Scalable Design of Space-Time Trellis Code with Low Decoding Complexity

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Abstract—**Design of space-time codes that scale with the number of transmit antennas is a difficult problem. In this paper, we introduce a new family of space-time trellis codes (STTC) that can be applied to any arbitrary number of transmit antennas. This family is constructed by utilizing QPSK STTCs as component codes to construct STTCs with larger constellation size. Unlike the design of existing STTC, the search space in our design does not grow exponentially with the constellation size or the number of transmit antennas. Additionally, we propose a practical approach to reduce the computational complexity of our proposed scheme using interference mitigation techniques. Simulation results compare the performance of our approach with that of space-time block codes for the case of two transmit antennas and several different number of receive antennas, a spectral efficiency of 4 bits/s/Hz, and slow Rayleigh fading channels.**

I. INTRODUCTION

The design of multiple-input multiple-output (MIMO) systems continues to attract the attention of researchers [1]-[5]. All existing research activities concentrate on the optimum design of space-time codes when the number of transmit antennas is fixed. For a STTC [2], when the number of transmit antennas increases, the search space for designing the code increases exponentially. For this reason, STTC designs are lacking for cases with a large number of transmit antennas. Further, the decoding complexity of such systems seriously limits the utility of such codes for practical applications. On the other hand, space-time block codes (STBC) [6] have simple decoding complexity and if combined with trellis codes [3], they exhibit good performance. For these codes, when the number of transmit antennas in the system changes, the design of the STBC changes. Moreover, full rate maximum diversity orthogonal codes are only available for a limited number of cases. Consequently, one important question is whether it is possible to design space-time codes such that the design is scalable to any number of transmit antennas and the decoding complexity does not grow with the number of transmit antennas.

Recently, we introduced a new approach [7] to design STTC for any arbitrary $n \times m$ MIMO system with a search space that does not increase exponentially with the constellation size or with the number of transmit antennas. The idea is to transmit more than one symbol from a single transmit antenna during a symbol period by superimposing the encoded symbols from the output of a low constellation size STTC encoder on top of each other. To achieve good performance, we induced randomness into the system to create additional channel paths, called virtual paths. Inducing randomness into a physical channel has been proposed in the past [8],[9],[10]. The main objective of these techniques is to induce more fluctuations into the channel. We propose to induce randomness into the physical channel in a new way. The goal is to explore the rich diversity capabilities of MIMO systems and we derived the optimum condition to attain maximum coding and diversity gains for slow fading channels.

This paper introduces a computationally efficient approach to decode our proposed STTC by utilizing a successive interference cancellation technique [11]. This proposed scheme is applicable to any number of transmit antennas without any requirement to change the encoder design.

The outline of the paper is as follows. In Section II, we will review the system model and our proposed STTC design for slow Rayleigh fading channels. The low decoding complexity STTC design is proposed in Section III. We compare the performance of the proposed algorithm with that of a STBC in section IV. Section V concludes the paper.

II. SYSTEM MODEL AND RANDOMIZATION TECHNIQUE

We consider a wireless communication system utilizing n transmit and m receive antennas. The channel path gain from transmit antenna i to receive antenna j is denoted by $h_{i,j}$ and is a complex Gaussian random variable with zero mean

This material is based on research supported by University of California, Office of president under agreement No. SC-05-33, by the AFRL under agreement No. FA9550-06-1-0041, by the NSF under Grant No. CCR-0112501, and by the University Information Technology Research Center (ITRC) Program of the Korean government.

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and variance 0.5 per complex dimension. We assume that different channel path gains are statistically independent. We also assume that the channel coefficients are constant during one block of data and change independently from one block to another. The received data r_t^j at antenna j and time t can be written as

$$r_t^j = \sum_{i=1}^{n} h_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m, \tag{1}$$

where c_t^i is the complex symbol with unit average power sent from antenna i at time t, n_t^j is the additive white complex Gaussian noise sample with zero mean and variance $N_0/2$ per dimension, and E_s is the contraction factor of the signal constellation.

