

# Joint Adaptive Compensation for Amplifier Nonlinearity and Quadrature Modulation Errors

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**Abstract**—This paper proposes a technique that jointly compensates the amplifier nonlinearity and quadrature modulation (QM) errors. The proposed method is derived based on the indirect learning architecture, which has been originally developed for the predistortion (PD). The proposed compensator updates the parameter vector based on the recursive least squares (RLS) criterion. The advantage of the proposed adaptive technique is demonstrated through computer simulation.

## I. INTRODUCTION

With demanding high spectral efficiency in mobile communications, the transmitted signal tends to have a high peak to average ratio, and linearization of a power amplifier (PA) has become an important issue. A popular linearization technique is the baseband digital predistortion (PD) which is highly cost-effective [1]–[4]. One difficulty that can be encountered when using a PD technique is its vulnerability to quadrature modulation (QM) errors, including amplitude/phase imbalances and dc offset. This fact makes the PD difficult to be applied to some transmission architectures such as the direct conversion scheme. To overcome the difficulty, a feedback loop for QM compensation (QMC) is employed in addition to the feedback loop for PD [5]–[7]. This approach is effective but considerably increases hardware cost. The method in [8] jointly compensates for amplifier nonlinearity and QM errors, while using only one feedback loop, but it is suffered by slow adaptation. In [9], two-step compensation approach is proposed where the QMC parameters are adjusted first by bypassing the PA and then the PD parameters are found. This method also need one feedback path, but it is difficult to cope

with time-varying QM errors.

In this paper, we propose an alternative method for the joint compensation based on the indirect learning architecture, which has been originally developed for PD [2]–[4]. The proposed method requires only one feedback loop, yet exhibits fast adaptation characteristics.

The organization of this paper is as follows. The proposed compensation technique is derived in Section II and it is extended to the memory polynomial case in Section III. Finally, Section IV presents simulation results demonstrating the advantage of the proposed method.

## II. DERIVATION OF THE PROPOSED COMPENSATION TECHNIQUE

Fig. 1 shows the structure of the proposed transmission scheme. It employs the PD and QMC block, in which the PD and QMC subblocks are connected in cascade, whose parameters are adaptively adjusted based on the indirect learning architecture. Specifically, the PD and QMC training block is employed in the feedback path, and the coefficients obtained by the training block are used for the PD and QMC. All the signals shown in Fig. 1 are complex-valued: the outputs of the QM and PA, denoted by  $v(n)$  and  $y(n)$ , respectively, are the baseband complex envelope of the corresponding bandpass signal. It is assumed that the QM for frequency up-conversion causes errors but the quadrature demodulation (QDM) for frequency down-conversion is ideal. The ideal QDM can be realized if the superheterodyne structure together with the digital intermediate frequency technique is employed.

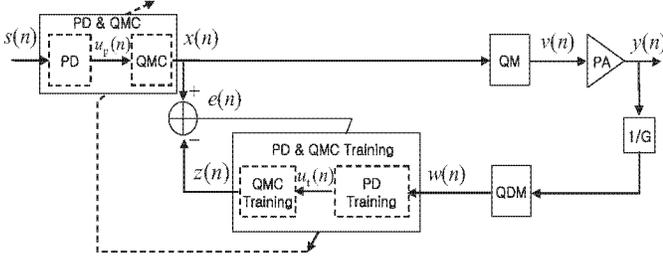


Fig. 1. Baseband model of a transmitter employing PD and QMC which are adaptively adjusted.

Suppose that a polynomial PD scheme is employed. Then the PD output  $u_p(n)$  is given by

$$u_p(n) = \mathbf{a}^T(n)\phi(s(n)) \quad (1)$$

where  $s(n)$  is the input to the PD,  $\mathbf{a}(n) = [a_1(n), a_3(n), \dots, a_{2P+1}(n)]^T$  is the  $P+1$ -dimensional coefficient vector and  $\phi(s(n)) = [\phi_1(s(n)), \phi_3(s(n)), \dots, \phi_{2P+1}(s(n))]^T$  with  $\phi_{2p+1}(x) = |x|^{2p}x$  [2],[3]. The QMC block is designed so that perfect compensation can be achieved under ideal situation. Let  $\epsilon$  and  $\theta$  denote the amplitude and phase imbalances, respectively. Then the QM output  $v(n)$  is written as

$$v(n) = \beta x(n) + \alpha x^*(n) + c \quad (2)$$

where

$$\begin{aligned} \beta &= (1/2)[1 + (1 + \epsilon)e^{j\theta}] \\ \alpha &= (1/2)[1 - (1 + \epsilon)e^{-j\theta}] \end{aligned}$$

and  $c$  represents the dc offset [10]. The condition for perfect QMC can be stated as follows.

