Low Complexity Design of Space-Time Convolutional Codes with High Spectral Efficiencies

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ABSTRACT
Space time convolutional codes (STCCs) are an effective way to combine transmit diversity with coding. The computational complexity of designing STCCs generally increases exponentially with the constellation size of the transmitted symbols. In this paper, we first present an innovative approach to design STCCs with high spectral efficiencies by utilizing QPSK STCCs as component codes and consequently. Unlike existing techniques, the search space does not grow exponentially with the constellation size. Then, we present two approaches to reduce the computational complexity of our proposed scheme. This scheme is applicable to cases with any number of transmit antennas without any requirement to change the encoder design. Simulation results evaluate the performance of our approach for the case of two transmit antennas and several different number of receive antennas, a spectral efficiency of 4 bits/s/Hz, and slow Rayleigh fading channels.

Categories and Subject Descriptors
B.4.1 [Data Communications Devices]: Receivers and Transmitters.

General Terms
Algorithms, Design.

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Space-time codes, MIMO systems.

1. INTRODUCTION
The design of multiple-input multiple-output (MIMO) systems continues to attract the attention of researchers. A system with MIMO capability has much higher spectral efficiency than that of single-input single-output (SISO) and single-input multiple-output (SIMO) systems [1]. The high capacity of a MIMO system stems from the fact that a MIMO system is mathematically equivalent to a set of parallel independent channels. We can exploit this idea to send redundant data to achieve diversity to combat channel fading and increase transmission reliability. Alternatively, we can increase data rate. These two aspects of MIMO channels, namely diversity and multiplexing and their tradeoffs, were investigated by many researchers [2], [3]. Thus, the concept of sending different data from separate transmit antennas (spatial multiplexing) or coded data from more than one transmit antenna (spatial diversity) has been explored [4], [5]. Even with the extensive research on space time convolutional codes (STCCs), STCC designs are lacking for cases with large constellation size and/or a large number of transmit antennas.

We will introduce a new approach to design STCCs for any arbitrary $n \times m$ MIMO system with a search space that does not increase exponentially with the constellation size and does not increase at all with the number of transmit antennas.

In this context, for n transmit antennas, one can transmit at most n different symbols at a time. One can also view an $n \times m$ MIMO system as an equivalent of n distinct $1 \times m$ systems. Then by producing a common design for each individual $1 \times m$ system, we arrive at an approach whose complexity does not grow with the number of transmit antennas. Further, we will build our STCCs from combining small (QPSK) constellation size STCCs such that our search complexity grows slowly with constellation size. To describe our approach, we first demonstrate that a system employing a STCC can be implemented with only a single transmit antenna when there are multiple receive antennas. The idea is to transmit more than one symbol from a single transmit antenna during a symbol period by superimposing the encoded symbols on top of each other. This objective is achieved by inducing randomness into the system to create additional channel paths, called virtual paths. Inducing randomness into a physical channel has been proposed by many authors [6], [7],[13],[14],[15]. The main objective of these techniques is to induce more fluctuations into the channel. The randomization concept has been also proposed for space-time code applications [7],[15].

In this paper, we propose to induce randomness into the

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physical channel in a new way. The goal is to explore the rich diversity capabilities of MIMO systems. We will derive the optimum condition to attain maximum coding and diversity gain for slow fading channels. Our optimality criterion is based on an upper bound on the pairwise block error probability [4]. Our objective is to design STCCs for SIMO systems and to extend this approach to design efficient STCCs for MIMO systems by modeling a MIMO system as a combination of multiple SIMO systems.

The outline of the paper is as follows. In Section II, we will review the system model and formulate our proposed randomization technique in STCC systems under slow fading wireless channels. The induced random variables that depend on information data using a mapping approach is described in Section III. The combined array processing based design is described briefly in Section IV. We show the simulation results for the performance of the proposed algorithms in section V. Section VI contains the conclusion.

2. System Model and Randomization Technique

We consider a wireless communication system utilizing n transmit and m receive antennas. The channel path gain from transmit antenna i to receive antenna j is denoted by $h_{i,j}$ and is a complex Gaussian random variable with zero mean and variance 0.5 per complex dimension. We assume that different channel path gains are statistically independent. We also assume that the channel coefficients are constant during one block of data and change independently from one block to another. The received data r_t^j at antenna j and time t can be written as

$$r_t^j = \sum_{i=1}^{n} h_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m \tag{1}$$

where c_t^i is the complex transmit symbol with unit average power sent from antenna i at time t, n_t^j is the additive white complex Gaussian noise sample with zero mean and variance $N_0/2$ per dimension, and E_s is the contraction factor of the signal constellation.