STTCs were originally designed to achieve diversity and coding gain in wireless fading channels utilizing multiple transmit antennas. The search space for designing these codes increases exponentially with the constellation size. It has been shown that one can apply STTCs in systems with only a single transmit antenna by using randomization techniques as long as we have multiple receive antennas, i.e., $n=1$ [7]. In this case [7], the received signal model, similar to (1), becomes

$$r_t^j = h_{1,j} C_t \sqrt{E_s} + n_t^j, \quad 1 \le j \le m. \tag{2}$$

Our objective is to design the transmit signal (C_t) such that we can utilize STTCs in the absence of multiple transmit antennas. We propose to use the transmitted signal

$$C_t = A_t^1 c_t^1 + A_t^2 c_t^2 + \ldots + A_t^n c_t^n, \tag{3}$$

where the A_t^i's are called the induced random variables and the c_t^i's are from the output of a STTC encoder. The A_t^i's are also independent of the physical channel path gains, the $h_{1,j}$'s. Combining (2) and (3) leads to

$$r_t^j = \sum_{i=1}^{n} h_{i,j}^{'} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m, \tag{4}$$

where $h_{i,j}^{'} = h_{1,j} A_t^i$ is called a virtual path gain. We call this a virtual path gain because only m physical paths exist in this system and by inducing random data at the transmitter, we have created $n \times m$ virtual paths. Of course, some of these virtual paths are statistically dependent, but this approach will allow us to employ a STTC in a setting with a single transmit antenna.

One interpretation of our approach is to form an $n \times m$ MIMO system using a group of n distinct $1 \times m$ system. This interpretation results in an architecture very similar to the V-BLAST architecture [5] with the exception that here STTC is implemented for the protection of the symbols in each path.

The induced random variables (A_t^i's) can either change from symbol to symbol or they can be constant during one data frame. Our intention is to derive conditions under which one can obtain the minimum upper bound on the block error probability. A block error occurs when the decoded data sequence

$$\underline{\mathbf{E}} = e_1^1 \ldots e_1^n \ldots e_N^1 \ldots e_N^n,$$

is different from the transmit sequence

$$\underline{\mathbf{C}} = c_1^1 \ldots c_1^n \ldots c_N^1 \ldots c_N^n,$$

where N is the number of symbols in one block. By applying an upper bound on the conditional pairwise block error probability (slow Rayleigh fading channel) for a maximum likelihood receiver matched to the virtual paths ($h_{i,j}^{'}$'s), we see that the upper bound on the conditional pairwise block error probability is

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \middle| A_t^i, h_{1,j}, 1 \le i \le n, 1 \le j \le m\right) \le \prod_{j=1}^{m} \exp\left(-\left|h_{1,j}\right|^2 A B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) A^* \frac{E_s}{4N_0}\right), \tag{5}$$

where $A = \left[A_t^1, A_t^2, \ldots, A_t^n\right]$ is the vector whose elements are the induced random variables. It can be shown [2] that $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = V^* D V$ is a Hermitian matrix, V is an unitary matrix whose rows $v_i, 1 \le i \le n$ are the eigenvectors of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$, and $D = diag(\lambda_1, \ldots, \lambda_n)$ where the λ_i's are the eigenvalues of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$.

It has been shown in [7] that the result is a set of discrete induced random variables that statistically depend on the STTC encoder output. Therefore, the new conditional upper bound on the pairwise block error probability case can be derived as [7]

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \middle| h_{1,j}, 1 \le j \le m\right) \le \prod_{j=1}^{m} \exp\left(-\left|h_{1,j}\right|^2 \sum_{i=1}^{N} \left|C_t - E_t\right|^2 \frac{E_s}{4N_0}\right), \tag{6}$$

where E_t is the error signal defined in (3). Note that in this case, the induced random variables, the A_t^i's, are statistically dependent on the output of the STTC encoder and for that reason, it is not feasible to separate them in this equation. Averaging (6) with respect to the channel coefficients, we arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left(\frac{1}{1 + \sum_{i=1}^{N} \left|C_t - E_t\right|^2 (E_s / 4N_0)}\right)^m. \tag{7}$$

This equation suggests that in order to minimize the upper bound on the pairwise block error probability, we need to maximize the minimum Euclidean distance between the modi-

