i - \zeta_1) \quad (A.1)$$

$$= 0, \quad (A.2)$$

where the first inequality comes from Lemma A.1. In (A.1), the equality holds when $\mathbf{b} = \mathbf{0}$. This completes the proof. ■

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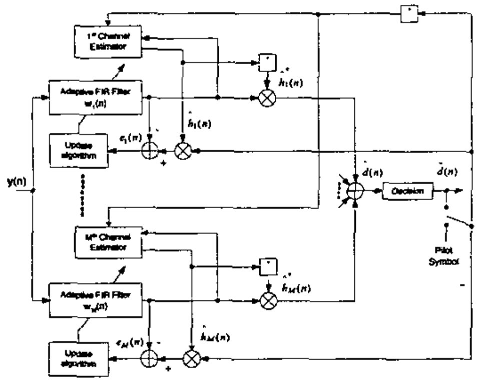

Fig. 1. The basic CMMSE-RAKE receiver.

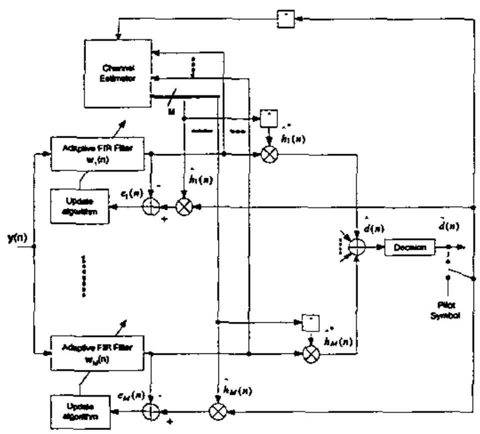

Fig. 2. The structure of the proposed receiver.

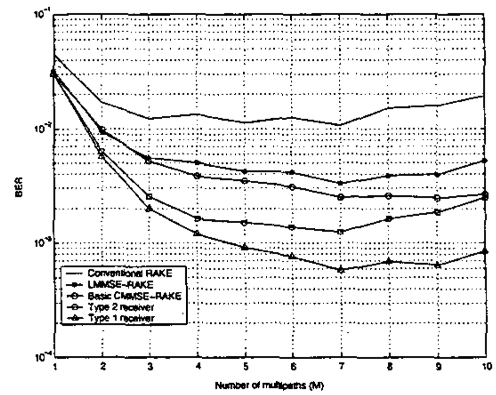

Fig. 3. BER versus $M(K{=}3, E_b/N_o = 15\text{dB})$.

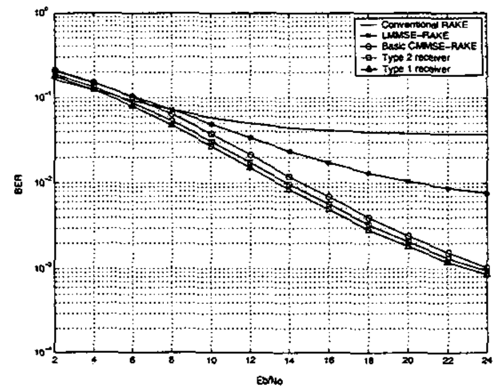

Fig. 4. BER versus $E_b/N_o(K{=}6, M{=}3)$.

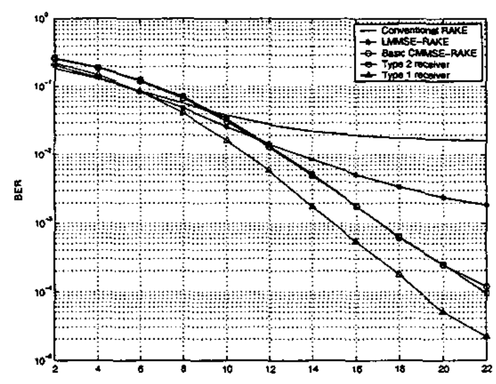

Fig. 5. BER versus $E_b/N_o(K{=}3, M{=}9)$.