STCCs were originally designed to achieve diversity and coding gain in wireless fading channels utilizing multiple transmit antennas. The search space for designing these codes increases exponentially with the constellation size. In this paper, we will first show that one can apply STCCs in systems with only a single transmit antenna by using randomization techniques as long as we have multiple receive antennas. Then we use this approach to design STCCs with high spectral efficiencies by properly employing STCCs with smaller constellation size, such as QPSK. Therefore, we assume for now that the number of transmit antennas is equal to 1, i.e., $n=1$. Using this assumption, (1) can be written as

$$r_t^j = h_{1,j} C_t \sqrt{E_s} + n_t^j, \quad 1 \le j \le m \tag{2}$$

Here again, the physical channel path gains, the $h_{1,j}$'s, are independent complex normal random variables with zero mean and variance 0.5 per dimension. Our objective is to design the transmit signal (C_t) such that we can utilize STCCs in the absence of multiple transmit antennas. We propose to use the transmitted signal

$$C_t = A_t^1 c_t^1 + A_t^2 c_t^2 + \ldots + A_t^n c_t^n \ , \tag{3}$$

where the A_t^i 's are called the induced random variables and the c_t^i 's are from the output of a STCC encoder. The A_t^i 's are also independent of the physical channel path gains, the $h_{1,j}$'s. Combining (2) and (3) leads to

$$r_t^j = \sum_{i=1}^{n} h'_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m \tag{4}$$

where $h'_{i,j} = h_{1,j} A_t^i$ is called virtual path gain. We call this the virtual path because only m physical paths exist in this system and by inducing random data at the transmitter, we have created $n \times m$ virtual paths. Of course, some of these virtual paths are statistically dependent, but this approach will allow us to employ a STCC in a setting with a single transmit antenna.

One immediate application of the approach outlined above is to model an $n \times m$ MIMO system as an equivalent group of n distinct $1 \times m$ system. This interpretation results in an architecture very similar to the V-BLAST architecture [12] with the exception that STCC is implemented for the protection of the symbols in each path.

The induced random variables (A_t^i 's) can either change from symbol to symbol or they can be constant during one data frame. Our intention is to derive conditions under which one can obtain the minimum upper bound on the block error probability. A block error occurs when the decoded data sequence

$$\underline{\mathbf{E}} = e_1^1 \ldots e_1^n \ldots e_N^1 \ldots e_N^n$$

is different from the transmit sequence

$$\underline{\mathbf{C}} = c_1^1 \ldots c_1^n \ldots c_N^1 \ldots c_N^n \ ,$$

where N is the number of symbols in one block. Applying an upper bound on the conditional pairwise block error probability (slow fading channel) for a maximum likelihood receiver in [4] to the virtual paths ($h'_{i,j}$'s), we see that the conditional pairwise upper bound on the block error probability is

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \middle| A_t^i, h_{1,j}, 1 \le i \le n, 1 \le j \le m\right) \le \prod_{j=1}^{m} \exp\left(-\left|h_{1,j}\right|^2 A B_s\left(\underline{\mathbf{C}}, \underline{\mathbf{E}}\right) A^* \frac{E_s}{4N_0}\right) \quad (5)$$

where $A = \left[A_t^1, A_t^2, ..., A_t^n\right]$ is the vector whose elements are the induced random variables. It can be shown that [4] $B_s\left(\underline{\mathbf{C}}, \underline{\mathbf{E}}\right) = V^* D V$ is a Hermitian matrix, V is an unitary matrix whose rows $v_i, 1 \le j \le n$ are the eigenvectors of $B_s\left(\underline{\mathbf{C}}, \underline{\mathbf{E}}\right)$ and $D = diag(\lambda_1, ..., \lambda_n)$ where the λ_j s are the eigenvalues of $B_s\left(\underline{\mathbf{C}}, \underline{\mathbf{E}}\right)$. Let $[T_1, ..., T_n] = AV^*$, then (5) can be written as

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \middle| A_t^i, h_{1,j}, 1 \le i \le n, 1 \le j \le m\right) \le \prod_{j=1}^{m} \exp\left(-\left|h_{1,j}\right|^2 \sum_{i=1}^{n} \lambda_i \left|T_i\right|^2 \frac{E_s}{4N_0}\right) \quad (6)$$

By taking the average over (6) with respect to channel coefficients and induced random variables, assuming Rayleigh fading, we arrive at

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}}\right) \le \left\langle \left(\frac{1}{1 + \sum_{i=1}^{n} \lambda_i \left|T_i\right|^2 \left(E_s / 4N_0\right)} \right)^m \right\rangle \quad (7)$$

where $\langle \cdot \rangle$ denotes the expected value with respect to $T_1, ..., T_n$.

