Dynamic Subchannel and Bit Allocation in multiuser MIMO/OFDMA Systems

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***Abstract*— Adaptive subchannel allocation and modulation scheme is developed for multiuser multiple-input multiple-output (MIMO)/orthogonal frequency division multiple access (OFDMA) systems through some modification of the approach for OFDMA systems. In the multiuser MIMO/OFDMA system, there are SDMA subchannels, in which each user occupies a single-user MIMO subchannel, for each OFDM subcarrier. An optimal procedure for allocating the SDMA subchannels to users and loading bits to single-user MIMO subchannels is formulated as an integer programming (IP) problem. Then a suboptimal algorithm that separately performs subchannel allocation and bit loading is proposed. Computer simulation results indicate that the performance of the suboptimal algorithm is reasonably close to that of the optimal IP.**

I. Introduction

Recently, it is shown that a multiuser MIMO downlink system can be decomposed into multiple parallel independent single-user MIMO downlink systems [1],[2]. Combined with the orthogonal frequency division multiple access (OFDMA) technique, it becomes the multiuser MIMO/OFDMA system in which the users are spatially and spectrally multiplexed (SDMA, OFDMA) [3]-[5] and each user receives spatially multiplexed bit streams (MIMO) [6]-[8]. In the multiuser MIMO/OFDMA system, there are SDMA subchannels, in which each user occupies a single-user MIMO subchannel, for each OFDM subcarrier (Fig. 1). Allocating the SDMA subchannels to each user and loading the bits to the single-user MIMO subchannels for adaptive modulation are important design issues of the multiuser MIMO/OFDMA.

In this paper, we develop adaptive subchannel allocation and bit loading scheme for the multiuser MIMO/OFDMA system through some modification of the approach for OFDMA systems in [9]. It is shown that the optimal subchannel allocation and bit loading problem, which minimizes the overall transmit power under the data rate constraint, can be solved by integer programming (IP). A suboptimal algorithm that separately performs subchannel allocation and bit loading is also proposed. Using computer simulation, we demonstrate that the performance of the suboptimal algorithm is reasonably close to that of the optimal IP.

II. Multiuser MIMO/OFDMA System

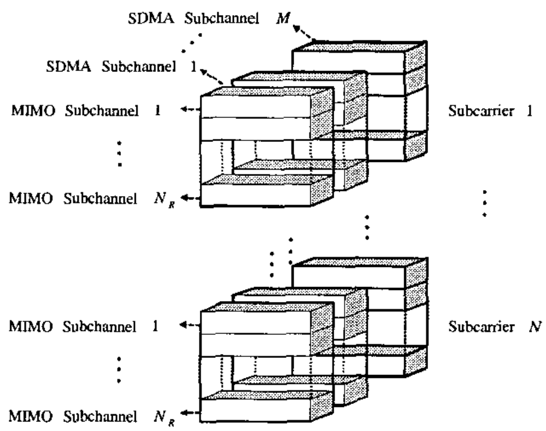

Fig. 1. Subchannels of the multiuser MIMO/OFDMA.

Fig. 2 shows the block diagram of the multiuser MIMO/OFDMA base station with N_T transmitter antennas and N subcarriers. We assume that the channel is flat fading. The base station receives downlink channel information from all K users, and using this information, it assigns a set of subchannels to each user and determines the number of bits to be transmitted through each subchannel. The subchannel and bit allocation information is sent to the mobile users via a separate control channel. When every user unit has N_R receiver antennas and $N_T = M \cdot N_R$, where M is an integer which is greater than or equal to two, M different users occupy M SDMA subchannels per each subcarrier and MIMO processing provides each user with N_R spatially multiplexed bit streams, which are referred to as MIMO subchannels. To be specific, let Ω_n be the set of indices of the M users who occupy the M SDMA subchannels of the n-th subcarrier. Then Ω_n consists of M integers chosen from $\{1, 2, \cdots, K\}$. We define an $N_R \times 1$ vector $X_{n,k}$, given by

$$X_{n,k} = \begin{cases} \text{information symbol vector if } k \in \Omega_n \\ 0, \text{ otherwise.} \end{cases} \quad (1)$$

This vector is multiplied with the $N_T \times N_R$ precoding matrix $F_{n,k}$ (of course, such multiplication is unnecessary when $X_{n,k} = 0$) to yield $S_{n,k} = F_{n,k} X_{n,k}$, and $\sum_{k \in \Omega_n} S_{n,k}$ is transmitted over the n-th subcarrier. By properly choosing