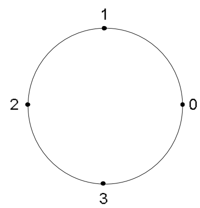

Fig. 1. Mapping of QPSK constellation

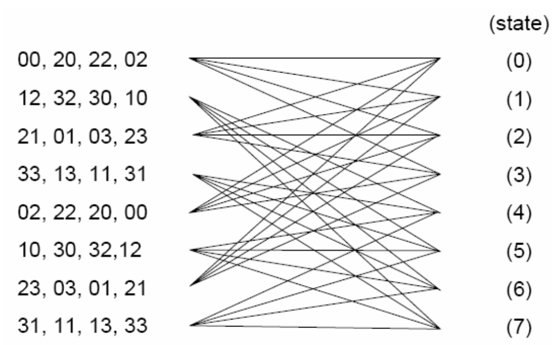

Fig. 2. 8-state QPSK-STTC of [12]

fied codeword $(C_t, t=1,...,N)$ and the codeword chosen in error $(E_t, t=1,...,N)$. Therefore, we need to design the induced random variables such that this minimum Euclidean distance is maximized.

There are several different design techniques for QPSK STTCs that consider maximizing the minimum Euclidean distance between the pair of codewords [4], [12], [13] while providing the maximum coding and diversity gains. In this paper, we use the QPSK STTC designs of [12] with 8 states for simulation and code design. Figs. 1 and 2 demonstrate the signal constellation and trellis diagram of the 8-state QPSK STTC design of [12]. The code is designed to have the maximum diversity and coding gains as well as maximizing the minimum Euclidean distance between the codeword pairs.

Our objective is to design induced random variables such that when we combine two QPSK signals, the new transmitted signal has a finite number of constellation points, i.e., equivalent to a 16-PSK constellation in this example, and at the same time maximizes the minimum Euclidean distance between codewords of the new transmitted signal (C_t). Fig. 3 illustrates this mapping for two QPSK signals. In this construction and mapping, we have chosen to use only induced random variables that have unit amplitude [7] and statistically dependent random phases. These random phases depend on the signals produced by the STTC encoder as illustrated in Fig. 3.

The construction of the transmitted signals (C_t) is accomplished in two steps. First, since all different combinations of two QPSK signals yield a maximum of 16 possible choices, we use the trellis diagram of Fig. 2 to assign

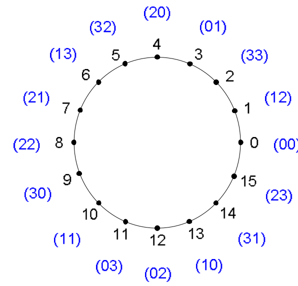

Fig. 3. Description of the mapping of two QPSK signals into 16-PSK for the 8-state QPSK STTC of [12]

these 16 combinations such that the minimum Euclidean distance between any two codewords is maximized. This objective is achieved by assigning every two QPSK symbols into a point in the 16-PSK constellation for all 16 possible choices and then computing the distance properties of that particular code. The signal assignment that maximizes the minimum Euclidean distance of the code will be selected. In the second step, we compute the A_t^i's such that the linear combination of two QPSK symbols using (3) will create the appropriate C_t as shown in Fig. 3. Utilizing this construction, for any two QPSK signals at the output of STTC encoder of Fig. 2, the values of these induced random variables are known. However, since the signals from the encoder output are random and unknown at the receiver, the induced random variables are also unknown and random at the receiver.

Note that for this particular construction of the signal, we no longer need to keep any table of induced random variables since there is a one-to-one mapping between any two QPSK signals from the output of STTC encoder and the A_t^i's. The design of these dependent induced random variables is based on the desire to maximize the minimum Euclidean distance between any two codewords to improve (7). We do not claim that such construction is necessarily the optimum solution.

III. Design Based on Concatenated Interference Suppression (IS)-Viterbi Algorithm (VA) Approach

In this section, we will show a practical approach to reduce the computational complexity in the system using a combined array processing technique [11]. This scheme is based on the group interference suppression (IS) method and effectively reduces the computational complexity by partitioning antennas at the transmitter into small groups, and using individual STTCs to transmit information from each group of antennas. Note that our unique representation of a MIMO system by n distinct SIMO systems allows us to separate the decoding of the signals sent from each transmit antenna.