3. Mapping-Based Design of Induced Random Variables

In our previous work [8], we found the optimum solution for $T_1, ..., T_n$. The result was a set of discrete random variables that statistically depend on the STCC encoder output. The objective was two fold. First, to reduce the number of points in the constellation for the transmit signal C_t that was defined in (3), and second, to force the amplitude of the transmit signal to be constant. This approach is also more suitable for practical applications.

The conditional upper bound on the pairwise block error probability in this case can be derived as [8]

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}} \middle| h_{1,j}, 1 \le j \le m\right) \le \prod_{j=1}^{m} \exp\left(-\left|h_{1,j}\right|^2 \sum_{i=1}^{N} \left|C_t - E_t\right|^2 \frac{E_s}{4N_0}\right) \quad (8)$$

where E_t is the error signal defined similar to (3). Note that in this case, the induced random variables, the A_t^i 's, are statistically dependent on the output of the STCC encoder and for that reason, it is not feasible to separate them in this equation. Averaging over (8) with respect to the channel coefficients, we arrive at

$$P\left(\underline{\mathbf{C}} \to \underline{\mathbf{E}}\right) \le \left(\frac{1}{1 + \sum_{i=1}^{N} \left|C_t - E_t\right|^2 \left(E_s / 4N_0\right)} \right)^m \quad (9)$$

This equation suggests that in order to minimize the upper bound on the pairwise block error probability, we need to maximize the minimum Euclidean distance between the modified codeword $\left(C_t, t = 1, ..., N\right)$ and the codeword chosen in error $\left(E_t, t = 1, ..., N\right)$. Therefore, we need to design the induced random variables such that this minimum Euclidean distance is maximized.

There are several different design techniques for QPSK STCCs that consider maximizing the minimum Euclidean distance between the pair of codewords [9], [10] while providing the maximum coding and diversity gains. In this paper, we use the QPSK STCC design of [9] with 16 states for simulation and code design. Fig. 1 demonstrates the signal constellation of QPSK symbol. Its 16 states trellis diagram for STCC is described in [9]. This code is designed to have the maximum diversity and coding gains as well as maximizing the minimum Euclidean distance between the codeword pairs.

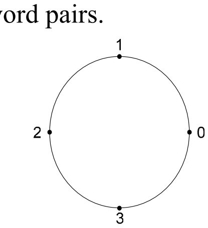
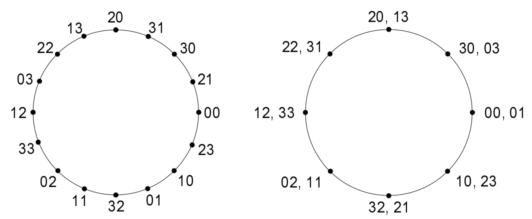


Fig. 1. Mapping of QPSK constellation

Our objective is to design induced random variables such that when we combine two QPSK signals, the new transmitted signal has a finite number of constellation points, i.e., equivalent to a 16-PSK constellation in this example, and at the same time maximizes the minimum Euclidean distance between codewords of the new transmitted signal $\left(C_t\right)$. Fig. 2(a) illustrates this mapping for two QPSK signals. In this construction and mapping, we have chosen to use only induced random variables that have unit amplitude [8] and statistically dependent random phases. These random phases depend on the signals produced by the STCC encoder as illustrated in Fig. 2(a).



(a) (b)

Fig. 2. Description of the mapping of two QPSK signals into (a) 16-PSK and (b) 8-PSK constellations.