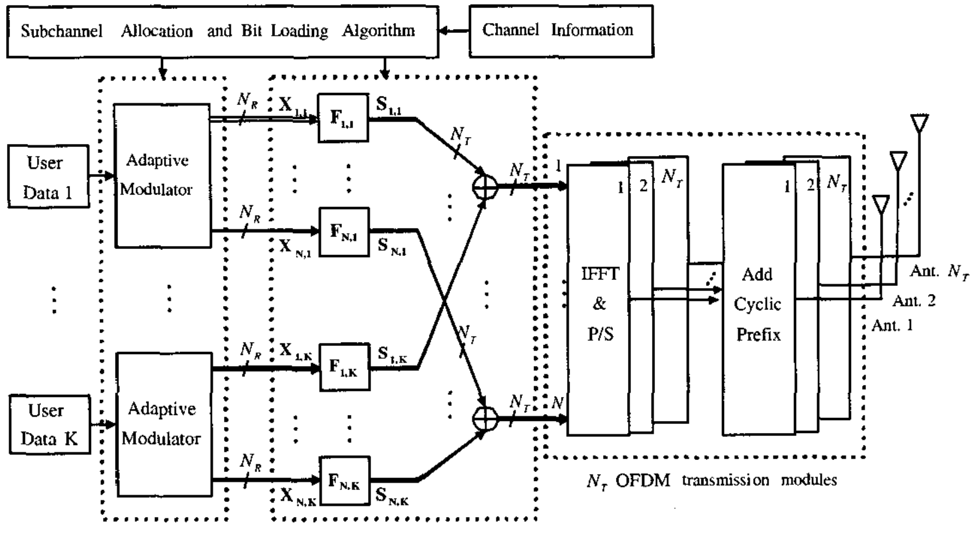

Fig. 2. Block diagram of the multiuser MIMO/OFDMA base station.

$F_{n,k}$, as shown in [1] and [2], the multiuser interference in $\sum_{k\in\Omega_n} S_{n,k}$ can be eliminated and each $S_{n,k}$, $k \in \Omega_n$ can be recovered at the receiver. Such an $F_{n,k}$ provides M SDMA subchannels in which each user occupies a single-user MIMO subchannel that enables the user to spatially multiplex N_R data streams. The transmitter sends the data of K users through a total of $M \cdot N \cdot N_R$ subchannels under the following conditions.

(C.1) The M SDMA subchannels per each subcarrier should be assigned to M different users.

(C.2) All N_R MIMO subchannels associated with an SDMA subchannel should be assigned to a user who occupies the SDMA subchannel.

Next, the optimal subchannel and bit allocation problem is formulated and solved by IP.

III. IP-BASED OPTIMAL SUBCHANNEL AND BIT ALLOCATION

A. Formulation

For simplicity, it is assumed that the number of SDMA subchannels for each OFDM subcarrier is equal to two (M=2)[1]. The data rate of the k-th user, R_k, can be expressed as

$$R_k = \sum_{n=1}^{N}\sum_{l=1}^{N_R}\sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C} c \cdot \rho_k(n,l,p,c) \quad (2)$$

$$\triangleq \sum_{n=1}^{N}\sum_{l=1}^{N_R} c_k(n,l) \quad (3)$$

where C is the maximum number of bits/symbol that can be loaded to a subchannel; $\rho_k(n,l,p,c)$ is equal to one if the k-th and p-th users occupy the two SDMA subchannels of the n-th subcarrier and c bits are loaded to the l-th MIMO subchannel associated with the SDMA subchannel of the k-th user; it is zero, otherwise. In (3), $c_k(n,l)$ represents the number of bits assigned to the k-th user's l-th MIMO subchannel when the k-th user occupies one of the SDMA subchannels of the n-th subcarrier. The transmission power allocated to the k-th user is expressed as

$$P_k = \sum_{n=1}^{N}\sum_{l=1}^{N_R}\sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C} \frac{f_k(c)}{\alpha_k^2(n,l,p)} \cdot \rho_k(n,l,p,c) \quad (4)$$

where $f_k(c)$ is the required received power for reliable reception of c bits/symbol when channel gain is equal to unity. $\alpha_k(n,l,p)$ is the equivalent channel gain[2] of the k-th user's l-th MIMO subchannel when the k-th and p-th users occupy the two SDMA subchannels of the n-th subcarrier. Without loss of generality, it is assumed that the MIMO subchannels are ordered depending on their equivalent channel gains, i.e. $|\alpha_k(n,1,p)| \geq |\alpha_k(n,2,p)|$ for any k, n, and p.

