Adaptive Modulation for MIMO Systems with V-BLAST Detection

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Abstract—**Adaptive modulation for multiple-input multiple-output (MIMO) systems with zero forcing (ZF) V-BLAST is considered. Motivated by the observation that the detection ordering of the original V-BLAST algorithm, called the *forward* ordering, is not suitable for adaptive modulation, a new detection ordering for V-BLAST-based adaptive modulation is proposed. This ordering, called the *reverse* ordering, selects the symbol corresponding to the minimum gain at each iteration, in contrast to the forward ordering which always gives priority to the maximum gain. The advantage of the reverse ordering over the forward ordering is demonstrated through analysis and simulation. It is shown that the reverse ordering can provide about 2dB gain as compared with the forward ordering and almost comparable to the optimal ordering which is found through an exhaustive search. The V-BLAST based adaptive modulation with reverse ordering performs somewhat worse than the singular value decomposition (SVD) based method [1], but the former is shown to be considerably simpler to implement than the latter.**

I. Introduction

Recently, adaptive modulation techniques have been proposed for MIMO systems in [1]-[3]. It is shown in [1] that MIMO channels can be transformed into parallel flat fading channels through singular value decomposition (SVD), and that conventional bit loading techniques such as the greedy algorithm [4]-[5] can be applied after such transformation. Although this approach can achieve a theoretical bound, it needs heavy computation for obtaining the singular matrices and evaluating SVD-based transformations. In frequency-division duplexing (FDD) mode, in which the channel information is not available at the transmitter, the SVD based adaptive modulation is also suffered by the abundance of information to be transferred from the receiver. As an alternative to the SVD-based approach, [2]-[3] propose the use of the V-BLAST algorithm for both adaptive modulation and detection: the method in [2] employs the conventional forward ordering that selects a symbol corresponding to the maximum gain at each iteration of detection procedure, and the one in [3] suggests that use of the optimal ordering that can be found through an exhaustive search. It is observed that the optimal ordering can provide a considerable performance gain over the forward ordering. However, implementing the V-BLAST with the optimal ordering is difficult due to the exhaustive search for finding such an ordering.

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In this paper, we propose the use of a new ordering policy, called the reverse ordering, for V-BLAST based adaptive modulation and detection. It is shown through some analysis and experiments that the proposed reverse ordering can provide significant performance improvement, and act like the optimal ordering. The performance and complexity of the V-BLAST based method with reverse ordering is compared with the SVD based method. The results indicate that the proposed scheme can be a useful alternative to the optimal method.

II. V-BLAST based Adaptive Modulation and Detection

Fig. 1 shows the V-BLAST based adaptive modulator and demodulator. The system has N_T transmitter antennas and N_R receiver antennas ($N_T \leq N_R$). Given the channel state information at the transmitter, the adaptive modulator determines the number of bits (constellation size) and power levels to be transmitted through each antenna. The receiver employs an adaptive V-BLAST detector which is essentially identical to the V-BLAST scheme with the exception that decision for each symbol is adaptively made depending on the constellation size and transmission power.

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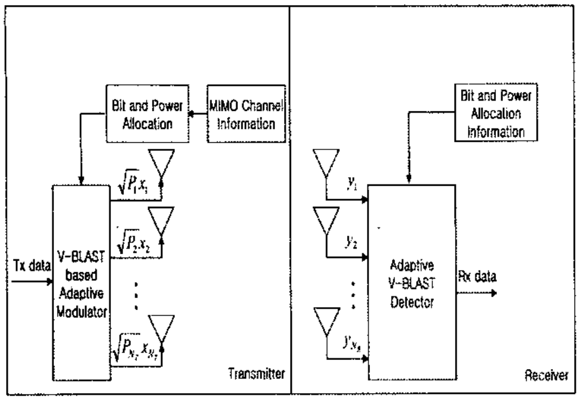

Fig. 1. V-BLAST based adaptive modulation and detection.