**Observation 1.** The QMC becomes perfect ( $v(n) = u_p(n)$ ) if the QMC output  $x(n)$  is given by

$$x(n) = d_\beta u_p(n) + d_\alpha u_p^*(n) + d_c \quad (3)$$

where  $d_\beta = \beta^*/(|\beta|^2 - |\alpha|^2)$ ,  $d_\alpha = -\alpha/(|\beta|^2 - |\alpha|^2)$ , and  $d_c = (\alpha c^* - \beta^* c)/(|\beta|^2 - |\alpha|^2)$ .

This can be proved by using (3) in (2). Based on observation 1, the QMC is modelled as

$$x(n) = d_1(n)u_p(n) + d_2(n)u_p^*(n) + d_3(n), \quad (4)$$

and our objective is to find an adaptive algorithm that makes the coefficient vector  $[d_1(n), d_2(n), d_3(n)]$  in (4) converges to  $[d_\beta, d_\alpha, d_c]$  in (3). Using (1) in (4),  $x(n)$  is rewritten as

$$\begin{aligned} x(n) &= d_1(n)\mathbf{a}^T(n)\phi(s(n)) + d_2(n)\mathbf{a}^H(n)\phi^*(s(n)) + d_3(n) \\ &\triangleq \mathbf{b}_{\text{qm}}^T(n)\phi_{\text{qm}}(s(n)) \end{aligned} \quad (5)$$

where  $\mathbf{b}_{\text{qm}}(n)$  and  $\phi_{\text{qm}}(s(n))$  are  $2(P+1)+1$ -dimensional parameter vectors given by  $\mathbf{b}_{\text{qm}}(n) = [d_1(n)\mathbf{a}^T(n), d_2(n)\mathbf{a}^H(n), d_3(n)]^T$  and  $\phi_{\text{qm}}(s(n)) = [\phi^T(s(n)), \phi^H(s(n)), 1]^T$ , respectively. Since the coefficients of the PD and QMC training block in the feedback path are identical to those of the PD and QMC block in the feed forward path, the output  $z(n)$  of the training block can be written as

$$z(n) = \mathbf{b}_{\text{qm}}^T(n)\phi_{\text{qm}}(w(n)) \quad (6)$$

where  $w(n)$  is the output of the QDM. To make  $z(n)$  close to  $x(n)$ , the following LS cost function is defined.

$$\begin{aligned} J(n) &= \sum_{l=1}^n \lambda^{n-l} |x(l) - z(l)|^2 \\ &= \sum_{l=1}^n \lambda^{n-l} |x(l) - \mathbf{b}_{\text{qm}}^T(n)\phi_{\text{qm}}(w(l))|^2 \end{aligned} \quad (7)$$

where  $\lambda$  is the forgetting factor ( $0 < \lambda \leq 1$ ). An RLS algorithm for obtaining the optimal  $\mathbf{b}_{\text{qm}}(n)$ , that minimizes  $J(n)$ , is summarized as follows.

#### RLS algorithm for PD and QMC

Initialize the algorithm by setting

$$\Phi^{-1}(0) = \delta^{-1}\mathbf{I}, \quad \mathbf{b}_{\text{qm}}(0) = [1, 0, \dots, 0]^T$$

