A Bandwidth Efficient Precode to Reduce Intercarrier Interference in OFDM

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Abstract— **A bandwidth efficient precode that can reduce the intercarrier interference (ICI) in orthogonal frequency division multiplexing (OFDM) systems is developed by modifying the existing rate-$1/2$ precodes. It is a rate-$(k-1)/k$ code which is based on the assumption that neighboring off-diagonal elements of frequency domain channel matrix are almost identical. The effect of precoding on OFDM system performance is examined through computer simulation. The results indicate that considerable performance improvement can be achieved by employing the proposed precoding, as compared with the conventional systems with/without precoding.**

Index Terms– OFDM, intercarrier interference, precoding, fast fading channel, channel estimation

I. Introduction

It has been recognized that the performance of an OFDM system can be severely degraded if the channel varies within an OFDM symbol period [1], [2]. This happens because such a fast varying channel causes intercarrier interference (ICI). To compensate for the ICI, frequency-domain equalization methods were proposed in [3], [4]. As an alternative, ICI suppression techniques based on precoding have been investigated [5]–[8]. A half-rate code is introduced in [5] and its extension to $1/k$-rate code is considered in [6]. A frequency-domain partial-response coding (PRC) with rate one was proposed in [7], [8].

In this paper, we propose a new precode with rate $(k-1)/k$ which enables significant ICI suppression as well as high data rate transmission. The new code is derived by extending the half-rate code in [5]. It is shown through computer simulation that the OFDM systems employing the proposed precodes can outperform the conventional systems in [5], [7], [8].

II. System Model

Fig. 1 illustrates an OFDM system with precoding. The $(N \cdot l/k)$-dimensional input vector $\mathbf{s}$ is multiplied with the N-by-$(N \cdot l/k)$ matrix $\mathbf{\Theta}$ where N is the number of OFDM subcarriers and l/k is the code rate. The resulting N-dimensional vector, $\mathbf{x} = \mathbf{\Theta}\mathbf{s}$, is passed through the OFDM transmitter and

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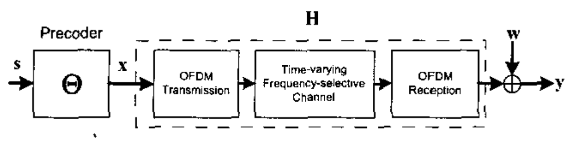

Fig. 1. OFDM system with precoding technique.

a channel. The received vector $\mathbf{y}$ after DFT in the OFDM receiver can be expressed as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{w} && (1) \\ &= \mathbf{H}\mathbf{\Theta}\mathbf{s} + \mathbf{w} && (2) \end{aligned}$$

where $\mathbf{H}$ is the N-by-N channel matrix whose entries represent flat fading channel gains and $\mathbf{w}$ is an N-dimensional noise vector. When the channel is fixed over an OFDM symbol period, the channel matrix $\mathbf{H}$ becomes a diagonal matrix and there is no ICI. For time-varying channels, $\mathbf{H}$ has off-diagonal terms representing ICI. The precoding matrix $\mathbf{\Theta}$ is designed under the following assumptions on $\mathbf{H}$:

(A.1) The magnitudes of the diagonal terms of $\mathbf{H}$ are considerably larger than those of the off-diagonal terms.

(A.2) Any two neighboring off-diagonal terms of $\mathbf{H}$ are close to each other.

III. Design of Precode Matrix $\mathbf{\Theta}$

In this section, we first examine the ICI cancellation scheme in [5], which employs a half-rate precode ($l/k = 1/2$). When $N = 4$, the precode from [5] is given by

$$\mathbf{\Theta} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \quad (3)$$

and $\mathbf{H}\mathbf{\Theta}$ is represented as

$$\mathbf{H}\mathbf{\Theta} = \begin{bmatrix} h(1,1) - h(1,2) & h(1,3) - h(1,4) \\ h(2,1) - h(2,2) & h(2,3) - h(2,4) \\ h(3,1) - h(3,2) & h(3,3) - h(3,4) \\ h(4,1) - h(4,2) & h(4,3) - h(4,4) \end{bmatrix}. \quad (4)$$

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TABLE I

OFDM SYSTEMS THAT CAN SUPPRESS ICI. P DENOTES THE NUMBER OF PILOT SUBCARRIERS AND D REPRESENTS THE NUMBER OF DATA SYMBOLS USING THE SPECIFIED MODULATION SCHEME.