To be specific, let $\mathbf{x} = [x_1, x_2, \cdots, x_{N_T}]^T$ denote the transmitted symbol vector in which each symbol has a unit average power. The received signal vector can be written in matrix form as

$$\mathbf{y} = \mathbf{HPx} + \mathbf{v} \tag{1}$$

where $\mathbf{y} = [y_1, y_2, \cdots, y_{N_R}]^T$; $\mathbf{H}$ is an N_R-by-N_T channel matrix representing flat fading channels; $\mathbf{P} = \mathrm{diag}[\sqrt{P_1}, \sqrt{P_2}, \cdots, \sqrt{P_{N_T}}]$ where P_i denotes the transmission power at the ith antenna; and $\mathbf{v} = [v_1, v_2, \cdots, v_{N_R}]^T$ is a zero-mean Gaussian vector with covariance matrix $E[\mathbf{v}\mathbf{v}^H] = \sigma^2\mathbf{I}_{N_R}$.

To detect $\mathbf{x}$ from $\mathbf{y}$, the V-BLAST algorithm performs iterative nulling and cancellation: at each iteration the optimal order of detection (or nulling and cancellation) is determined and nulling vectors are computed. Let x_{k_i} be the symbol which is detected at the i-th iteration where $k_i \in \{1, 2, \cdots N_T\}$ and the permutation $[k_1, k_2, \cdots, k_{N_T}]$ represents the order of detection. When the zero-forcing (ZF) criterion is used for nulling, the decision statistic z_{k_i} for determining x_{k_i} can be written as

$$z_{k_i} = \mathbf{w}_{k_i}^H \mathbf{y} = \sqrt{P_{k_i}} x_{k_i} + \mathbf{w}_{k_i}^H \mathbf{v} \tag{2}$$

where $\mathbf{w}_{k_i}^H$ is the nulling vector. The decision is made by considering the constellation size of x_{k_i}. Now to explain the V-BLAST based adaptive modulation, we normalize both sides of (2) by $||\mathbf{w}_{k_i}||$. This yields

$$z_{k_i}' = \frac{\sqrt{P_{k_i}} x_{k_i}}{||\mathbf{w}_{k_i}||} + \mathbf{v}' \tag{3}$$

where $z_{k_i}' = z_{k_i}/||\mathbf{w}_{k_i}||$ and $\mathbf{v}' = \mathbf{w}_{k_i}^H\mathbf{v}/||\mathbf{w}_{k_i}||$ with $E[\mathbf{v}'\mathbf{v}'^H] = \sigma^2\mathbf{I}_{N_R}$. (3) indicates that the inverse of the normalization factor $||\mathbf{w}_{k_i}||$ can be thought of as a channel gain between the transmitted symbol x_{k_i} and the decision statistic z_{k_i}'. The ZF V-BLAST yields a total of N_T such gains $\{||w_{k_1}||^{-1}, ||w_{k_2}||^{-1}, \cdots, ||w_{k_{N_T}}||^{-1}\}$, which will be referred to as the *equivalent* gains. The V-BLAST based adaptive modulation utilizes the equivalent gains for bit loading and power allocation: since the channel matrix $\mathbf{H}$ is known at the transmitter, then all equivalent gains $\{||\mathbf{w}_{k_i}||^{-1}\}$ can be evaluated and an adaptive modulator can be designed based on the equivalent gains, using the greedy algorithm. The V-BLAST based adaptive modulation and detection algorithm is summarized below.

Transmitter: Adaptive modulation

- Given the channel matrix $\mathbf{H}$, evaluate equivalent gains (inverse of nulling vector norms) using ZF V-BLAST algorithm.
- Allocate bit and power based on the equivalent gains, using the greedy algorithm.

Receiver: Detection

- Given $\mathbf{H}$, $\mathbf{y}$, constellation sizes and transmission powers, evaluate the nulling vectors and make decisions using the ZF V-BLAST.

The performance of the V-BLAST based adaptive modulation, which has been originally proposed in [2] and [3], varies depending on the order $[k_1, k_2, \cdots, k_{N_T}]$ for evaluating equivalent gains. In [2], it is suggested to follow the detection order of the V-BLAST algorithm: at the i-th iteration, k_i is given by

$$k_i = \arg\min_{j \in S_i} ||\mathbf{w}_{i,j}|| \tag{4}$$

where $S_i = \{1, 2, \cdots, N_T\} - \{k_1, k_2, \cdots, k_{i-1}\}$ and $\mathbf{w}_{i,j}$ is the nulling vector for x_j at the i-th iteration (S_i is a set collecting indices of the symbols to be detected from the i-th iteration). The ordering in (4), which will be referred to as the *forward* ordering, chooses the symbol corresponding to the maximum equivalent gain at each iteration. [3] proposes to follow the optimal ordering that exhibits the best performance among $N_T!$ possible orderings; however, finding such an ordering requires an exhaustive search, and thus its implementation is difficult. Next, we propose an alternative ordering policy called the *reverse* ordering.