In Fig. 4, the STTC encoder outputs are divided into two groups (G_1 and G_2) where G_1 and G_2 include odd and even

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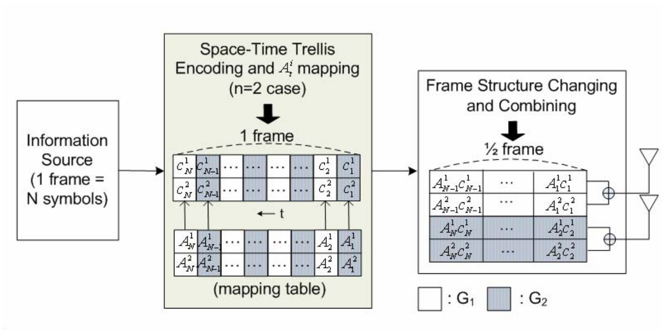

Fig. 4. Block diagram of the transmitter based on the IS-VA approach

slots respectively. Every two STTC encoder outputs are mapped into a symbol using the induced random variables, the A_t^i s. We utilize the results from the previous section, where we designed a set of induced random variables that statistically depend on the STTC encoder output. Through the mapping table, STTC encoder outputs are multiplied by A_t^i s and combined. Finally, the mapped signals are transmitted by the transmit antennas. Here, for the simplicity, we assume the number of the transmit antennas, n , is 2 and the transmit signal for each transmit antenna can be given

$$\begin{bmatrix} C_t^1 \\ C_t^2 \end{bmatrix} = \begin{bmatrix} A_{2t-1}^1 c_{2t-1}^1 + A_{2t-1}^2 c_{2t-1}^2 \\ A_{2t}^1 c_{2t}^1 + A_{2t}^2 c_{2t}^2 \end{bmatrix}. \tag{8}$$

Therefore, the odd slot signals (G_1) are transmitted by the first antenna while the even slot signals (G_2) are transmitted by the second antenna. Extension of this idea to more than two transmit antennas is trivial.

The received signals can be derived in the vector form as

$$\mathbf{r}_t = \mathbf{H}\mathbf{s}_t + \mathbf{n}_t , \tag{9}$$

Where

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{2,1} \\ h_{1,2} & h_{2,2} \\ \vdots & \vdots \\ h_{1,m} & h_{2,m} \end{bmatrix},$$

$$\mathbf{r}_t = \begin{bmatrix} r_t^1 \\ r_t^2 \\ \vdots \\ r_t^m \end{bmatrix}, \ \mathbf{s}_t = \begin{bmatrix} C_t^1 \\ C_t^2 \end{bmatrix}, \text{ and } \mathbf{n}_t = \begin{bmatrix} n_t^1 \\ n_t^2 \\ \vdots \\ n_t^m \end{bmatrix}.$$

Here, the number of physical channel paths is $2m$, while there are $4m$ virtual channel paths. Let's denote the number of virtual transmit antennas as n' , i.e., $n' = 4$ for this example. For each transmit antenna, two signals originally encoded in adjacent odd and even slots are mapped into one symbol and then transmitted. Therefore, each antenna receives four encoded signals. This means that we have two trellis steps corresponding to the received signal during one time slot and, to decode the set of $\left(C_t^1, C_t^2\right)^T$, we need to compute the branch metric as

$$\sum_{j=1}^{m} \left| r_t^j - \sum_{i=1}^{2} h_{i,j} C_t^i \right|^2 . \tag{10}$$

Therefore, we propose a two-stage decoding method using the concatenated IS-VA for selecting the survivor path of the trellis using the following technique.

A. First Stage (IS for G_1 decoding)

Consider the n' transmit antennas divided into two groups, each group has two virtual transmit antennas. To decode G_1 , we suppress the interfering signal (G_2) as described in [5] and [11]. We let $\mathbf{H}_{\text{sup}}$ denote the matrix as

$$\mathbf{H}_{\text{sup}} = \begin{bmatrix} h_{2,1} \\ h_{2,2} \\ \vdots \\ h_{2,m} \end{bmatrix}. \tag{11}$$

In the matrix $\mathbf{H}_{\text{sup}}$, the rank of $\mathbf{H}_{\text{sup}}$ is equal to 1. Therefore, we can compute $(m-1)$ orthonormal vectors in the left null space of $\mathbf{H}_{\text{sup}}$. Let $\mathbf{\Phi}$ denote $(m-1)\times m$ matrix composed of these vectors. By multiplying $\mathbf{\Phi}$ to both sides of (9), we arrive at

$$\begin{aligned} \mathbf{\Phi}\mathbf{r}_t &= \mathbf{\Phi}\mathbf{H}\mathbf{s}_t + \mathbf{\Phi}\mathbf{n}_t \\ \hat{\mathbf{r}}_t &= \hat{\mathbf{H}} C_t^1 + \hat{\mathbf{n}}_t \end{aligned}. \tag{12}$$

