COMPRESS-FORWARD RELAYING OVER PARALLEL GAUSSIAN CHANNELS

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ABSTRACT

We propose a compress-forward (CF) strategy for relaying over parallel Gaussian channels. The proposed CF strategy compresses received signals at the relay under a constraint on the product of their allowable distortions and forwards them to the destination over all subchannels. Power allocation over all subchannels at both the source and the relay and bit allocation for compressing received signals at the relay are jointly optimized under a total power constraint on each node. The proposed CF strategy is shown to generalize a previous CF strategy in which the relay compresses the received signal from the i-th incoming subchannel and forwards it only through the i-th outgoing subchannel. We show that, if the incoming and outgoing subchannels at the relay are suitably matched, both CF strategies have similar performance. Otherwise, the gap between their performances becomes significant.

Index Terms— Relay, compress-forward, resource allocation, OFDM, parallel Gaussian channels

1. INTRODUCTION

Recently, relaying and cooperative communications have received significant attention, with potential applications in cellular networks, sensor networks, and ad hoc networks. Relaying was first comprehensively studied by Cover and El Gamal in [1], in which they introduced two relaying strategies, namely, a *decode-forward (DF)* strategy and a *compress-forward (CF)* strategy. These two relaying strategies together with an *amplify-forward (AF)* strategy are now the basic building blocks of relaying and cooperative communications. Although the DF strategy is optimal for degraded relay channels [1], the CF strategy is preferable if the channel between the relay and the destination nodes is relatively strong. For *Gaussian* relay channels, an achievable rate for the CF strategy is explicitly shown in [2, 3].

In this paper, we consider a CF relaying strategy for *parallel* Gaussian channels, as might arise, for example, in orthogonal frequency-division multiplex (OFDM) systems. In [4], a CF strategy is proposed for parallel Gaussian channels in which the relay is restricted to transmit the compressed version of the received signal on its i-th incoming subchannel only through its i-th outgoing subchannel. Such a restriction is removed here. In [5], a CF strategy is considered for more general vector, instead of parallel, Gaussian channels in which subchannels may not be orthogonal to each other. Although we consider simpler cases in this paper, we optimize transmit powers at both the source and the relay, assuming transmit covariance matrices are given by diagonal matrices; such an optimization is not examined in [5]. We also observe that the product of squared distortions over all subchannels is more important than their sum, and hence the optimal distortions are apparently different from well-known reverse water-filling solutions [6].

The rest of the paper is organized as follows. Section 2 introduces the channel model considered as well as an upper bound on its channel capacity. Section 3 develops the proposed CF strategy. Section 4 provides some numerical results.

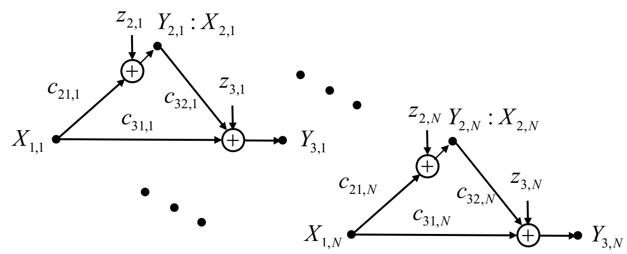

Fig. 1. Parallel Gaussian channel with one relay.

2. CHANNEL MODEL

We consider a parallel Gaussian channel with one relay in Fig. 1. For simplicity, we assume a full-duplex relay. The received signals at the i-th subchannel of the relay and the destination are given by

$$Y_{2,i} = c_{21,i}X_{1,i} + Z_{2,i}, \tag{1}$$

$$Y_{3,i} = c_{31,i}X_{1,i} + c_{32,i}X_{2,i} + Z_{3,i}, \tag{2}$$

respectively, where $c_{21,i}$, $c_{31,i}$, and $c_{32,i}$ are real constants, $Z_{2,i} \sim \mathcal{N}(0,1)$ and $Z_{3,i} \sim \mathcal{N}(0,1)$ are noises at the relay and at the destination, respectively, that are independent of each other and independent from transmitted signals, $\mathbb{E}[|X_{1,i}|^2] = P_{1,i}$, $\mathbb{E}[|X_{2,i}|^2] = P_{2,i}$, $\sum_{i=1}^N P_{1,i} \le P_1$ and $\sum_{i=1}^N P_{2,i} \le P_2$.