The optimal subchannel allocation and bit loading problem, which minimizes the total transmission power required for transmitting the data with rate $\{R_1,\ldots,R_K\}$, is formulated as follows:

Minimize

$$P_T = \sum_{k=1}^{K}[\sum_{n=1}^{N}\sum_{l=1}^{N_R}\sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C} \frac{f_k(c)}{\alpha_k^2(n,l,p)} \cdot \rho_k(n,l,p,c)] \quad (5)$$

$$\text{for } \rho_k(n,l,p,c) \in \{0,1\}$$

[1]Extension to the case where $M \geq 3$ is straightforward but tedious; thus it will not be considered in this paper.

[2]The equivalent channel gain can be obtained from $F_{n,k}H_{n,k}$ following the procedure in [1] and [2], where $H_{n,k}$ is the physical channel matrix associated with the k-th user occupying the n-th subcarrier.

subject to

$$R_k = \sum_{n=1}^{N}\sum_{l=1}^{N_R}\sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C} c\cdot \rho_k(n,l,p,c), \forall k \quad (6)$$

$$0 \le \sum_{c=1}^{C}\rho_k(n,l,p,c) \le 1, \forall k,n,l,p \quad (7)$$

$$0 \le \sum_{k=1}^{K}\sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C}\rho_k(n,l,p,c) \le 2, \forall n,l \quad (8)$$

$$0 \le \sum_{l=1}^{N_R}\sum_{c=1}^{C}\rho_k(n,l,p,c) \le N_R\cdot\delta_k(n,p), \forall k,n,p \quad (9)$$

$$\sum_{k=1}^{K}\sum_{p=1,p\neq k}^{K}\delta_k(n,p) = 2, \forall n \quad (10)$$

$$\delta_k(n,p) = \delta_p(n,k), \forall k,n,p \quad (11)$$

$$\delta_k(n,p) \in \{0,1\}, \forall k,n,p \quad (12)$$

where P_T denotes the total transmission power and $\delta_k(n,p)$ is equal to one if the k-th and p-th users occupy the two SDMA subcahnnels associated with the n-th subcarrier and zero, otherwise. Inequalities (7) and (8) are included to satisfy the condition (C.1). Similarly, (9)-(12) are added to meet (C.2). This problem can be solved by IP having $\rho_k(n,l,p,c)$ and $\delta_k(n,p)$ as binary variables. In general, IP needs an exponential time algorithm whose complexity increases exponentially with the number of constraints and variables.

In the following subsection, the behavior of the optimal IP algorithm is examined through computer simulation and some observations are made regarding bit loading. The proposed suboptimal algorithm is then developed based in these observations.

B. Behavior of IP: Simulation Results

The optimal IP algorithm was applied to the multiuser MIMO/OFDMA system. The parameters were as follows: N=64, N_T=4, N_R=2, and M=2. Thus, the total number of subchannels to be assigned was 256. The number of users K was 4; the maximum number of loaded bits $C = 12$; the required BER was 10^{-4}. 8-tap frequency selective Rayleigh fading channels with exponentially decaying power profiles were assumed. The channel parameters were fixed during one frame period. The average channel magnitudes of the different users were generally unequal and the difference, denoted by γ, between the strongest and weakest channels of the four users was either zero or 30dB.

Table I summarizes the resulting number of assigned subchannels for each user and number of loaded bits for each subchannel when $R_1 = R_2 = R_3 = R_4 = 256$. Due to the fact that the equivalent channel gains of the two MIMO subchannels (N_R=2), $\{\alpha_k(n,l,p), l = 1,2\}$, are different in most cases, the results are sorted into two groups: the 1st group encompasses the results associated with the MIMO subchannel having the larger equivalent channel gain $\alpha_k(n,1,p)$ and the 2nd group contains the rest. For the channels with an identical average gain ($\gamma = 0$), the subchannels were almost equally distributed to the users (28 to 34 and 27 to 32 subchannels of the groups 1 and 2, respectively, were assigned per user). When γ = 30 dB, the most subchannels were assigned to user 1 with the lowest average channel magnitude and the least subchannels were assigned to user 4 with the highest average channel magnitude. This occurred because of the constraint $R_1 = R_2 = R_3 = R_4$. It is important to observe that the optimal IP tended to load the same number of bits (constellation size) to the subchannels allocated to a user. This indicates that $c_k(n,l)$ in (3) tends to be the same irrespective of n. To be specific, let $c_k(l)$ denote the *dominant* constellation. Then when $\gamma = 0$ dB, $c_1(1) = 6$ because 21 out of 34 MIMO subchannels were loaded 6 bits. Note that the remaining 13 subchannels were loaded 4 bits. In this manner, we can find a dominant constellation for each pair of (k,l). It was observed, by repeating the same experiment 1000 times, that about 70% of $c_k(n,l)$ were equal to their corresponding dominant constellation $c_k(l)$, and that almost all $c_k(n,l)$ were in the neighborhood of $c_k(l)$. This observation regarding the dominant constellation is essential in deriving the suboptimal algorithms.