The construction of the transmitted signals (C_t) is accomplished in two steps. First, since all different combinations of two QPSK signals yield a maximum of 16 possible choices, we use the trellis diagram of STTC to assign these 16 combinations such that the minimum Euclidean distance between any two codewords is maximized. This objective is achieved by assigning every two QPSK symbols into a point in the 16-PSK constellation for all 16 possible choices and then computing the distance properties of that particular code. The signal assignment that maximizes the minimum Euclidean distance of the code will be selected. In the second step, we compute the A_t^i's such that the linear combination of two QPSK symbols using (3) will create the appropriate C_t as shown in Fig. 2(a). Utilizing this construction, for any two QPSK signals at the output of STCC encoder, the values of these induced random variables are known. However, since the signals from the encoder output are random and unknown at the receiver, the induced random variables are also unknown and random at the receiver.

Note that for this particular construction of the signal, we no longer need to keep any table of induced random variables since there is a one-to-one mapping between any two QPSK signals from the output of STCC encoder and A_i's. The design of these dependent induced random variables is based on the desire to maximize the minimum Euclidean distance between any two codewords to improve (9). We do not claim that such construction is necessarily the optimum solution, however, our simulation results will show that this construction outperforms the 16-QAM STCC of [4]. Here, if we assign every two QPSK symbols to a point in a lower constellation size, we can effectively reduce the computational complexity of the receiver. Note that we can map two QPSK symbols into an M-PSK constellation where $4 \le M \le 16$. By mapping every two QPSK symbols into a point in a smaller constellation size, we can dramatically reduce the computational complexity of the receiver. Since every point in the constellation belongs to a unique position in the trellis diagram, reducing the constellation size of the transmitted symbol will not create any ambiguity at the receiver.

Fig. 2(b) is a 8-PSK constellation for mapping the combined two QPSK signals. In this case, we design the induced random variables such that the new combined two QPSK transmitted signals are mapped to a point in an 8-PSK constellation and the assigned signal for each point does not create any ambiguity at the receiver. We design this mapping to maximize the minimum Euclidean distance between any two codeword pairs and the simulation result for this 8-PSK mapping demonstrates small performance degradation compared to the 16-PSK mapping.

4. Design Based on Successive interference cancellation Approach

Another approach to reduce the computational complexity in the system is the group interference suppression method using a combined array processing technique [11]. This method effectively reduces the computational complexity by partitioning antennas at the transmitter into small groups, and using individual STCCs to transmit information from each group of antennas. At the receiver, the individual STCCs can be decoded while suppressing signals from other groups of antennas by treating them as interference [11].

Note that this unique representation of a MIMO system by n distinct SIMO systems allows us to separate the decoding of the signals sent from each transmit antenna.

In Fig. 3, the STCC encoder outputs are divided into two groups (G_1 , G_2) with equal size. Every two STCC encoder outputs are mapped into a symbol using the induced random variables, A_t^i s. We utilize the results from the previous section, where we designed a set of induced random variables that statistically depend on the STCC encoder output. Finally, the mapped signals are transmitted by the transmit antennas.

The received signals can be derived in the vector form as

$$\mathbf{r}_t = \Phi \mathbf{s}_t + \mathbf{n}_t \ , \tag{10}$$

where

$$\Phi = \begin{pmatrix} h_{1,1}A_t^1 & h_{1,1}A_t^2 & h_{2,1}A_{t+N/2}^1 & h_{2,1}A_{t+N/2}^2 \\ h_{1,2}A_t^1 & h_{1,2}A_t^2 & h_{2,2}A_{t+N/2}^1 & h_{2,2}A_{t+N/2}^2 \\ \vdots & \vdots & \vdots & \vdots \\ h_{1,m}A_t^1 & h_{1,m}A_t^2 & h_{2,m}A_{t+N/2}^1 & h_{2,m}A_{t+N/2}^2 \end{pmatrix},$$

$$\mathbf{s}_t = \begin{pmatrix} c_t^1 \\ c_t^2 \\ c_{t+N/2}^1 \\ c_{t+N/2}^2 \end{pmatrix}, \text{ and } \mathbf{n}_t = \begin{pmatrix} n_t^1 \\ n_t^2 \\ \vdots \\ n_t^m \end{pmatrix}. \tag{11}$$

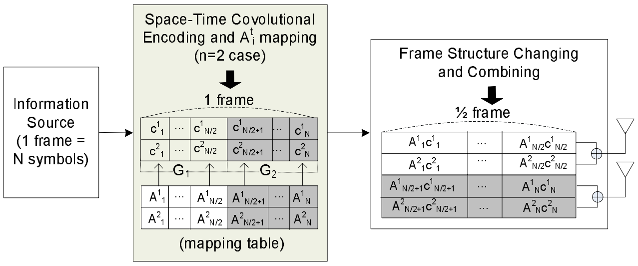


Fig. 3. Block diagram of the transmitter (combined array process based approach)