From Lemma 4 in [1], the channel capacity on the above parallel Gaussian channel with one relay is upper bounded by

$$C^+ = \max_{P_{1,i},P_{2,i},\rho_i} \min\left\{C_{11}^+, C_{12}^+\right\} \tag{3}$$

subject to $\sum_{i=1}^N P_{s,i} \le P_s$ for $1 \le s \le 2$, where $\rho_i = \frac{\mathbb{E}\left[X_{1,i}X_{2,i}\right]}{\sqrt{P_{1,i}P_{2,i}}}$, $C_{11}^+ = \sum_{i=1}^N I\left(X_{1,i}, X_{2,i}; Y_{1,i}\right)$, and $C_{12}^+ = \sum_{i=1}^N I\left(X_{1,i}; Y_{2,i}, Y_{3,i}|X_{2,i}\right)$.

3. COMPRESS-FORWARD RELAYING OVER PARALLEL GAUSSIAN CHANNELS

We first assume that $X_{1,i} \sim \mathcal{N}(0, P_{1,i})$, $X_{2,i} \sim \mathcal{N}(0, P_{2,i})$, and covariance matrices of $\underline{X}_1 = (X_{1,1}, \cdots, X_{1,N})$ and $\underline{X}_2 = (X_{2,1}, \cdots, X_{2,N})$ are given by $\boldsymbol{R}_{\underline{X}_1} = \mathrm{diag}(P_{1,i}, \cdots, P_{1,N})$ and $\boldsymbol{R}_{\underline{X}_2} = \mathrm{diag}(P_{2,i}, \cdots, P_{2,N})$, respectively. Then, $Y_{2,1}, \cdots Y_{2,N}$ are independent of each other, and from [1], the achievable rate for the CF strategy is given by

$$R_{CF,V} = \sup \sum_{i=1}^{N} I(X_{1,i}; \hat{Y}_{2,i}, Y_{3,i}|X_{2,i}) \tag{4a}$$

subject to

$$\sum_{i=1}^{N} I(X_{2,i}; Y_{3,i}) \ge \sum_{i=1}^{N} I(Y_{2,i}; \hat{Y}_{2,i}|X_{2,i}, Y_{3,i}), \tag{4b}$$

where $\hat{Y}_{2,i}$ is an estimate of $Y_{2,i}$ and the supremum is taken over all joint probability distributions of the form $p(x_{1,i}, x_{2,i}, y_{2,i}, y_{3,i}, \hat{y}_{2,i}) = p(x_{1,i})p(x_{2,i})p(\hat{y}_{2,i}|x_{2,i}, y_{2,i})p(y_{2,i}, y_{3,i}| x_{1,i}, x_{2,i})$. The term $I(X_{1,i}; \hat{Y}_{2,i}, Y_{3,i}|X_{2,i})$ can be interpreted as the achievable rate for a 1×2 SIMO channel, and (4b) implies that the bit-rate used for compressing $Y_{2,i}$ should be less than or equal to the bit-rate that we can transmit data reliably from the relay to the destination. At the destination, $X_{2,i}$ is first decoded regarding $X_{1,i}$ as interference, and then the decoded information is used to reconstruct $\hat{Y}_{2,i}$ through Wyner-Ziv source decoding [7]. Since block-Markov encoding is used at the source, the Wyner-Ziv source decoder exploits $Y'_{3,i} = Y_{3,i} - c_{32,i}X_{2,i}$ as its side information where $Y_{3,i}$ is the received signal in the *previous* block and $X_{2,i}$ is subtracted from $Y_{3,i}$ since $X_{2,i}$ was already decoded.

