Outage Analysis for MIMO Rician Channels and Channels with Partial CSI

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Abstract— We analyze the outage performance and diversity-multiplexing tradeoff (DMT), originally introduced by Zheng and Tse, for multiple antenna Rician channels or channels with partial channel state information at the transmitter (CSIT). The asymptotic behaviors of the outage are analyzed in the limit of high signal-to-noise ratio (SNR). We also analyze the SNR where the maximum diversity order (MDO) is obtained for Rician channels, which serves as a desired operation point. In addition, by applying the outage analysis for Rician channels we present the differential DMT (DDMT) to develop a better understanding of the asymptotic relationship between the outage probability, transmission rate, SNR, and Rician factor.

I. INTRODUCTION

Fading can degrade the performance of wireless communication systems significantly. Multiple-input multiple-output (MIMO) antenna systems can improve the performance by providing diversity and/or multiplexing gains [1]–[3]. Zheng and Tse discovered a fundamental tradeoff between the two gains [4] assuming independent and identically distributed (i.i.d.) Rayleigh fading channels without any CSIT. Such assumptions can be pessimistic. More realistic scenarios would be to have partial CSIT by limited feedback, where the transmitter can have an average information about the mean of channel instances with some uncertainty. Many practical systems already have some form of limited feedback for MIMO [5] and it is therefore important to analyze the performance for such systems. Such a model is similar to the Rician fading with a line-of-sight (LOS) path between the transmitter and the receiver, which is another important class of fading channels.

The limits of communication over fading channels are often analyzed through outage performance. For MIMO Rician fading or non-zero mean channels, some recent work shows the ergodic capacity [6]–[8], but there are not many analytical results on the outage capacity, see e.g., [9]. In this paper, we analyze several distinguishing features of the outage behavior in MIMO Rician channels. In particular, we define and quantify the asymptotic SNR gap (ASG), MDO, and finite-SNR gap (FSG). The ASG shows that the Ricianness can improve the outage performance by a constant dB gap in SNR, but it cannot improve the diversity order at high SNR, where the Rayleighness becomes dominant. The MDO shows that there

exists an SNR where the diversity order is maximized, which can be very large as the Rician factor increases. Since this provides the best diversity order, it can be a desired operating point and finding it is of practical importance.

In addition to the above outage analysis, we also analyze the DMT for Rician. We define the DDMT that is more suitable for capturing the DMT for Rician. It helps us develop a better understanding of the asymptotic relationship between the outage probability, transmission rate, SNR and Rician factor when the change in SNR, rate, and the outage probabilities are small. Our work is based on recently refined results on DMT called the throughput-reliability tradeoff (TRT) [10]. Besides, the DMT for other practical channel models, which include rank-deficient and spatially correlated MIMO channels, was recently analyzed [11].

The rest of this paper is organized as follows. Section II describes the MIMO Rician fading channel model. In Section III, we show asymptotic analysis of the outage behavior. Section IV presents some analytical results on the proposed DDMT.

II. CHANNEL AND SYSTEM MODEL

We assume a flat fading mode, where its discrete-time equivalent model is given by

$$\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where n_T and n_R are the number of transmit and receive antennas, respectively, $\mathbf{x} \in \mathbb{C}^{n_T}$ is the transmitted signal vector and $\mathbf{y} \in \mathbb{C}^{n_R}$ is the received signal vector. $\mathbf{n} \in \mathbb{C}^{n_R}$ is the additive white Gaussian noise (AWGN) vector. $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix whose element h_{ij} represents the complex channel gain between the jth transmit antenna and the ith receive antenna. Finally, ρ corresponds to the average SNR at each receive antenna.