Here, the number of physical channel paths is $2m$, while there are $4m$ virtual channel paths. Let's denote the number of virtual transmit antennas as n', i.e., $n' = 4$ for this example. Considering the n' transmit antennas divided into two groups, each group has two virtual transmit antennas. To decode G_1, we suppress the interfering signal (G_2) as described in [11],[12]. We let $\Delta(G_1)$ denote the matrix as

$$\Delta(G_1) = \begin{pmatrix} h_{2,1}A^1_{t+N/2} & h_{2,1}A^2_{t+N/2} \\ \vdots & \vdots \\ h_{2,1}A^1_{t+N/2} & h_{2,m}A^2_{t+N/2} \end{pmatrix} \qquad (12)$$

In the matrix $\Delta(G_1)$, the column vectors are not independent each other and, hence, the rank of $\Delta(G_1)$ is equal to 1. Therefore, we can compute three orthonormal vectors in the null space of $\Delta(G_1)$. Let $\Omega(G_1)$ denote $3 \times m$ matrix composed of these vectors. By multiplying $\Omega(G_1)$ to both sides of (10), we arrive at

$$\hat{\mathbf{r}}_t = \hat{\Phi}\mathbf{s}^1_t + \hat{\mathbf{n}}_t \qquad (13)$$

where $\mathbf{s}^1_t = \left(c^1_t, c^2_t\right)^T$. Finally, we can decode G_1 for the first half frame using the minimum decision metric [11] and, after decoding G_1, subtract the contribution of G_1 from the received signals. The remaining received signal corresponds to G_2, from the second half of the frame and can be decoded following the technique described in [11].

5. Simulation Results

In the simulations presented in this section, a coherent detection is assumed along with perfect knowledge of the channel coefficients at the receiver. For applying the proposed algorithm (PA), we design a STCC system with spectral efficiency of 4 bits/s/Hz, using a QPSK STCC for each antenna, and compare it with the 16-QAM STCC of [4] when there are two transmit antennas.

Fig. 4 clearly shows that our approach can perform better than that of [4] for a $2 \times m$ MIMO systems with $3 \le m \le 5$ and a block length of 520 bits for slow Rayleigh fading channels. Note that the 8-PSK constellation mapping produces little degradation in performance compared to that for the 16-PSK constellation while reducing the computational complexity considerably. Note that the implementation of the Viterbi Algorithm (VA) for the new scheme is slightly different from the original VA. In the new PA, at each time interval, the equivalent of n data symbols are transmitted simultaneously from n different transmit antennas. In this case, each path metric for the VA using the PA will compute the equivalent of n path metrics from the original trellis diagram. Accordingly, we need to incorporate this feature when we compute the survivor paths for each STCC block.

In Fig. 5, we show the performance of the new STCC system based on the combined array processing (CAP) approach under similar condition as in Fig. 5. When the number of receive antennas is increased, the performance gap between the CAP-based PA and [4] is reduced. Finally, the proposed CAP-based design is another low complexity solution for applying our randomization techniques to STCC systems. In this approach, the complexity of the receiver increases linearly with an increase in constellation size, which is similar to STBC.

6. Conclusion

In this paper, we showed the application of randomization techniques to STCCs and introduced a new approach to induce randomness into the channel. We introduced two techniques to reduce the computational complexity. One approach is based on reducing the constellation size of the combined transmitted signal in our proposed algorithm. We also proposed using the successive interference cancellation concept to design a low complexity receiver. Note that the complexity of the latter case is comparable to that of space-time block codes (STBC). Besides, this approach can be applied to any number of transmit antennas while the design of STBC depends on the number of transmit antennas. Future work should concentrate on comparing this approach with STBCs combined with trellis coded modulation schemes.

7. ACKNOWLEDGMENTS

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8. REFERENCES

[1] G.J. Foschini, Jr. and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple

antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311-335, March 1998.

[2] R.W. Heath and A.J. Paulraj, "Switching between diversity and multiplexing in MIMO Systems," *IEEE Transaction on Communications*, vol. 53, no. 6, pp. 962-969, June 2005.

[3] L. Zheng and D.N. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-Antenna channels," *IEEE Transaction on Information Theory*, pp.1073-1096, May 2003.

