DESIGN OF POWERS-OF-TWO COEFFICIENT FIR FILTERS WITH MINIMUM ARITHMETIC COMPLEXITY

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ABSTRACT

A mixed-integer-linear-programming (MILP)-based algorithm is proposed for designing canonic-signed-digit (CSD) FIR filters with minimal complexity. The proposed algorithm reduces the number of variables in MILP through certain modifications of the scheme in [2]. Design examples demonstrate that the proposed algorithm can design a CSD FIR filter with minimum complexity and requires less computation than the existing MILP algorithm.

1. INTRODUCTION

Recently, in [1] and [2] new methods were presented for designing multiplierless FIR filters with CSD coefficients, represented by sums of signed-powers-of-two (SPT) terms. In contrast to the conventional schemes in [3]-[5], which try to design a CSD FIR filter with an optimum frequency response, these new methods focus on minimizing the implementation complexity for a given frequency response specification. They are suboptimal algorithms, which are based on a trellis search [1] and MILP [2].

The current paper proposes an alternative approach to the design of CSD FIR filters with minimal complexity. In particular, a suboptimal algorithm based on MILP is developed by modifying the scheme in [2]. The advantages of the proposed algorithm over existing ones are demonstrated through various design examples.

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2. PROPOSED ALGORITHM

Consider the design of a linear phase FIR filter with CSD coefficients. Suppose, for the sake of illustration, that the impulse response $\{h_{csd}(n), n = 0, 1, \ldots, 2N - 1\}$ is symmetric and its length is even $(2N)$. The frequency response is given by (omitting the the linear phase term $\exp(-j(N-1)\omega/2)$)

$$H(\omega) = 2\sum_{n=0}^{N-1} h_{csd}(N-1-n)\cos(n+\frac{1}{2})\omega. \quad (1)$$

The coefficient $h_{csd}(n)$ is expressed as a sum of SPT terms:

$$h_{csd}(n) = \sum_{i=1}^{M} s_i(n)2^{-i} \quad (2)$$

where $s_i(n) \in \{-1, 0, 1\}$ and M denotes the wordlength. The number of SPT terms in $h_{csd}(n)$ is given by $\sum_{i=1}^{M} |s_i(n)|$. The objective of the CSD FIR filter design is to determine $\{s_i(n)\}$ such that the total number of SPT terms $2\sum_{n=0}^{N-1}\sum_{i=1}^{M} |s_i(n)|$ is minimized, while $H(\omega)$ in (1) satisfies the desired frequency response specification. The design problem is described

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as follows:

$$\text{Minimize} \quad J = \sum_{n=0}^{N-1} \sum_{i=1}^{M} |s_i(n)| \tag{3}$$

subject to

$$\begin{array}{ll} |H(\omega) - 1| \leq \delta(\omega) & \text{when } \omega \in \text{passband} \\ |H(\omega)| \leq \delta(\omega) & \text{when } \omega \in \text{stopband} \\ \sum_{i=1}^{M} |s_i(n)| \leq L_{\max} & \text{for } n = 0, 1, \ldots, N-1 \\ -1 \leq s_i(n) \leq 1 & \text{for integer } s_i(n). \end{array}$$

Here $\delta(\omega)$ denotes the maximum allowable error and $L_{\max}$ is the maximum number of SPT terms that can be assigned to each $h_{\text{csd}}(n)$. This problem can be solved by MILP, treating $s_i(n)$ as variables—the number of variables is NM. In the conventional scheme [3] used to design CSD FIR filters, the coefficients $\{h_{\text{csd}}(n)\}$ are treated as variables, thus there are N variables in the MILP. As a consequence, the MILP associated with (3) requires considerably heavier computation than the MILP in [3]. Accordingly, to alleviate this difficulty, in [2] only D least significant digits (LSDs) are considered during optimization by replacing the objective function in (3) with

$$J = \sum_{n=0}^{N-1} \sum_{i=M-D+1}^{M} |s_i(n)|. \tag{4}$$

This is justified by the fact that the most significant digits (MSDs) of $h_{\text{csd}}(n)$ from (3) are usually identical to those of the CSD coefficients, $\text{csd}[h_{\text{d}}(n)]$, which are obtained by quantizing the infinite-precision optimal filter coefficients, $h_{\text{d}}(n)$. However, the probability that the MSD values of $h_{\text{csd}}(n)$ are different from those of $\text{csd}[h_{\text{d}}(n)]$ may not be negligible when $|h_{\text{d}}(n)|$ is large. Therefore, instead of the constant D in (4), it is desirable to employ a variable $D(n)$, which is determined depending on $|h_{\text{d}}(n)|$. A simple way of achieving this effect is to add the following constraint to the optimization in (3):

$$\begin{array}{ll} |h_{\text{csd}}(n) - h_{\text{d}}(n)| \leq \alpha |h_{\text{d}}(n)|, & \text{if } |h_{\text{d}}(n)| \geq \frac{2^{-M+\beta}}{1+\alpha} \\ |h_{\text{csd}}(n)| < 2^{-M+\beta}, & \text{otherwise} \end{array} \tag{5}$$

where α and β are positive constants. As such, the proposed algorithm is described by (3) and (5). It attains $\{h_{\text{d}}(n)\}$ in the initial stage and then searches for $\{h_{\text{csd}}(n)\}$ in the neighborhood of $\{h_{\text{d}}(n)\}$. Due to (5), the search range is adaptively adjusted depending on $|h_{\text{d}}(n)|$.