The entries of $\mathbf{H}$ are modeled as independent random variables. More precisely,

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{H}} + \sqrt{\frac{1}{K+1}}\mathbf{H}_w \tag{2}$$

where $K \geq 0$ denotes the Rician factor, $\bar{\mathbf{H}}$ is deterministic and the entries of $\mathbf{H}_w$ are independent zero-mean unit-variance complex Gaussian. This can also be considered as a channel model with partial CSIT where the transmitter knows the mean

$\bar{\mathbf{H}}$ of the channel $\mathbf{H}$. The channel matrix $\mathbf{H}$ is normalized such that

$$E[\text{tr}(\mathbf{H}\mathbf{H}^\dagger)] = n_R n_T. \quad (3)$$

For notational convenience, we define

$$m = \min\{n_T, n_R\} \quad \text{and} \quad n = \max\{n_T, n_R\}.$$

We assume the channel $\mathbf{H}$ is known at the receiver, but not at the transmitter. It is also assumed that the transmitter knows the average channel condition $\bar{\mathbf{H}}$ and the Rician factor K, which can also be a model for the case where the transmitter has a partial CSI for when the channel is Rayleigh.

III. ASYMPTOTIC BEHAVIOR OF OUTAGE PERFORMANCE

In the following, we briefly describe outage probability and other terminologies and derive some asymptotic results. Unless otherwise stated, all logarithms are assumed to be to the base 2 throughout the paper.

A. Outage Formulation

For the channel described by (1), an outage is defined as the event when the corresponding mutual information does not support the target rate, i.e.,

$$\{H \in \mathbb{C}^{n_R \times n_T} | I(\mathbf{x}; \mathbf{y}|\mathbf{H} = H) < R\}$$

where H indicates the channel realization and $p(\mathbf{x})$ is the input distribution. For this channel, optimizing over all input distributions, which can be taken to be Gaussian with a covariance matrix $\mathbf{Q}$ without loss of optimality, the optimal outage probability is defined as

$$P^*_{out}(R, \rho) \triangleq \inf_{\mathbf{Q} \geq 0, \text{tr}(\mathbf{Q}) \leq n_T} \Pr\left\{ \log\det\left(\mathbf{I}_{n_R} + \frac{\rho}{n_T}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right) < R\right\} \quad (4)$$

It is shown in [4] that we get an upper and a lower bound on the outage probability by picking $\mathbf{Q} = \mathbf{I}_{n_T}$ and $\mathbf{Q} = n_T\mathbf{I}_{n_T}$, respectively. Since the optimal input distribution for general channel distribution is unknown to date, we will use $\mathbf{Q} = \mathbf{I}_{n_T}$ throughout the paper, which is physically realizable. Furthermore, such a choice is without loss of generality for asymptotic analysis such as DMT [4]. Let $\phi_1, \phi_2, \cdots, \phi_m$ denote the m eigenvalues of $\bar{\mathbf{H}}$. For this choice of $\mathbf{Q}$, the outage probability can be written as

$$P_{out}(R, \rho, K, \mathbf{\Phi}) = \Pr\left\{ \log\det\left(\mathbf{I}_{n_R} + \frac{\rho}{n_T}\mathbf{H}\mathbf{H}^H\right) < R\right\} \quad (5)$$

where $\mathbf{\Phi} = \text{diag}\{\phi_i\}$. First, we need the following results to derive the asymptotic outage probability and other formulas.

Lemma 1 ([12]): Suppose $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_m$ be the ordered nonzero eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$, then the joint probability density function (pdf) of λ_i's is

$$f(\mathbf{\Lambda}) = \frac{e^{-\text{tr}\{K\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}\}}(K+1)^{mn}}{\prod_{l=1}^{m}(n-l)!(m-l)!} \prod_{k=1}^{m} \lambda_k^{n-m} \prod_{k<l}^{m} (\lambda_k - \lambda_l)^2$$
$$\cdot {}_0F_1(n, K\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}, (K+1)\mathbf{\Lambda})e^{-\text{tr}\{(K+1)\mathbf{\Lambda}\}} \quad (6)$$

where $\mathbf{\Lambda} = \text{diag}\{\lambda_i\}$. For the case of unequal ϕ_i's, we can express the hypergeometric function of matrix arguments ${}_0F_1(n, K\bar{\mathbf{H}}^\dagger\bar{\mathbf{H}}, (K+1)\mathbf{\Lambda})$ as the following [13]