To apply the Wyner-Ziv source code, we first find an equivalent *forward channel* for the Gaussian source $Y_{2,i}$ and its side information $Y'_{3,i}$. Using the forward channel model shown in Fig. 4 of [7] and the fact that

$$\mathbb{E}[Y_{2,i}|X_{2,i}, Y_{3,i} = y_{3,i}] = \mathbb{E}[Y_{2,i}|Y'_{3,i} = y'_{3,i}]$$

$$= \frac{\mathbb{E}[Y_{2,i}Y'_{3,i}]}{\mathbb{E}[Y'^2_{3,i}]} y'_{3,i} = \frac{c_{21,i}c_{31,i}P_{1,i}}{1 + c_{31,i}^2 P_{1,i}} y'_{3,i}, \tag{5}$$

$$\mathrm{Var}[Y_{2,i}|X_{2,i}, Y_{3,i} = y_{3,i}] = \mathrm{Var}[Y_{2,i}|Y'_{3,i} = y'_{3,i}]$$

$$= \mathbb{E}[Y_{2,i}^2] - \frac{\mathbb{E}^2[Y_{2,i}Y'_{3,i}]}{\mathbb{E}[Y'^2_{3,i}]} = \frac{1 + (c_{21,i}^2 + c_{31,i}^2)P_{1,i}}{1 + c_{31,i}^2 P_{1,i}}, \tag{6}$$

we reconstruct $\hat{Y}_{2,i}$ as $\hat{Y}_{2,i} = W_i + k_i Y_{3,i} = a_i(Y_{2,i} + Z_{w,i}) + k_i Y_{3,i}$, where $W_i = a_i(Y_{2,i} + Z_{w,i})$, $k_i = \frac{c_{21,i}c_{31,i}P_{1,i}}{1+c_{31,i}^2 P_{1,i}}(1 - a_i)$, $a_i = \left(\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{1+c_{31,i}^2 P_{1,i}} - d_i\right) / \left(\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{1+c_{31,i}^2 P_{1,i}}\right)$, $d_i = \mathbb{E}[|\hat{Y}_{2,i} - Y_{2,i}|^2]$ and $Z_{w,i} \sim \mathcal{N}(0, \sigma_{w,i}^2)$ with

$$\sigma_{w,i}^2 = \frac{\frac{1+(c_{21,i}^2+c_{31,i}^2)P_1}{1+c_{31,i}^2 P_{1,i}} \cdot d_i}{\frac{1+(c_{21,i}^2+c_{31,i}^2)P_{1,i}}{1+c_{31,i}^2 P_{1,i}} - d_i}. \tag{7}$$

Then,

$$I(X_{1,i}; \hat{Y}_{2,i}, Y_{3,i}|X_{2,i}) = \frac{1}{2}\log_2 \frac{\left(1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}\right)^2}{1 + \left(c_{21,i}^2(1 + d_i) + c_{31,i}^2\right) P_{1,i}}, \tag{8}$$

$$I(Y_{2,i}; \hat{Y}_{2,i}|X_{2,i}, Y_{3,i}) = \frac{1}{2}\log_2 \frac{1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}}{\left(1 + c_{31,i}^2 P_{1,i}\right) d_i}, \tag{9}$$

$$I(X_{2,i}; Y_{3,i}) = \frac{1}{2}\log_2 \left(1 + \frac{c_{32,i}^2 P_{2,i}}{1 + c_{31,i}^2 P_{1,i}}\right), \tag{10}$$

which do not depend upon a_i or k_i. $I(Y_{2,i}; \hat{Y}_{2,i}|X_{2,i}, Y_{3,i})$ is the bit-rate used for compressing $Y_{2,i}$. Note that (9) can be also directly derived from (6) by $\frac{1}{2}\log_2 \frac{\mathrm{Var}[Y_{2,i}|X_{2,i}, Y_{3,i} = y_{3,i}]}{d_i}$. Then, from (4), we obtain

$$R_{CF,V} = \max_{P_{1,i},P_{2,i},d_i} \sum_{i=1}^{N} \frac{1}{2}\log_2 \frac{\left(1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}\right)^2}{1 + \left(c_{21,i}^2(1 + d_i) + c_{31,i}^2\right) P_{1,i}} \tag{11a}$$

subject to

$$\sum_{i=1}^{N} \log_2 d_i \ge \sum_{i=1}^{N} \log_2 \frac{1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}}{1 + c_{31,i}^2 P_{1,i} + c_{32,i}^2 P_{2,i}} \tag{11b}$$