III. Reverse Ordering for V-BLAST Based Adaptive Modulation

For adaptive modulation, it is desirable to maximize the equivalent gains— this will increase the number of bits that

TABLE I.
$\{||\mathbf{w}_{i,j}||^{-1}\}$ AND EQUIVALENT GAINS FOR FORWARD ORDERING

	x_1	x_2	x_3
1st iteration $\|\|\mathbf{w}_{1,j}\|\|^{-1}$	0.457	0.355	**0.678**
2nd iteration $\|\|\mathbf{w}_{2,j}\|\|^{-1}$	**1.031**	0.505	-
3rd iteration $\|\|\mathbf{w}_{3,j}\|\|^{-1}$	-	**1.136**	-
equivalent gains	1.031	1.136	0.678

TABLE II.
$\{||\mathbf{w}_{i,j}||^{-1}\}$ AND EQUIVALENT GAINS FOR REVERSE ORDERING

	x_1	x_2	x_3
1st iteration $\|\|\mathbf{w}_{1,j}\|\|^{-1}$	0.457	**0.355**	0.678
2nd iteration $\|\|\mathbf{w}_{2,j}\|\|^{-1}$	1.143	-	**0.967**
3rd iteration $\|\|\mathbf{w}_{3,j}\|\|^{-1}$	**2.325**	-	-
equivalent gains	2.325	0.355	0.967

can be loaded. A property that is useful for increasing the equivalent gains is as follows: if j is an element of both S_i and S_{i+1}, then

$$||\mathbf{w}_{i,j}||^{-1} \leqq ||\mathbf{w}_{i+1,j}||^{-1} \tag{5}$$

where S_i and $||\mathbf{w}_{i,j}||$ are defined in (4). The inequality in (5), which is shown in [6], suggests to keep larger $||\mathbf{w}_{i,j}||^{-1}$ values at the i-th iteration so that they can be increased further at the $(i+1)$-th iteration. This leads to the following ordering policy: at the i-th iteration, k_i is given by

$$k_i = \arg\max_{j \in S_i} ||\mathbf{w}_{i,j}||. \tag{6}$$

The ordering in (6) is called the reverse ordering. It selects the symbol corresponding to the minimum equivalent gains at each iteration and as a result, tends to increase the equivalent gains. As an example, consider a MIMO-system with the following channel matrix:

$$\mathbf{H} = \begin{bmatrix} 1.2 & 0.5 & 1.5 \\ 1.5 & 1.0 & 0.4 \\ 1.3 & 0.2 & 1.2 \end{bmatrix}. \tag{7}$$

Tables I and II show the corresponding $\{||\mathbf{w}_{i,j}||^{-1}\}$ and equivalent gains for forward and reverse orderings, respectively. It is seen that the maximum and average equivalent gains associated with the reverse ordering are considerably greater than those from the forward ordering.

The advantage of the reverse ordering over the forward ordering can also be shown analytically. In what follows, it will be proved that the reverse ordering results in a larger average equivalent gain than the forward ordering when $N_T = 2$ and 3. This analysis can be extended for $N_T = 4$, but there are a few exceptions—some cases for which forward ordering yields a larger average equivalent gain will be observed. To proceed, we define the sum of equivalent gains corresponding to an order $[k_1, k_2, \cdots, k_{N_T}]$:

$$J(k_1, k_2, \cdots, k_{N_T}) = \sum_{i=1}^{N_T} ||\mathbf{w}_{k_i}||^{-1}. \tag{8}$$

The lemma stated below are useful for comparing the forward and reverse orderings.