IV. Suboptimal Subchannel Allocation and Bit Loading

In an attempt to simplify the subchannel allocation and bit loading problem, we consider the following two step approach. In the first step, subchannels are assigned under the assumption of dominant constellation, i.e.

$$c_k(n,l) = \begin{cases} c_k(l), & \text{if } \sum_{p=1,p\neq k}^{K}\sum_{c=1}^{C}\rho_k(n,l,p,c) = 1 \\ 0, & \text{otherwise.}\end{cases} \quad (13)$$

In the second, bits are distributed to the subchannels which are assigned in the first step. Of course, the assumption in the first step is not assumed in this step. The subchannel allocation in the first step can be performed via a suboptimal algorithm, while the bit assignment in the second step can adopt the greedy algorithm [10]. By performing subchannel allocation and bit loading separately, this yields a suboptimal algorithm that is considerably simpler to implement than the optimal IP approach.

A. Subchannel Allocation

Using (13), the subchannel allocation problem in the first step can be written as

Minimize

$$P_T = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{p=1,p\neq k}^{K}\sum_{l=1}^{N_R}\frac{f_k(c_k(l))}{\alpha_k^2(n,l,p)}\cdot\delta_k(n,p) \quad (14)$$

for $\delta_k(n,p) \in \{0,1\}$

subject to

$$\sum_{n=1}^{N}\sum_{p=1,p\neq k}^{K}\delta_k(n,p) = \frac{R_k}{\sum_{l=1}^{N_R}c_k(l)}, \forall k \quad (15)$$

TABLE I

NUMBER OF ASSIGNED SUBCHANNELS FOR EACH USER AND NUMBER OF LOADED BITS FOR EACH SUBCHANNEL WHEN $R_1 = R_2 = R_3 = R_4 = 256$.

γ (dB)	MIMO subchannel group	User k	# of assigned MIMO subchannels	# of MIMO subchannels with $c_k(n,l) = i$ bits				
				$i=2$	$i=4$	$i=6$	$i=8$	$i=10$
0	l=1	1	34	0	13	21	0	0
		2	28	0	0	28	0	0
		3	32	0	9	23	0	0
		4	34	0	15	19	0	0
		Total	128	0	37	91	0	0
	l=2	1	30	21	9	0	0	0
		2	27	10	17	0	0	0
		3	31	21	10	0	0	0
		4	32	23	9	0	0	0
		Total	120	75	45	0	0	0
30	l=1	1	63	31	32	0	0	0
		2	28	0	1	27	0	0
		3	20	0	0	4	16	0
		4	17	0	0	0	10	7
		Total	128	31	33	31	26	7
	l=2	1	33	33	0	0	0	0
		2	25	6	18	1	0	0
		3	20	0	8	12	0	0
		4	17	0	0	15	2	0
		Total	95	39	26	28	2	0

$$\sum_{k=1}^{K} \sum_{p=1, p \neq k}^{K} \delta_k(n,p) = 2, \forall n \tag{16}$$

$$\delta_k(n,p) = \delta_p(n,k), \forall k, n, p. \tag{17}$$

This is again an IP problem, but it is considerably simpler than the original IP in (5)-(12). The number of integer variables in (14)-(16) is less than that in (5)-(12), and furthermore $\{c_k(l)\}$ in (13) can be obtained during an initial phase, prior to solving the IP. Evaluation of $\{c_k(l)\}$ requires the following assumption on the channel gain:

$$\alpha_k^2(n,l,p) = \begin{cases} \alpha_k^2(l), & \text{if } \delta_k(n,p) = 1 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where

$$\alpha_k^2(l) = \frac{1}{N(K-1)} \sum_{n=1}^{N} \sum_{p=1, p \neq k}^{K} \alpha_k^2(n,l,p). \tag{19}$$