$$\sum_{i=1}^{N} P_{s,i} \le P_s \text{ for } 1 \le s \le 2, \tag{11c}$$

$$d_i \le \frac{1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}}{1 + c_{31,i}^2 P_{1,i}} \text{ for } 1 \le i \le N, \tag{11d}$$

where (11d) results from that $I(Y_{2,i}; \hat{Y}_{2,i}|X_{2,i}, Y_{3,i})$ in (9) should be positive. Let $L(\underline{P_1}, \underline{P_2}, \underline{d}, \lambda_1, \lambda_2, \lambda_3, \underline{\lambda_4})$ be the Lagrangian given by

$$L(\cdot) = \sum_{i=1}^{N} \frac{1}{2}\log_2 \frac{\left(1 + \left(c_{21,i}^2 + c_{31,i}^2\right) P_{1,i}\right)^2}{1 + \left(c_{21,i}^2(1 + d_i) + c_{31,i}^2\right) P_{1,i}}$$

$$+\sum_{i=1}^{N}\frac{\lambda_1}{2}\left(\log_2 d_i-\log_2\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{1+c_{31,i}^2P_{1,i}+c_{32,i}^2P_{2,i}}\right)$$
$$+\lambda_2\left(P_1-\sum_{i=1}^{N}P_{1,i}\right)+\lambda_3\left(P_2-\sum_{i=1}^{N}P_{2,i}\right)$$
$$+\sum_{i=1}^{N}\lambda_{4,i}\left(\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{1+c_{31,i}^2P_{1,i}}-d_i\right). \quad (12)$$

Although (11) is a nonconvex optimization problem, we can obtain at least necessary conditions for the optimal solution using the Karush-Khun-Tucker conditions, yielding

$$\frac{c_{21,i}^2(1-d_i^*)+c_{31,i}^2+\left(c_{21,i}^2+c_{31,i}^2\right)\left(c_{21,i}^2(1+d_i^*)+c_{31,i}^2\right)P_{1,i}^*}{2\left(1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}^*\right)\left(1+\left(c_{21,i}^2(1+d_i^*)+c_{31,i}^2\right)P_{1,i}^*\right)}$$
$$-\frac{\lambda_1^*\left(c_{21,i}^2+\left(c_{21,i}^2+c_{31,i}^2\right)c_{32,i}^2P_{2,i}^*\right)}{2\left(1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}^*\right)\left(1+c_{31,i}^2P_{1,i}^*+c_{32,i}^2P_{2,i}^*\right)}$$
$$-\lambda_2^*+\frac{\lambda_{4,i}^*c_{21,i}^2}{\left(1+c_{31,i}^2P_{1,i}^*\right)^2}=0, \quad (13)$$

$$P_{2,i}^*=\left[\frac{\lambda_1^*}{2\lambda_3^*}-\frac{1}{c_{32,i}^2}\left(1+c_{31,i}^2P_{1,i}^*\right)\right]^+, \quad (14)$$

$$\frac{\lambda_1^*}{2d_i^*}-\lambda_{4,i}^*-\frac{c_{21,i}^2P_{1,i}^*}{2\left(1+\left(c_{21,i}^2(1+d_i^*)+c_{31,i}^2\right)P_{1,i}^*\right)}=0, \quad (15)$$

where $[x]^+=\max\{x,0\}$. If (11d) is active ($\lambda_{4,i}^*>0$), then

$$d_i^*=\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}^*}{1+c_{31}^2P_{1,i}^*}, \quad (16)$$

and if (11d) is inactive ($\lambda_{4,i}^*=0$), from (15), we obtain

$$d_i^*=\left[\frac{\lambda_1^*}{1-\lambda_1^*}\left(1+\frac{1+c_{31,i}^2P_{1,i}^*}{c_{21,i}^2P_{1,i}^*}\right)\right]^+. \quad (17)$$

Following [4], if we let $d_i=\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{1+c_{31,i}^2P_{1,i}+c_{32,i}^2P_{2,i}}$, then (11) reduces as follows.