For each instant of time  $n = 1, 2, \dots$ , evaluate

$$\begin{aligned} \xi_{\text{qm}}(n) &= x(n) - \mathbf{b}_{\text{qm}}^T(n-1)\phi_{\text{qm}}(w(n)) \\ &= \mathbf{b}_{\text{qm}}^T(n-1)(\phi_{\text{qm}}(s(n)) - \phi_{\text{qm}}(w(n))) \\ &\quad + \lambda^{-1}\Phi^{-1}(n-1)\phi_{\text{qm}}^*(w(n)) \\ \mathbf{k}(n) &= \frac{\lambda^{-1}\Phi^{-1}(n-1)\phi_{\text{qm}}^*(w(n))}{1 + \lambda^{-1}\phi_{\text{qm}}^T(w(n))\Phi^{-1}(n-1)\phi_{\text{qm}}^*(w(n))} \end{aligned}$$

$$\begin{aligned} \mathbf{b}_{\text{qm}}(n) &= \mathbf{b}_{\text{qm}}(n-1) + \mathbf{k}(n)\xi_{\text{qm}}(n) \\ \Phi^{-1}(n) &= \lambda^{-1}\Phi^{-1}(n-1) - \lambda^{-1}\mathbf{k}(n)\phi_{\text{qm}}^T(w(n))\Phi^{-1}(n-1) \end{aligned}$$

where  $\delta$  is a small positive constant,  $\mathbf{k}(n)$  is the gain vector, and  $\xi_{\text{qm}}(n)$  is a priori estimation error.

### III. EXTENSION TO THE MEMORY POLYNOMIAL CASE

To consider PA memory effects, which cannot be ignored for wideband applications, the PD is designed based on a memory polynomial [4]. The PD output is represented as

$$\begin{aligned} u_p(n) &= \sum_{p=0}^P \sum_{q=0}^Q a_{2p+1,q}(n) |s(n-q)|^{2p} s(n-q) \\ &= \sum_{p=0}^P \sum_{q=0}^Q a_{2p+1,q}(n) \phi_{2p+1,q}(s(n)) \\ &= \sum_{q=0}^Q \mathbf{a}_q^T(n) \phi_q(s(n)) \end{aligned} \quad (8)$$

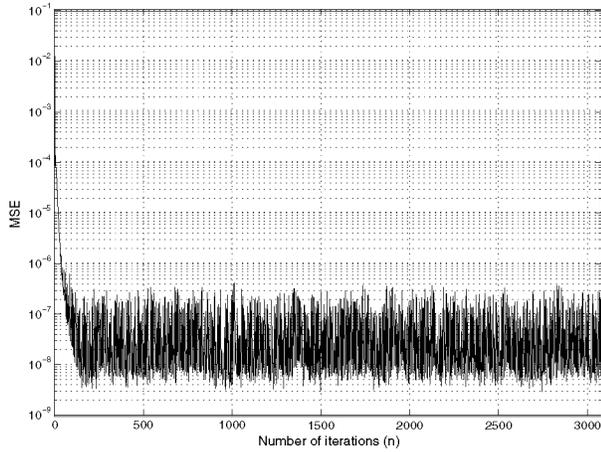


Fig. 2. Learning curve for PD and QMC (case 1).

where  $\phi_q(s(n)) = [\phi_{1,q}(s(n)), \phi_{3,q}(s(n)), \dots, \phi_{2P+1,q}(s(n))]^T$ ,  $\phi_{2p+1,q}(s(n)) = |s(n-q)|^{2p} s(n-q)$ ,  $\mathbf{a}_q(n) = [a_{1,q}(n), a_{3,q}(n), \dots, a_{2P+1,q}(n)]^T$ , and  $Q$  represents the memory size. Rewriting (8) in a vector form yields

$$u_p(n) = \underline{\mathbf{a}}^T(n) \underline{\phi}(s(n)) \quad (9)$$

where  $\underline{\phi}(s(n)) = [\phi_0^T(s(n)), \phi_1^T(s(n)), \dots, \phi_Q^T(s(n))]^T$ , and  $\underline{\mathbf{a}}(n) = [\mathbf{a}_0^T(n), \mathbf{a}_1^T(n), \dots, \mathbf{a}_Q^T(n)]^T$ . Using (9) in (4),  $x(n)$  can be represented as

$$x(n) = d_1(n) \underline{\mathbf{a}}^T(n) \underline{\phi}(s(n)) + d_2(n) \underline{\mathbf{a}}^H(n) \underline{\phi}^*(s(n)) + d_3(n) \triangleq \underline{\mathbf{b}}_{\text{qm}}^T(n) \underline{\phi}_{\text{qm}}(s(n)) \quad (10)$$