$$ {}_0F_1(n, K\bar{\mathbf{H}}^H\bar{\mathbf{H}}, (K+1)\mathbf{\Lambda}) = \frac{\prod_{l=1}^{m-1}(m-l)^l(n-l)^l}{[K(K+1)]^{m(m-1)/2}\prod_{k<l}^{m}(\phi_k - \phi_l)}$$
$$\cdot \frac{\det(\{{}_0F_1(n-m+1, K(K+1)\phi_i\lambda_j)\})}{\prod_{k<l}^{m}(\lambda_k - \lambda_l)} \quad (7)$$

where $\det(\{a_{ij}\})$ indicates the determinant of a matrix whose (i, j)th entry is a_{ij} and ${}_0F_1(\cdot, \cdot)$ is the scalar hypergeometric function which is defined in [14] as

$$ {}_0F_1(n-m+1, x) = (n-m)!x^{-\frac{(n-m)}{2}}I_{n-m}(2\sqrt{x}) \quad (8)$$

where $I_{n-m}(\cdot)$ is a modified Bessel function of the first kind.

For the special case of $m = 1$, corresponding either to a multiple-input single-output (MISO) or to a single-input multiple-output (SIMO) system we have the following simpler expressions. Here, the channel matrix collapses into either a row or a column vector $\mathbf{h}$ whose ith element is denoted as h_i, and the outage probability can be written as

$$P_{out}(R, \rho, K) = \Pr\left\{ \log\left(1 + \frac{\rho}{n_T}\sum_{l=1}^{n}|h_l|^2\right) < R\right\}. \quad (9)$$

The sum $U = \sum_{l=1}^{n}|h_l|^2$ is a complex noncentral chi-squared random variable with n degrees of freedom and non-centrality parameter Kn. The pdf of U is given in [15] as

$$f_U(u) = (K+1)\left[\frac{(K+1)u}{Kn}\right]^{(n-1)/2} e^{-Kn-(K+1)u}$$
$$\cdot I_{n-1}\left(2\sqrt{K(K+1)nu}\right). \quad (10)$$

B. The Asymptotic SNR Gap (ASG)

In this section, we show the asymptotic behavior in the limit of high SNR. As we see in Fig. 1 and 2 (see Examples 1 and 2), the Rician effect is dominant at the low or medium SNR regime but the outage curve for Rician channels behaves as that of the Rayleigh fading at high SNR. We define the diversity order for given R, ρ, K, and $\mathbf{\Phi}$ as

$$d_o(R, \rho, K, \mathbf{\Phi}) = -\frac{d\log P_{out}(R, \rho, K, \mathbf{\Phi})}{d\log \rho}.$$

In Theorem 1, we prove that the diversity order, which is the slope of the outage probability (on a log-log scale), remains the same but the ASG between Rayleigh and Rician fading channels exists at high SNR.

Definition 1: In the limit of high SNR, the ASG $\Delta_a(R, K, \mathbf{\Phi})$, if it exists, is defined as

$$\Delta_a(R, K, \mathbf{\Phi}) \triangleq \lim_{\rho \to \infty} \frac{P_o^I(R, P_{out}(R, \rho, K, \mathbf{\Phi}), 0, \mathbf{\Phi})}{\rho} \quad (11)$$

where P_o^I is the inverse function of P_{out} on the second argument, i.e., $P_o^I(R, P_{out}(R, \rho, K, \mathbf{\Phi}), K, \mathbf{\Phi}) = \rho$.

Theorem 1: The ASG is given by

$$\Delta_a(R, K, \mathbf{\Phi}) = \frac{e^{K\text{tr}(\mathbf{\Phi})/mn}}{K+1}. \quad (12)$$
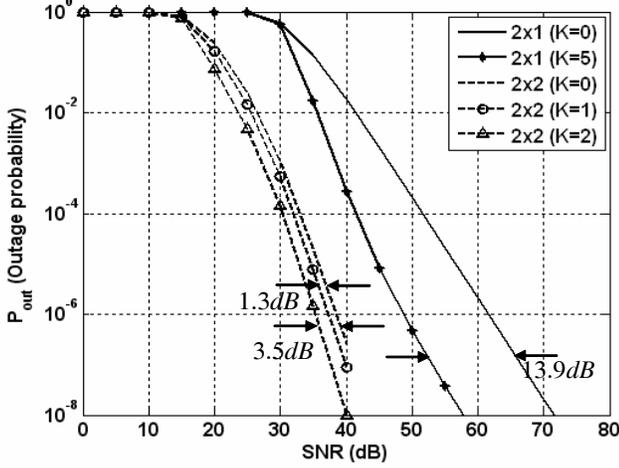
Fig. 1. Outage probability for $R = 10$ and $K = 0, 1, 2, 5$ (MISO and MIMO cases). The ASGs are shown.