	P	Precodes	Code rate	Modulation (D)	Detection
SYSTEM 1	32	The proposed $\mathbf{C}$ in (8)	$3/4$	QPSK (8) 8QAM (16)	V-BLAST detector with MMSE nulling
SYSTEM 2	24	The proposed $\mathbf{C}$	$4/5$	QPSK (32)	V-BLAST detector with MMSE nulling
SYSTEM 3 [5]	32	ICI cancelling mod & demodulation	$1/2$	16QAM (16)	1-tap equalizer + symbol by symbol detector
SYSTEM 4 [8]	32	2-tap partial response coding	1	QPSK (32)	1-tap equalizer + MLSD
SYSTEM 5 [11]	32	$\times$	1	QPSK (32)	Equalizer in [11] + symbol by symbol detector

Note that Θ in (3) is a block-diagonal matrix whose diagonal blocks are identical. Under the assumptions (A.1) and (A.2), the off-diagonal blocks of $\mathbf{H}\Theta$ in (4) are close to the zero vector, which indicates the suppression of most ICI. The proposed precode is an extension of the half-rate code in [5]. The new code is a $(k-1)/k$ code ($l = k-1$) which is derived under the following assumptions on Θ.

(A.3) Θ=diag$(\mathbf{C}, \mathbf{C}, \cdots, \mathbf{C})$ where $\mathbf{C}$ is a k-by-l matrix.[1]

In (A.3), Θ becomes the precode from [5] if $\mathbf{C} = [1, -1]^T$. To rewrite (2) in terms of $\mathbf{C}$, the vector $\mathbf{y}$, $\mathbf{s}$, and $\mathbf{w}$ are partitioned into sub-block vectors. Specifically, let $\mathbf{y} = [\underline{y}^T(0), \cdots, \underline{y}^T(N/k-1)]^T$, $\mathbf{s} = [\underline{s}^T(0), \cdots, \underline{s}^T(N/k-1)]^T$, and $\mathbf{w} = [\underline{w}^T(0), \cdots, \underline{w}^T(N/k-1)]^T$ where $\underline{y}(n)$ and $\underline{w}(n)$ are k-dimensional vectors and $\underline{s}(n)$ is a l-dimensional vector which represent the n-th sub-blocks of the $\mathbf{y}$, $\mathbf{w}$, and $\mathbf{s}$, respectively. Then (2) can be expressed as

$$\underline{y}(n) = \underline{H}(n,n)\mathbf{C}\underline{s}(n) + \sum_{i=0, i\neq n}^{N/k-1} \underline{H}(n,i)\mathbf{C}\underline{s}(i) + \underline{w}(n). \quad (5)$$

for each $n = 0, 1, \cdots, N/k-1$, and

$$\mathbf{H} = \begin{bmatrix} \underline{H}(0,0) & \underline{H}(0,1) & \cdots & \underline{H}(0, N/k-1) \\ \underline{H}(1,0) & \underline{H}(1,1) & \cdots & \underline{H}(1, N/k-1) \\ \underline{H}(N/k-1, 0) & \underline{H}(N/k-1, 1) & \cdots & \underline{H}(N/k-1, N/k-1) \end{bmatrix} \quad (6)$$

where $\underline{H}(n, m)$ is a k-by-k matrix. In the right-hand-side (RHS) of (5), the first term $\underline{H}(n,n)\mathbf{C}\underline{s}(n)$ is the signal of interest and the second term represents the undesirable ICI. Thus the design of $\mathbf{C}$ is focused on forcing $\underline{H}(n,i)\mathbf{C}$ to zero whenever $i \neq n$. Once the ICI term is removed, the signal model in (5) reduces to

$$\underline{y}(n) = \underline{H}(n,n)\mathbf{C}\underline{s}(n) + \underline{w}(n). \quad (7)$$

If $\underline{H}(n,n)$ and $\mathbf{C}$ are known at the receiver and $\underline{H}(n,n)\mathbf{C}$ is of full-rank, $\underline{s}(n)$ in (7) can be detected using standard techniques such as the maximum likelihood (ML) and V-BLAST detectors [9], [10]. This imposes the following condition on $\mathbf{C}$.

(C.1) The rank of $\mathbf{C}$ is $l = k-1$.

[1] diag$(\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_n)$ denotes a block diagonal matrix with diagonal blocks $\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_n$.

In addition, to keep the transmission power to a constant, another condition is necessary.

(C.2) Trace of $\mathbf{CC}^H$ is equal to k.

Finally, we need a condition for ICI cancellation.

(C.3) All column vectors of $\mathbf{C}$ have only two non-zero elements at two consecutive rows, and their sum is equal to zero.

The validity of (C.3) is shown in the observation presented below.

Observation 1. For any (n, i), $n \neq i$, $\underline{H}(n,i)\mathbf{C} \simeq 0$ if (C.3) is satisfied.