Finally, we can compute the branch metric for the G_1 elements using the minimum decision metric [11] and continue with the second trellis step. Note that we cannot decide on G_1 before decoding the G_2 elements, this means that the value of G_1 is different for each state when we subtract its values from the received vector to compute G_2 . The branch metric is given by

$$\sum_{j=1}^{m-1} \left| \hat{r}_t^j - \hat{h}_{1,j} C_t^1 \right|^2, \ \ m \geq 2 . \tag{13}$$

B. Second Stage (VA for G_2 decoding)

We next consider the decoding for the G_2 elements. After finding the survivor path for each state in the first step of decoding G_1 , subtract the contribution of G_1 elements from the received signals. Note that this contribution is different in each state of the trellis. The remaining received signal elements $\left(c_{2t}^1, c_{2t}^2\right)^T$ correspond to the G_2 contribution from the even slots

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of a frame and can be used to compute the survivor path using the VA. After computing the path metric corresponding to G_2, we then choose the survivor paths for all the states based on the results from the first and second decoding stage.

IV. SIMULATION RESULTS

In the simulations presented in this section, coherent detection is assumed along with perfect knowledge of the channel coefficients at the receiver. To apply the proposed algorithm (PA), we design a scalable STTC system with spectral efficiency of 4 bits/s/Hz, using a QPSK STTC for each transmit antenna when there are two transmit antennas. Note that the implementation of the IS-VA decoding for the new scheme is slightly different from the conventional decoding methods. In the new PA, at each time interval, the equivalent of n data symbols are transmitted simultaneously from n different transmit antennas. In this case, each path metric for IS-VA will compute the equivalent of n path metrics from the original trellis diagram (see Figs. 2, 3, and 4). Accordingly, we need to incorporate this feature when we compute the survivor paths for each STTC block.

In Fig. 5, we compare our STTC system based on IS-VA approach with the 16-PSK space-time block code (STBC) of [3] that combines STBC with a multiple trellis coded modulation (MTCM) scheme. When the number of receive antennas is increased, the performance gap between the PA and [3] is reduced and finally our technique outperforms the STBC-MTCM based system of [3]. Note that the complexity of our PA is less than that of STBC-MTCM [3]. Besides, this approach can be applied to any number of transmit antennas while the design of STBC depends on the number of transmit antennas. The main disadvantage of this algorithm is the fact that by using IS technique, we will not have full transmit diversity for all the symbols [11]. However, for all practical values of frame error rates (FER), when the number of receive antennas is large, our approach outperforms STBC-MTCM of [3].

V. CONCLUSION

In this paper, we showed the application of randomization techniques to construct scalable STTCs. We proposed using the successive interference cancellation concept to design a newly constructed receiver with low decoding complexity. This approach can be applied to any number of transmit antennas while the design of STBC-MTCM depends on the number of transmit antennas. The complexity of our scheme is less than that of STBC-MTCM.

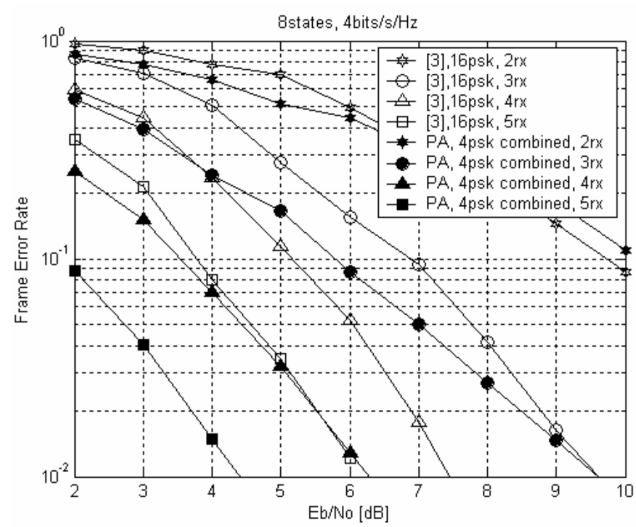

Fig. 5. Frame error rate comparison between 16PSK-8state STBC-MTCM of [3] and PA for $2\times2, 3, 4$ and 5 MIMO systems

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