Fig. 2. Outage probability for $R = 12$, $K = 10, 13$, and $n_T = 2$ (MISO case). The MDO and FSG are shown.

Proof: We refer readers to the full paper [16]. ∎

Note that this gap is irrelevant to the target data rate R. Furthermore in the limit of high SNR regime, it is ascertained that the diversity order for a given R of the Rician channel is the same that of the Rayleigh channel.

Example 1: Consider 2×2 MIMO and 2×1 MISO systems with $K = 0, 1, 2$ and $K = 0, 5$ for Rician-faded channels, respectively, where the target data rate $R = 10$ is used. Fig. 1 shows that as ρ becomes high, there is a fixed gap between two outage curves for Rayleigh and Rician channels. The analytical calculation in Theorem 1 shows: $\Delta_a \approx 1.33, 3.91, 13.93$ dB for $K = 1$, $K = 2$ cases in 2×2 MIMO and $K = 5$ case in 2×1 MISO, respectively. Our analytical results match the numerical results at finite-SNR's very well.

C. The Maximum Diversity Order (MDO) and the Finite-SNR Gap (FSG)

In this section, we show that the outage behavior of the Rician channel exhibits the maximum diversity order at a certain SNR. Since such an operating condition can give us the best diversity order, it is of practical interest to find when and how much gain is obtained.

Definition 2: Let β denote $\beta = \log \rho$. The MDO $d_{\max}(R, K, \mathbf{\Phi})$ is defined as

$$d_{\max}(R, K, \mathbf{\Phi}) \triangleq d_o(R, \rho^*, K, \mathbf{\Phi}) \qquad (13)$$

at the corresponding desired SNR

$$\beta_{op}(R, K, \mathbf{\Phi}) \triangleq \log \operatorname*{argmax}_{\rho} d_o(R, \rho, K, \mathbf{\Phi}) \qquad (14)$$

where $\log \rho^* = \beta_{op}(R, K, \mathbf{\Phi})$.

Unlike a Rayleigh-faded channel, the diversity order for a given SNR in Rician channels can also be affected by the Rician parameters K and $\mathbf{\Phi}$ as well as the target data rate R. In AWGN channels, i.e., deterministic channels, there exists a minimum SNR threshold denoted as $\beta_G(R)$ in the log scale, above which the outage probability in (5) tends to
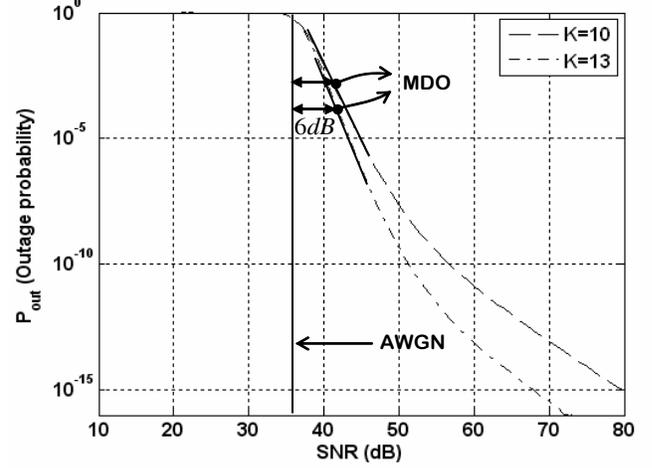
zero. It would be meaningful to examine the gap between $\beta_{op}(R, K, \mathbf{\Phi})$ and $\beta_G(R)$, which is defined as the FSG.