3. DESIGN EXAMPLES

To compare the proposed algorithm with existing ones, the design examples in [1] were considered. Specifically, lowpass and halfband filters were designed using the proposed MILP described by (3) and (5), the optimal MILP in (3), conventional MILP [2], and the trellis-based algorithm [1]. The normalized peak ripple (NPR)[1] and minimum number of SPT terms, N_{SPT} needed for the filter implementation were compared. In addition, for the MILP algorithms, the number of variables in the MILP, N_{MILP}, was examined.

Example 1 (Lowpass Filter). The normalized passband and stopband edge frequencies were 0.15 and 0.25, respectively, with an equal weighting on the passband and stopband ripples. The filter length and word-length were: $2N = 28$ and $M = 12$, respectively. The desired NPR was -50 dB. Table 1 summarizes the results. When the maximum allowed number of SPT terms per coefficient $L_{\max}$ was limited to three, the filter designed by the optimal MILP required 56 SPT terms. Thus the minimum value of N_{SPT} in this case was 56. The proposed and conventional MILP achieved the minimum

[1] NPR is given by $\delta_{NPR} = max\{\delta_p w_p/g, \delta_s w_s/g\}$ where g is the average passband gain and δ_p, δ_s, w_p, and w_s are the passband and stopband peak ripples and error weighting, respectively.

N_{SPT} when ($\alpha = 0.3, \beta = 2$) and $D = 10$, respectively. The filter designed using the trellis-based algorithm required four additional SPT terms compared to the filter designed using the optimal MILP. Among the MILP algorithms that achieved the minimum N_{SPT}, the proposed algorithm required considerably less computation than the others. This was because N_{MILP} for the proposed MILP was 98, while for the others it was 168 and 140.

When $L_{\max} = 4$, the MILP in (3) could not be solved due to the excessive computational load. However, the proposed and conventional MILPs were completed. The filters designed by these algorithms required four fewer SPT terms than the trellis-based algorithm. The N_{MILP} values for the proposed ($\alpha = 0.1875, \beta = 3$) and conventional ($D = 8$) MILP were 99 and 112, respectively, thus the former required less computation than the latter.

Example 2 (Halfband Filter). The normalized passband and stopband edge frequencies were 0.1 and 0.4, respectively, with an equal weighting on the passband and stopband ripples. The filter length and wordlength were 15 and 14, respectively. The desired NPR was -80 dB and $L_{\max} = 4$. The results are shown in Table 2. The minimum N_{SPT} was 20, which was achieved by the optimal, proposed ($\alpha = 0.0125, \beta = 3$), and conventional ($D = 11$) MILP algorithms. Among these algorithms, the proposed MILP required the least computation because its N_{MILP} was 48, while the N_{MILP} values for the others were 70 and 55. The filter designed using the trellis-based algorithm required three additional SPT terms compared to the filter designed using the optimal MILP.

4. CONCLUSION

An alternative approach to the design of CSD FIR filters with minimal complexity was developed by modifying the MILP algorithm in [2]. Various design examples confirmed that the proposed algorithm could design a CSD FIR filter with minimum complexity and required less computation than the existing MILP algorithm.

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Table 1: Performance Comparison for Example 1

	NPR (dB)	N_{SPT}	N_{MILP}
$L_{\max} = 3$			
Optimal (3)	-50.01	56	168
Proposed (3) & (5)	-50.01	56	98 ($\alpha = 0.3, \beta = 2$)
Conventional [2]	-50.04	62	126 ($D = 9$)
	-50.01	56	140 ($D = 10$)
Trellis [1]	-50.12	60	
$L_{\max} = 4$			
Optimal (3)	-	-	168
Proposed (3) & (5)	-50.01	54	99 ($\alpha = 0.1875, \beta = 3$)
Conventional [2]	-50.10	56	98 ($D = 7$)
	-50.01	54	112 ($D = 8$)
Trellis [1]	-50.07	58	

Table 2: Performance Comparison for Example 2

	NPR (dB)	N_{SPT}	N_{MILP}
Optimal (3)	-84.89	20	70
Proposed (3) & (5)	-84.89	20	48 ($\alpha = 0.0125, \beta = 3$)
Conventional [2]	-80.12	23	50 ($D = 10$)
	-84.89	20	55 ($D = 11$)
Trellis [1]	-85.62	23	