where  $\underline{\mathbf{b}}_{\text{qm}}(n) = [d_1(n) \underline{\mathbf{a}}^T(n), d_2(n) \underline{\mathbf{a}}^H(n), d_3(n)]^T$  and  $\underline{\phi}_{\text{qm}}(s(n)) = [\underline{\phi}^T(s(n)), \underline{\phi}^H(s(n)), 1]^T$ . The RLS solution for obtaining the optimal  $\underline{\mathbf{b}}_{\text{qm}}(n)$  is identical to the RLS algorithm for the memoryless case with the exception that  $\mathbf{b}_{\text{qm}}(n)$  and  $\phi_{\text{qm}}(s(n))$  of the original algorithm are replaced with  $\underline{\mathbf{b}}_{\text{qm}}(n)$  and  $\underline{\phi}_{\text{qm}}(s(n))$ , respectively.

#### IV. SIMULATION RESULTS

Computer simulations were conducted to examine the performance of the proposed schemes. In the simulation, the following parameters are assumed: 16-QAM signal constellation is used for transmission; the pulse shaping filter is the 8-times oversampled raised-cosine filter with a rolloff of 0.22; and the forgetting factor  $\lambda$  is 0.95. The amplitude and phase imbalances are  $\epsilon = 0.03$  and  $\theta = 3^\circ$ , respectively, which correspond to  $\beta = 1.0143 + j0.027$  and  $\alpha = -0.0143 - j0.027$ . The dc offset is given by  $c = 0.03 + j0.01$ .

1) *Saleh PA Model and Polynomial PD*: The PA input  $v(n)$  and output  $y(n)$  are assumed to obey the Saleh model [11]

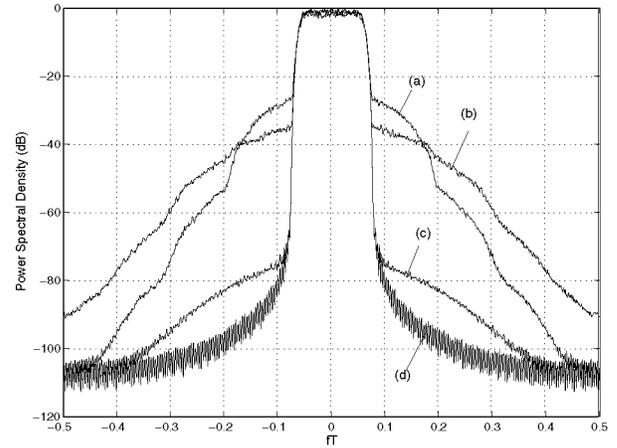


Fig. 3. Performance comparison (case 1). (a) without PD and QMC; (b) PD without QMC; (c) proposed; and (d) original.

given by

$$y(n) = \frac{\alpha_a |v(n)|}{1 + \beta_a |v(n)|^2} \exp \left( j \left( \angle v(n) + \frac{\alpha_\phi |v(n)|^2}{1 + \beta_\phi |v(n)|^2} \right) \right)$$

with  $\alpha_a = 1.1$ ,  $\beta_a = 0.3$ ,  $\alpha_\phi = 1$ , and  $\beta_\phi = 1$ . Here, the linearized gain is set to  $G = 1$ , which is the gain of the linearized PA at  $|v(n)| = 1$ . The peak back-off (PBO) is set to 2dB. The PD with a polynomial order  $P = 3$  is employed. The learning curve corresponding to  $E[|\xi_{\text{qm}}(n)|^2]$  of the proposed algorithm is shown in Fig. 2. In the simulation, the empirical mean square error (MSE) values are obtained through 1000 independent trials. The MSE reaches a steady-state after about 200 iterations. The advantage of the proposed method is demonstrated through examining the power spectral density (PSD) of the PA output, as shown in Fig. 3. For comparison, the PSDs without PD and QMC and with only PD are also plotted and the PSD of the original input  $s(n)$  provides a performance bound. The proposed scheme considerably outperforms the PD without QMC. Specifically, the proposed method achieves the PSD of -80dB at the normalized frequency of  $fT=0.1$ , whereas the PD without QMC provides only -35dB at the same frequency. In the entire frequency range, the out-of-band PSD of the proposed method is lower than that without the PD and QMC. However for the PD without QMC, the PSD is even higher than that of amplifier output without the PD and QMC at  $fT > 0.18$ . This figure indicates that if the QM error is not compensated, the PD cannot provide a satisfactory PSD performance.