Definition 3: As the Rician factor K tends to infinity, the FSG is defined as

$$\Delta_f(R) \triangleq \lim_{K \to \infty} \left[2^{\beta_{op}(R, K, \mathbf{\Phi}) - \beta_G(R)} \right]. \qquad (15)$$

For the MISO and SIMO cases, we have the following theorem, where $\mathbf{\Phi}$ is dropped from parameters.

Theorem 2: In the limit of large K for MISO or SIMO systems, we get

$$\beta_{op}(R, K) = \log \left(\frac{4n_T(2^R - 1)}{n} \right) + O\left(\frac{1}{K} \right) \qquad (16)$$

$$d_{\max}(R, K) = \frac{n}{4}K + \frac{1 + 2n}{4} + O\left(\frac{1}{K} \right). \qquad (17)$$

Proof: We refer readers to the full paper [16]. ∎

This means $\beta_{op}(R, K)$ is irrelevant to K and $d_{\max}(R, K)$ is linear in K at high K. From Theorem 2, it is easily found that the outage probability at $\beta_{op}(R, K)$ is irrelevant to the target data rate R as well as the Rician factor K at large K.

Theorem 3: In MISO or SIMO systems, the FSG is given by

$$\Delta_f(R) = 4, \qquad (18)$$

which is equal to 6dB.

Proof: We refer readers to the full paper [16]. ∎

Note that it is irrelevant to the Rician factor K or the target data rate R.

Example 2: Consider a MISO system with 2 transmit antennas. Numerical results show the outage probability versus SNR per receive antenna in dB with parameters $R = 12$ and $K = 10, 13$ in Fig. 2. The irrelevance of $\beta_{op}(R, K)$ to K, linearity of the MDO with respect to K, and the FSG can be seen from this figure. Similar can be also observed in Fig. 3 where parameters $R = 12$ and $K = 3, 10, 13$ are used. Although the analytical results of the MDO are asymptotic, we
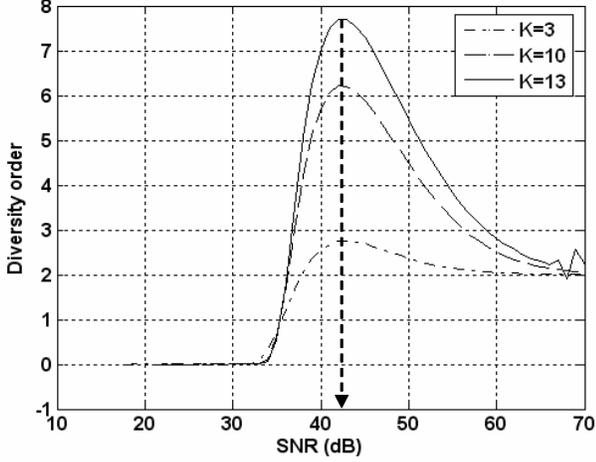
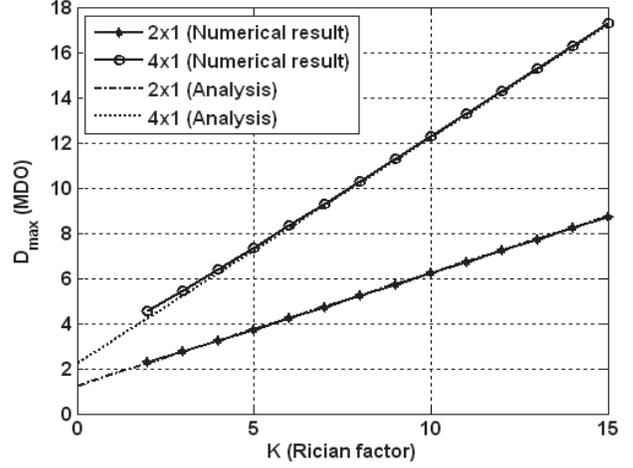Fig. 3. Diversity order for $R = 12$, $K = 3, 10, 13$, and $n_T = 2$ (MISO case). The MDO are also shown.