$$R_{CF,S}$$
$$=\max_{P_{1,i},P_{2,i}}\sum_{i=1}^{N}\frac{1}{2}\log_2\left(1+c_{31,i}^2P_{1,i}+\frac{c_{21,i}^2P_{1,i}}{1+\frac{1+\left(c_{21,i}^2+c_{31,i}^2\right)P_{1,i}}{c_{32,i}^2P_{2,i}}}\right) \quad (18)$$

subject to $\sum_{i=1}^{N}P_{s,i}\leq P_s$ for $1\leq s\leq 2$, where $R_{CF,V}\geq R_{CF,S}$. The most significant disadvantage of this CF strategy is that the compressed signal from the i-th incoming subchannel at the relay is always forwarded to the destination only through the i-th outgoing subchannel. However, for example, if $c_{21,i}^2$ is quite large while $c_{32,i}^2$ is quite small, then many bits are allocated to describe $Y_{2,i}$ and the relay should allocate large power to the i-th outgoing subchannel to transmit reliably to the destination. This results inefficient power and bit-rate allocations. On the other hand, in (11), after compressing $Y_{2,1},\cdots Y_{2,N}$, they are re-encoded by one channel encoder, and then forwarded to the destination over all N subchannels.

For point-to-point communication, simultaneous description of multiple independent Gaussian sources under a constraint on the sum of squared distortions is a well-known problem, and its solution is given by reverse water-filling [6]. However, (11b) corresponds to a constraint on the product of d_i's, and hence its solution is also apparently different from reverse water-filling.

Since $X_{2,i}$ is first decoded at the destination regarding $X_{1,i}$ as interference, its signal-to-interference-and-noise-ratio (SINR) becomes $\frac{c_{32,i}^2}{1+c_{31,i}^2P_{1,i}^*}$. Then, it is notable that $P_{2,i}^*$ is given by a water-filling solution for such a SINR and independent of $c_{21,i}^2$, i.e., the relay power allocation does not depend upon the characteristics of its incoming link. We can also see that d_i^* in (17) is independent of $c_{32,i}^2$ and decreases as $c_{21,i}^2$ increases, which implies that the relay bit-rate allocation does not depend upon the characteristics of its outgoing link, and as the relay receives less noisy signals, more bits should be allocated to describe them.

The effects of the variance of $Y_{3,i}'$ given by $1+c_{31,i}^2P_{1,i}^*$ is worth discussing. As $1+c_{31,i}^2P_{1,i}^*$ increases, $P_{2,i}^*$ decreases while d_i^* in (17) increases, which means that the relay allocates less resources to the subchannels that receive less noisy signals from the source at the destination. Note that (11a) increases as d_i decreases, and hence (11b) is always active and $\lambda_1^*>0$. When no signal is transmitted over the i-th subchannel at the source with $P_{1,i}^*=0$, (15) reduces to $d_i^*=\frac{\lambda_1^*}{2\lambda_{4,i}^*}$. Since $\lambda_1^*>0$ and $d_i^*\leq 1$ from (11d), $\lambda_{4,i}^*>0$ and $d_i^*=1=\text{Var}\left[Z_{2,i}\right]$, which means that no bits are allocated to compress $Y_{2,i}=Z_{2,i}$.

4. NUMERICAL RESULTS

For comparison, we first define an upper bound C_2^+ by $C_2^+\triangleq\max_{P_{1,i},P_{2,i},\rho_i}C_{12}^+$ which is obviously larger than or equal to C^+ in (3). Since

$$I\left(X_{1,i};Y_{2,i},Y_{3,i}|X_{2,i}\right)$$
$$=\frac{1}{2}\log_2\left(1+(c_{31,i}^2+c_{21,i}^2)P_{1,i}(1-\rho_i^2)\right)$$
$$\leq\frac{1}{2}\log_2\left(1+(c_{31,i}^2+c_{21,i}^2)P_{1,i}\right), \quad (19)$$

we can easily see that both $R_{CF,V}$ in (11) and $R_{CF,S}$ in (18) asymptotically achieve C_2^+ as $P_{2,i}\to\infty$. We also compare with a CF strategy proposed in [5] where only bit-rates are optimized for fixed transmit powers at both the source and the relay. Assuming uniform power allocation, its achievable rate is calculated and denoted by $R_{CF,R}$. (18) is also recalculated for uniform power allocation and denoted by $R_{CF,B}$.