2) *Memory Polynomial PA and PD Model*: We assume that the PA obeys a memory polynomial model [4] given by

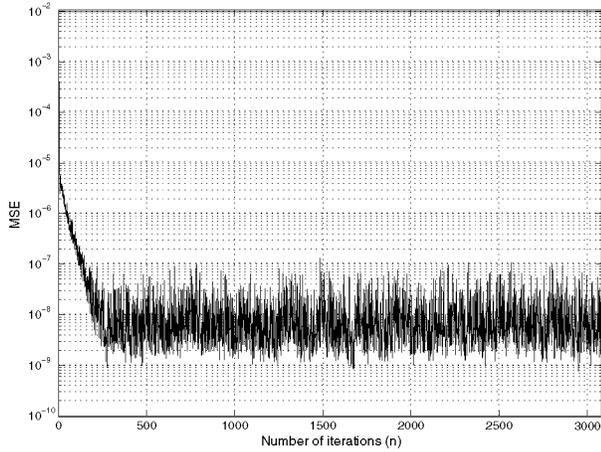


Fig. 4. Learning curve for PD and QMC (case 2).

$$y(n) = \sum_{p=0}^P \sum_{q=0}^Q c_{2p+1,q} |v(n-q)|^{2p} v(n-q).$$

In the simulation,  $P = 2$  and  $Q = 2$  are used as in [4]. For the PD in (8), polynomial order and memory size are set to  $P = 3$  and  $Q = 2$ , respectively. The learning curve corresponding to  $E[|\xi_{qm}(n)|^2]$  of the proposed algorithm is shown in Fig. 4. The experimental MSE reaches a steady-state after about 400 iterations. Note that the convergence rate of the proposed adaptive technique based on the memory polynomial PD is 2 times slower than that of the previous case for the memoryless PA and PD. This is because the rate of convergence decreases as the number of coefficients to be updated increases. Fig. 5 shows the PSD performance of the proposed compensator. Similarly to the memoryless PA and PD case, it is observed that the proposed scheme outperforms the PD without QMC and specifically, the PSD is reduced by approximately 45dB for  $fT=0.1$ . As a result, the proposed method achieves the PSD of -75dB at  $fT=0.1$ , while the PD without QMC provides only -30dB at the same frequency.

## V. CONCLUSION

A joint compensation algorithm for amplifier nonlinearity and quadrature modulation errors was proposed. For the development of the joint algorithm, two types of the predistortion schemes were considered: one is the memoryless polynomial and the other is the memory polynomial. It was demonstrated through computer simulation that the proposed methods can compensate both the amplifier nonlinearity and QM errors effectively. Further work will include the following.

- 1) Convergence analysis of the proposed algorithm.

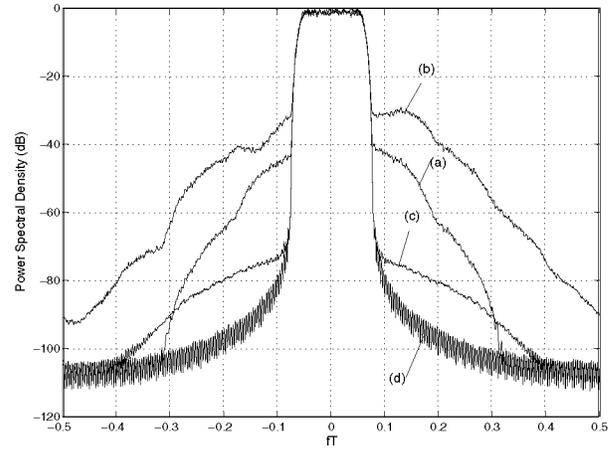


Fig. 5. Performance comparison (case 2). (a) without PD and QMC; (b) PD without QMC; (c) proposed; and (d) original.

- 2) Development of the compensation technique for amplifier nonlinearity and QDM errors.

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