Fig. 4. The MDO versus Rician factor K. The curves for the numerical results and analytical ones nearly coincide.

show in Fig. 4 that they are quite accurate, even for relatively small K.

IV. THE DIFFERENTIAL DIVERSITY-MULTIPLEXING TRADEOFF (DDMT)

In this section, we define DDMT that can explain the DMT when the variations in P_{out}, ρ, and R are small. This is also useful in analyzing the DMT for Rician channels as well as Rayleigh channels. Our work builds on the DMT and TRT formulation in [4], [10]. In [4], the multiplexing gain r and the diversity gain d are defined by

$$r = \lim_{\rho \to \infty} \frac{R(\rho)}{\log \rho} \qquad \text{and} \qquad d = -\lim_{\rho \to \infty} \frac{\log P_e(\rho)}{\log \rho}$$

where $\log P_e(\rho)$ is the average error probability.

The optimal DMT is given as a piecewise linear curve connecting the following corner points

$$d^*(r) = (n_T - r)(n_R - r)$$

where r is an integer provided that the block length $l \geq n_T + n_R - 1$. Furthermore, the TRT helps us to characterize the relationship between R, $\log \rho$, and P_e. The result considers, however, the limit of the high SNR and it fails to explain the finite-SNR behavior for Rician channels such as the MDO and TG analyzed in Section III. In this work, we formulate the DDMT that can also capture such finite-SNR behaviors.

A. Case 1: Rayleigh channel

The TRT results [10] show that the slope of the outage probability and the horizontal spacing behave differently in different operating regions on the R-$\log \rho$ plane. Motivated by this, we characterize the DDMT of Rayleigh channels for different operating regions.

Definition 4: The differential multiplexing gain $r_D(k)$ and the differential diversity gain $d_D(k)$ for the operating region given by $\mathcal{R}(k)$ are defined by

$$d_D(k) \triangleq -\lim_{\substack{\rho 1 \to \infty \\ R_1, R_2 \in \mathcal{R}(k)}} \frac{\log P_{\text{out}}(R_1, \rho_1) - \log P_{\text{out}}(R_2, \rho_2)}{\log \rho_1 - \log \rho_2} \quad (19)$$

$$r_D(k) \triangleq \lim_{\substack{\rho 1 \to \infty \\ R_1, R_2 \in \mathcal{R}(k)}} \frac{R_1 - R_2}{\log \rho_1 - \log \rho_2} \quad (20)$$

where

$$\log \rho_2 = (1 + \alpha) \log \rho_1 \quad (21)$$

for a positive constant α,

$$\mathcal{R}(k) = \left\{ R \big| k < \frac{R}{\log \rho} < k + 1 \right\} \quad (22)$$

for $0 \leq k < m$, $k \in \mathbb{Z}$, and $\mathcal{R}(m) = \{R | R > m \log \rho\}$.

We show there exist m DDMT curves in Rayleigh fading channels in the following theorem.

Theorem 4: The DDMT of the Rayleigh fading channels is given by

$$d_D(k) = g(k) - c(k)r_D(k) \quad (23)$$

where

$$c(k) = m + n - (2k + 1) \quad \text{and} \quad g(k) = mn - k(k + 1) \quad (24)$$

for $0 \leq k < m$.

Proof: We refer readers to the full paper [16]. ∎

Example 3: Consider a 3×3 Rayleigh-faded channel. The DDMT curve is plotted in Fig. 5. Each DDMT curve is a line connecting the maximum differential diversity gain $g(k)$ and the maximum differential multiplexing gain $g(k)/c(k)$ of each region. Note that the DMT coincides with the outer most boundary of the DDMT curves.