Lemma 1. Let F denote a permutation representing the forward ordering. Define a new permutation F' by moving one integer in F from its original position to a position in the left-hand-side (LHS) of the original position. Then,

$$J(F) \leq J(F'). \tag{9}$$

The proof of this lemma is lengthy, and thus it is omitted. For example, if $N_T = 3$ and $F = [1,2,3]$, then F' can be one of $\{[2,1,3],[3,1,2],[1,3,2]\}$. The inequality in (9) indicates that there always exists at least one ordering yielding a larger average equivalent gain than the forward ordering.

Lemma 2. Let R denote a permutation representing the reverse ordering. Define a new permutation R' by moving one integer in R from its original position to a position in the LHS of the original position. Then,

$$J(R) \geq J(R'). \tag{10}$$

This lemma can be proved by modifying the proof of Lemma 1. (10) confirms the efficiency of the reverse ordering. Using the two lemmas, $J(F)$ and $J(R)$ are compared.

Observation 1. For $N_T = 2$ and 3, $J(F) \leq J(R)$.

Proof. When $N_T = 2$, this observation is a direct consequence of Lemma 1. Let $N_T = 3$ and, without loss of generality, $F = [1,2,3]$. Then R can be one of $\{[2,1,3],[2,3,1],[3,1,2],[3,2,1]\}$ encompassing all permutations starting with either 2 or 3. When R is either $[2,1,3]$ or $[3,1,2]$, the proof is completed via Lemma 1. If $R = [2,3,1]$, then $J(R) \geq J(2,1,3) \geq J(1,2,3) = J(F)$ where 1st and 2nd inequalities result from Lemmas 2 and 1,

TABLE III.
EMPIRICAL MEAN VALUES OF $J(F)$ AND $J(R)$ FOR COMPLEX GAUSSIAN CHANNELS

N_T	2	3	4	8
mean of $J(F)$	2.635	4.932	8.061	28.5
mean of $J(R)$	3.368	6.988	11.78	42.57

respectively. The case with $R = [3, 2, 1]$ can be proved in a similar manner. ■

Observation 2. Let $N_T = 4$. For a given forward ordering, there are 18 candidate permutations for reverse ordering. The inequality $J(R) \geq J(F)$ is true for 14 permutations out of the 18 candidates. If we let $F = [1, 2, 3, 4]$, then the four permutations for which $J(R) \geq J(F)$ is not always true are $\{[3, 4, 1, 2], [3, 4, 2, 1], [4, 3, 1, 2], [4, 3, 2, 1]\}$.

The proof of this observation is rather cumbersome and is omitted. For the four permutations in Observation 2, it is possible to find channels (through some computer search) for which $J(F) \geq J(R)$. For example,

$$\mathbf{H} = \begin{bmatrix} 0.85 & 1.41 & 2.0 & 1.81 \\ 0.20 & 0.80 & 0.73 & 0.78 \\ 0.51 & 0.09 & 0.45 & 0.73 \\ 0.71 & 0.54 & 0.73 & 0.68 \end{bmatrix}. \tag{11}$$

results in $F = [1, 2, 3, 4]$, $R = [3, 4, 1, 2]$, $J(F) = 5.25$ and $J(R) = 3.56$. In spite of these exceptions, $J(R) \geq J(F)$ generally holds. Next this statement is confirmed through computer simulation.

The mean values of $J(F)$ and $J(R)$ were obtained empirically by generating $10,000$ $\mathbf{H}$ matrices ($N_T = N_R$) consisting of zero-mean complex Gaussian random variables. The results are shown in Table III. As expected, the mean values of $J(R)$ were greater than those of $J(F)$.

IV. Performance Evaluation

The performances of the V-BLAST based adaptive modulations are examined through computer simulation and compared with the SVD based scheme. In the simulation, the following parameters were assumed: $N_T = N_R = 4$; QAM-constellation set $\{0, 4, 16\}$ was used for carrying zero, two, and four bits/symbol; and the channel was complex Gaussian random variables with zero-mean. The bit error rate (BER) curves were empirically estimated through $10,000$ trials. The results are shown in Fig. 2. The proposed reverse ordering provided almost 2dB gain as compared with the forward ordering, and was almost comparable to the optimal ordering. Comparing the V-BLAST based methods with the SVD based method, the former exhibited at least 1dB loss. However, the former is considerably simpler to implement than the latter, as shown in the following section.