[4] V. Tarokh, N. Seshadri, R. Calderbank, "Space-time codes for high data rate wireless communications: performance criterion and code construction," *IEEE Transaction on Information Theory*, vol. 44, pp. 744-765, March 1998.

[5] G.L. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, pp. 41-59, Fall 1996

[6] P. Viswanath, D.N. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Transaction on Information Theory*, pp.1277-1294, June 2002.

[7] H. El Gammal, "On the robutstness of space-time coding," *IEEE Transaction on Signal Processing*, vol. 50, no. 10, pp.2417-2428, October 2002.

[8] H. R. Sadjadpour, K. Kim, H. Wang, R. Blum, and Y. Lee, "Application of randiomization techniques to space-time convolutional Codes," *IEEE WirelessCom 2005* and also accepted to *IEEE Transaction on Signal Processing*.

[9] Z. Chen, J. Yuan, and B. Vucetic, "An improved space-time trellis coded modulation scheme on slow Rayleigh fading channels," *IEEE International Conference on Communications*, pp. 1110-1116, June 2001.

[10] J. Yuan, Z. Chen, B. Vucetic, and W. Firmanto, "Performance and design of space-time coding in fading channels," *IEEE Transaction on Communications*, vol. 51, pp. 1991-1996, December 2003.

[11] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Combined array processing and space-time coding," *IEEE Transaction on Information Theory*, vol. 45, no.4, May 1999.

[12] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-BLAST: An architecture for realizing very high date rates over the rich-scattering wireless channels," *Proceedings of ISSSE,* Pisa, Italy, pp. 295-300, September 1998.

[13] W.Y. Kuo and M.P. Fitz, "Design and analysis of transmitter diversity using intentional frequency offset for wireless communications," *IEEE Transaction on Vehicular Technology*, pp. 871-881, 1997.

[14] A. Hottinen and O. Tirkkonen, "A Randomization Technique for Non-Orthogonal Space-Time Block Codes,' Proceedings of *IEEE Vehicular Technology Conference*, Rhodes, Greece, May 2001.

[15] E. Larsson, "Constellation Randomization (CoRa) for Outage improvement", *IEEE Globecom conference*, 2004.

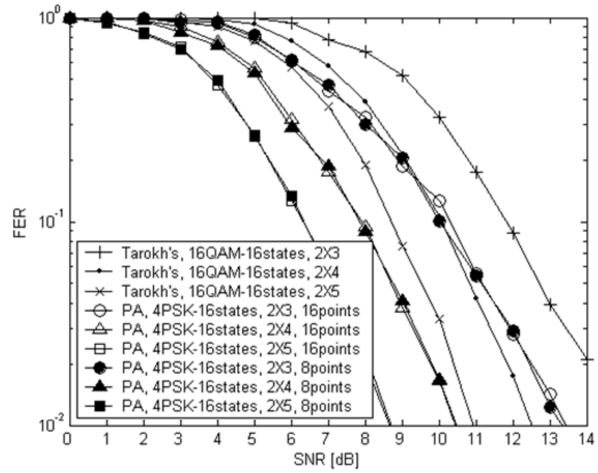

Fig. 4. Frame error rate comparison between 16-QAM STCC of [4] and PA (8 and 16 points-mapping) for $2\times 3,4,\text{and } 5$ MIMO systems.

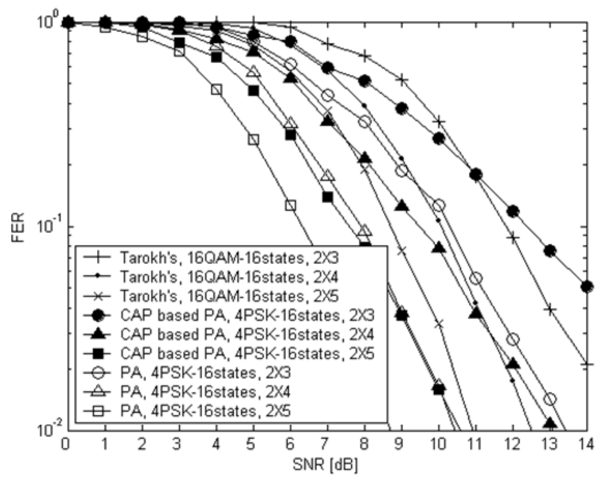

Fig. 5. Frame error rate comparison between 16-QAM STCC of [4] and PAs for $2\times 3,4,\text{and } 5$ MIMO systems.