B. Case 2: Rician channel

By applying the outage analysis for Rician channels we can also examine the DDMT curve for $\mathcal{R}(0)$ in Rician channels. For a Rician case, K or Φ is added to or dropped from parameters when needed.
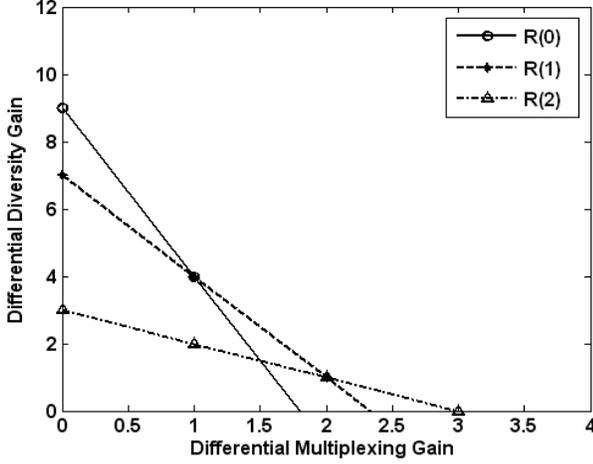
Fig. 5.   The DDMT curves for $n_T = n_R = 3$ in Rayleigh channels.

Fig. 6.   The DDMT curves in multiple antenna Rician channels.

Corollary 1: When $k = 0$, the DDMT of the Rician fading channels is the same as that of the Rayleigh case and is given by

$$d_D(0) = mn - (m + n - 1)r_D(0). \qquad (25)$$

Proof: We refer readers to the full paper [16]. ∎

As the Rician factor K tends to infinity, i.e., in AWGN channels, the DDMT is given by the following.

Theorem 5: The DDMT of the deterministic channel $\bar{\mathbf{H}}$ is given by

$$r_D = \text{rank}(\bar{\mathbf{H}}) \qquad (26)$$

where r_D is the differential multiplexing gain for the deterministic channels.

Proof: We refer readers to the full paper [16]. ∎

Note that the differential diversity gain $d_D \to \infty$ in this case. In addition to the above operating region, it is meaningful to investigate the DDMT curve for the operating region specified as follows.

Definition 5: The differential diversity gain $\bar{d}_D(\bar{r}_D, R, K)$ of MISO or SIMO systems is defined by

$$\bar{d}_D(\bar{r}_D, R, K) \triangleq \lim_{\delta \log \rho \to 0} \left[\frac{\log P_{out}(R, \rho^*, K)}{\delta \log \rho} \right.$$
$$\left. - \frac{\log P_{out}(R + \bar{r}_D \delta \log \rho, 2^{\log \rho^* + \delta \log \rho}, K)}{\delta \log \rho} \right] \quad (27)$$

at asymptotically large K where $\bar{r}_D$ is the differential multiplexing gain.

Theorem 6: Let $r_{\max}(R)$ denote the differential multiplexing gain satisfying $\bar{d}_D(\bar{r}_D, R, K) = 0$. For the operating region in the vicinity of the MDO, the DDMT of the Rician fading channels is obtained as

$$\bar{d}_D(\bar{r}_D, R, K) = d_{\max}(R, K) - \frac{d_{\max}(R, K)}{r_{\max}(R)} \bar{r}_D \qquad (28)$$

where $r_{\max}(R) = 1 - 2^{-R}$.

Proof: We refer readers to the full paper [16]. ∎

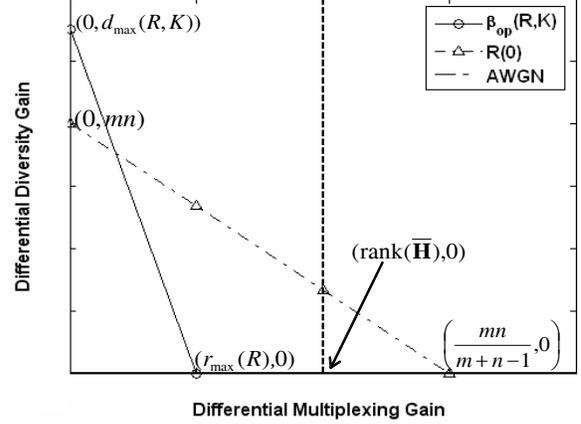Note that $r_{\max}(R)$ is irrelevant to K in the limit of large K.

Example 4: Fig. 6 shows the DDMT curves for 2 different operating regions in Rician channels corresponding to Corollary 1 and Theorem 6. It also shows the DDMT for the deterministic $\bar{\mathbf{H}}$.

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