Under the assumption in (18), the optimization in (14)-(17) can be simplified as

$$\text{Minimize } P_T = \sum_{k=1}^{K} \sum_{l=1}^{N_R} \frac{f_k(c_k(l))}{\alpha_k^2(l)} \cdot \frac{R_k}{\sum_{l=1}^{N_R} c_k(l)} \tag{20}$$

$$\text{subject to } \sum_{k=1}^{K} \frac{R_k}{\sum_{l=1}^{N_R} c_k(l)} = 2N \tag{21}$$

where $\sum_{n=1}^{N} \sum_{p=1, p \neq k}^{K} \delta_k(n,p)$ is replaced with $\frac{R_k}{\sum_{l=1}^{N_R} c_k(l)}$, due to (15). Suppose that $\{c_k(l)\}$ are not integers but real numbers. Then the solution $\{c_k^*(l)\}$ of (20) and (21) can be obtained by introducing a Lagrange multiplier μ and using a numerical technique, such as vector-form Newton's method [13]. Although the obtained $\{c_k^*(l)\}$ are not integers, they can still be used for the optimization in (14)-(17).

B. Bit Loading

After the subchannel allocation, the bit loading problem can be written as follows:

$$\text{Minimize } P_T = \sum_{k=1}^{K} [\sum_{n=1}^{N} \sum_{p=1, p \neq k}^{K} \sum_{l=1}^{N_R} \frac{f_k(c_k(n,l))}{\alpha_k^2(n,l,p)} \cdot \delta_k(n,p)] \tag{22}$$

$$\text{subject to } R_k = \sum_{n=1}^{N} \sum_{p=1, p \neq k}^{K} \sum_{l=1}^{N_R} c_k(n,l) \cdot \delta_k(n,p), \forall k \tag{23}$$

where only $\{c_k(n,l) \in \{0, \cdots, C\}\}$ are treated as variables since $\{\delta_k(n,p)\}$ are already determined through the subchannel allocation. Equations (22) and (23) reveal that the bit loading for each user can be independently preformed: minimizing $\sum_{n=1}^{N} \sum_{p=1, p \neq k}^{K} \sum_{l=1}^{N_R} \frac{f_k(c_k(n,l))}{\alpha_k^2(n,l,p)} \cdot \delta_k(n,p)$ for each k, under the constraint in (23), eventually minimizes the total power P_T. This fact indicates that the bit loading for each user can be performed as in the case of single-user OFDM, as such it is possible to apply the greedy algorithm proposed for single-user OFDM in [10].

V. SIMULATION RESULTS

The optimal IP and suboptimal algorithms were examined. For comparison, we also considered the random algorithm where subchannels were randomly allocated [5] and bits were loaded by the greedy algorithm. The parameters were the same as those in Section III-B. The whole simulation results were the average of 100 trials. Table II shows the resulting transmission power P_T for various $\{R_1, R_2, R_3, R_4\}$ and

TABLE II

REQUIRED TRANSMISSION POWER.

γ (dB)	bits/OFDM symbol R_1	R_2	R_3	R_4	Optimal IP (dB)	Suboptimal (dB)	Random (dB)
0	256	256	256	256	39.93	40.06	42.93
	128	128	384	384	40.38	40.56	43.10
	64	64	448	448	41.03	41.27	43.31
30	256	256	256	256	55.92	56.33	62.41
	128	128	384	384	50.74	51.17	57.08
	64	64	448	448	46.71	47.26	53.31

$\gamma \in \{0, 30\}$[dB]. As expected, the optimal IP yielded the minimum P_T for all cases. The P_T values from the suboptimal algorithm were reasonably close to the minimum values. The performance gap between the optimal IP and the suboptimal algorithm did not exceed 0.24 dB when $\gamma = 0$ dB and 0.55 dB when $\gamma = 30$ dB. The random algorithm was worse than the others: the maximum additionally required power were 3.00 dB when $\gamma = 0$ dB and 6.60 dB when $\gamma = 30$ dB.

VI. CONCLUSION

It was shown that subchannel allocation and bit loading for multiuser MIMO/OFDMA systems can be optimized using IP. An efficient suboptimal algorithm was proposed that separately performs subchannel allocation and bit loading. Simulation results indicated that preformance of the proposed suboptimal approach reasonably close to those of optimal IP, yet the former is considerably simpler to implement than the latter.

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