V. Complexity for Implementing Adaptive Modulation and Detection

Suppose that the channel matrix $\mathbf{H}$ is known at both the transmitter and receiver, as in some time-division duplexing (TDD) systems. Then adaptive modulation and detection can be independently processed at the transmitter and the receiver, respectively, without interchanging information. For this case, the number of complex multiplications and additions required by the adaptive modulation and detection is summarized in Table IV (the V-BLAST and SVD assume the algorithm in [8] and Golub-Reinsch algorithm in [9], respectively). The V-BLAST based method needs considerably less computation than the SVD based scheme. The latter needs $6N_T^3$ operations for SVD and, in addition, at the transmitter it requires computations that are proportional to the frame size for premultiplication of the left singular matrix.

When $\mathbf{H}$ is known only at the receiver, as in some FDD systems, it is desirable to evaluate the V-BLAST and SVD at the receiver and transfer the information which are necessary for adaptive modulation to the transmitter. The computational loads and the amount of information to be

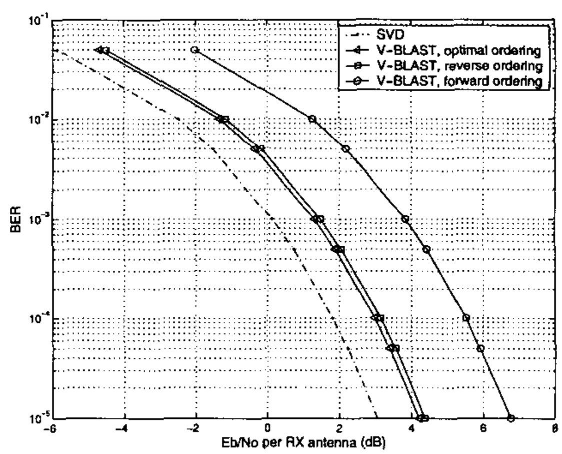

Fig. 2. Performance comparison.

TABLE IV.

COMPUTATIONAL LOADS WHEN $\mathbf{H}$ IS KNOWN AT BOTH THE TRANSMITTER AND RECEIVER. HERE N IS THE LENGTH OF A DATA FRAME

		V-BLAST	SVD
Complex Multiplications	Transmitter	$7N_T^3/3$	$6N_T^3 + NN_T^2$
	Receiver	$7N_T^3/3$	$6N_T^3$
Complex Additions	Transmitter	$5N_T^3/3$	$6N_T^3 + NN_T(N_T-1)$
	Receiver	$5N_T^3/3$	$6N_T^3$

TABLE V.

COMPUTATIONAL LOADS AND THE AMOUNT OF INFORMATION TO BE TRANSFERRED FROM THE RECEIVER TO THE TRANSMITTER WHEN $\mathbf{H}$ IS KNOWN ONLY AT THE RECEIVER

		V-BLAST	SVD
Complex Multiplications	Transmitter	-	NN_T^2
	Receiver	$7N_T^3/3$	$21N_T^3/2$
Complex Additions	Transmitter	-	$NN_T(N_T-1)$
	Receiver	$5N_T^3/3$	$21N_T^3/2$
Information from Rx to Tx		$2N_T$ samples/frame	$(2N_T + N_T^2)$ samples/frame

transferred in this case are summarized in Table V. The V-BLAST based method does not need any computation for adaptive modulation at the transmitter, because all necessary information for adaptive modulation (constellation sizes and transmission powers) can be evaluated at the receiver. In this case, $2N_T$ samples/frame should be delivered for adaptive modulation. The SVD based adaptive modulation requires calculations for premultiplying at the transmitter, and needs full SVD computations ($21N_T^3/2$) at the receiver. The amount of information to be transferred for adaptive modulation is $(2N_T + N_T^2)$ samples/frame that represent singular values and the left singular matrix. These results demonstrate that the V-BLAST based method is considerably simpler to implement than the SVD based method.

VI. CONCLUSION

In the paper, we proposed a new ordering policy, which is called reverse ordering, for V-BLAST based adaptive modulation. Analytical and simulation results showed that the proposed scheme outperforms the method in [2] and is comparable to the optimal ordering in [3] which is found through exhaustive search. The proposed V-BLAST based adaptive modulation exhibited about 1dB loss as compared with the SVD based method, but the simplicity of the former in implementation makes it a useful alternative to the latter in practical applications.

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