Table 1. Channel gains $c^2_{21,i}$, $c^2_{32,i}$ and $c^2_{31,i}$ for $i = 1 \cdots, 4$

	i	1	2	3	4
CH. A	$c^2_{21,i}$	0.01	0.1	1	1
	$c^2_{32,i}$	1	1	0.1	0.01
	$c^2_{31,i}$	1	1	1	1
CH. B	$c^2_{21,i}$	0.01	0.1	1	1
	$c^2_{32,i}$	0.01	0.1	1	1
	$c^2_{31,i}$	1	1	1	1

Fig. 2 shows C_2^+, $R_{CF,V}$, $R_{CF,S}$, $R_{CF,B}$ and $R_{CF,R}$ for various P_2 values when $P_1 = 1$, $N = 4$ and the channel gains are given by Channel A in Table 1. As $P_{2,i} \to 0$, all $R_{CF,V}$, $R_{CF,S}$, $R_{CF,B}$ and $R_{CF,R}$ approach $\max_{P_{1,i}, P_{2,i}} \sum_{i=1}^{N} \frac{1}{2} \log_2 \left(1 + c^2_{31,i} P_{1,i}\right)$. Although both $R_{CF,B}$ and $R_{CF,R}$ cannot achieve C_2^+ as $P_2 \to \infty$, both $R_{CF,S}$ and $R_{CF,V}$ asymptotically achieve it. This explains why the optimal power allocation is needed. Since the channel gains are given such that a good (bad) incoming link is matched to a bad (good) outgoing link, respectively, the gap between $R_{CF,S}$ and $R_{CF,V}$ is relatively large for moderate P_2 values. On the other hand, Fig. 3 shows C_2^+, $R_{CF,V}$, $R_{CF,S}$, $R_{CF,B}$ and $R_{CF,R}$ if the channel gains are given by Channel B in Table 1. In this case, the channel gains are such that a good (bad) incoming link is matched to a good (bad) outgoing link, respectively. Since it seems reasonable in this case to deliver each compressed signal for the received signal in the i-th incoming subchannel through the i-th outgoing subchannel, both $|R_{CF,V} - R_{CF,S}|$ and $|R_{CF,R} - R_{CF,B}|$ are negligible for most P_2 values.

Acknowledgments

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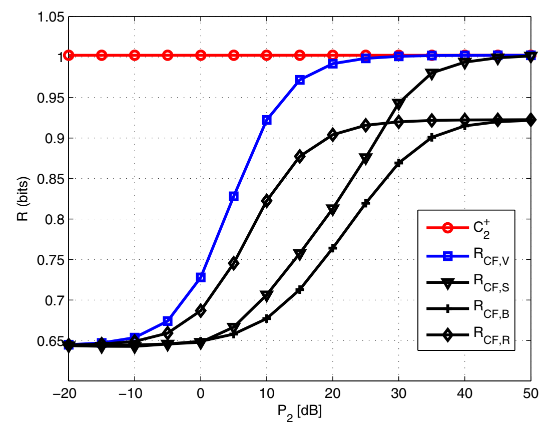

Fig. 2. Numerical results for Channel A in Table 1.

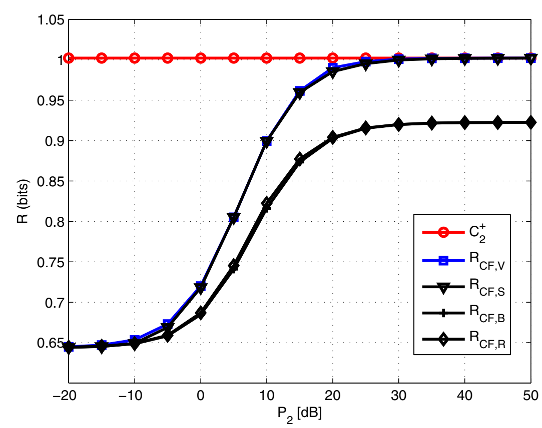

Fig. 3. Numerical results for Channel B in Table 1.