Proof. Suppose that $\mathbf{C}$ satisfies (C.3). Then it is straightforward to see that each element of $\underline{H}(n,i)\mathbf{C}$ is represented as the difference between two neighboring off-diagonal elements of $\mathbf{H}$, and $\underline{H}(n,i)\mathbf{C} \simeq 0$, due to (A.2). □

For example, the following $\mathbf{C}$ matrix meets the conditions (C.1), (C.2) and (C.3); it yields a $3/4$-rate precode.

$$\mathbf{C} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}. \quad (8)$$

IV. SIMULATION RESULTS

The effect of the proposed and existing codes on an OFDM system has been examined through computer simulation. The parameters of the OFDM system were as follows: the number of subcarriers $N = 64$; number of cyclic prefix was 8; OFDM symbol period $T_s = 240\mu sec$ and carrier frequency 5 GHz. The channel was a frequency selective Rayleigh fading channel with 8 taps and had exponentially decaying power profiles. The time-varying channel was estimated by using the data-aided channel estimator in [11] for coherent detection. Various systems, listed in Table I, were simulated to compare the performance of the proposed and existing codes. Systems 1 and 2 employ the proposed codes with rate $3/4$ and $4/5$, respectively. Systems 3, 4 and 5 are based on the techniques introduced in [5], [8] and [11], respectively. It was assumed that all systems transmit 64 information bits per one OFDM symbol. The modulation scheme and the number of pilot subcarriers were chosen so that the sum of data and pilot sucarriers was 64. The power

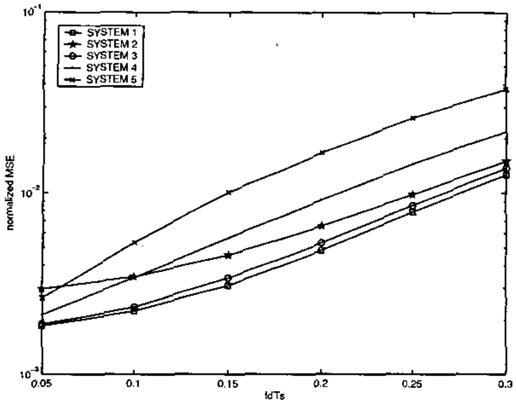

Fig. 2. Normalized mean square error of channel estimation versus f_dT_s when $SNR = 25$dB.

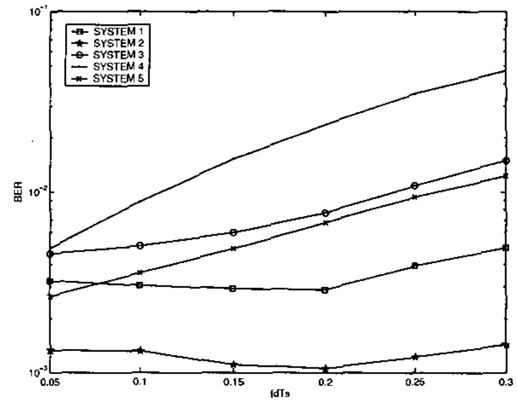

Fig. 3. BER versus f_dT_s when $SNR = 25$dB.

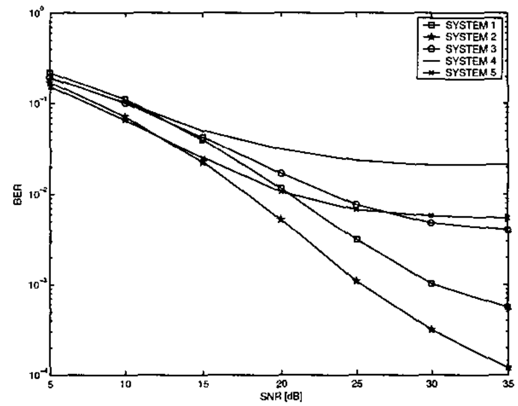

Fig. 4. BER versus SNR when $f_dT_s = 0.2$.

of all transmitted symbols, for both pilot and data subcarriers, was normalized to 1.

Fig. 2 shows the normalized mean square error (MSE) of channel estimation versus f_dT_s, when $SNR = 25$dB. Here f_d was the maximum Doppler shift. Comparing channel estimation errors, SYSTEM 1 which employ the proposed $3/4$ code performed the best and SYSTEM 5 which does not employ any precode performed the worst. The MSE of SYSTEM 2 employing the proposed $4/5$ code was somewhat larger than that of SYSTEM 1, because the former used 24 pilot subcarriers while the latter used 32 pilots.

Fig. 3 shows bit error rate (BER) performance against f_dT_s when $SNR = 25$dB. It is seen that systems employing the proposed precodes outperformed the others. In this case, SYSTEM 2 performed better than SYSTEM 1, because the former used 40 subcarriers for QPSK data transmission, while the latter used 32 subcarriers for transmission of either QPSK or 8QAM data. Both of the systems were reasonably robust to the normalized Doppler frequency f_dT_s.

Finally, Fig. 4 shows the BER performance against SNR when $f_dT_s = 0.2$. In general, SYSTEM 2 exhibited the best performance. In low SNR ($SNR \leq 15$dB), SYSTEM 5 acted well — it was comparable to SYSTEM 2. SYSTEM 1 performed better than SYSTEMS 3, 4 and 5 when $SNR \geq 20dB$

V. Conclusion

The bandwidth efficient precodes for suppressing ICI are derived under the assumption that neighboring off-diagonal elements of the frequency domain channel matrix are almost identical. Simulation results demonstrated that the proposed codes can provide performance improvements over the existing coding schemes.

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