# Predictive Transmit Beamforming for MIMO-OFDM in Time-varying Channels with Limited Feedback[†]

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## ABSTRACT

A limited feedback-based transmit beamforming technique for multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is investigated in time-varying channels. The performance of the system is significantly degraded by outdated feedback information even when the channel varies slowly. To compensate for the impairment in time-varying channels, the optimal transmit beamforming vector for a future channel, which maximizes the expected effective channel gain, is derived by applying the autoregressive (AR) model to the channels. These are obtained at the receiver. Following this, schemes for the selection of beamforming vectors are proposed to reduce the feedback amount. These can effectively reduce the amount of feedback information by utilizing both the frequency and time correlation of transmit beamforming vectors. Simulation results show that the proposed techniques outperform existing schemes in terms of the bit error rate (BER) performance with the same amount of feedback.

## Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

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## General Terms

Algorithms, Design, Performance

## Keywords

MIMO-OFDM, transmit beamforming, time-varying channel, prediction

## 1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems provide spatial diversity that can be used to combat the fading characteristic of the channel [1], [2]. In narrowband channels, when the channel state information (CSI) is available at the transmitter, the array gain as well as the diversity gain can be obtained using transmit beamforming and receive combining [3], [4]. This technique can be extended easily to wideband channels by employing orthogonal frequency division multiplexing (OFDM) [5], [6]. Transmit beamforming with receive combining for MIMO-OFDM (a combination of MIMO and OFDM) can be performed independently for each subcarrier of MIMO-OFDM. This approach, however, requires knowledge of the transmit beamforming vector for every active subcarrier at the transmitter. When the downlink and uplink channels are not reciprocal (as in a frequency division duplex system), the receiver must inform the transmitter of the desired transmit beamforming vector through a feedback control channel. Practically, the feedback rate can be managed using limited feedback techniques where beamforming vectors are quantized using a beamforming codebook [7], [8]. However, the feedback requirements still remain inadmissible due to the large number of subcarriers. Recently, techniques to reduce the amount of feedback information by utilizing the frequency-domain correlation of quantized beamforming vectors (or precoding matrices) have been proposed [9]– [13].

In this paper, limited feedback-based transmit beamforming for MIMO-OFDM is investigated in time-varying channels where the feedback information used by the transmitter would be outdated due to feedback delays. To mitigate performance degradation caused by the feedback delay, transmit beamformer selection schemes are proposed for use at the receiver based on autoregressive (AR) channel modeling and followed by a two-dimensional (2D) clustering at the transmitter. Simulation results show that the proposed schemes outperform the existing schemes in terms of the bit error rate (BER) performance with the same amount of feedback.

## 2. SYSTEM MODEL

A MIMO-OFDM system with transmit beamforming and receive combining that uses $M_t$ transmit antennas, $M_r$ receive antennas, and $N$ subcarriers is illustrated in Fig. 1. The symbol $s(k,n)$ is transmitted through the $k$-th subcarrier at time index $n$ using the beamforming vector $\mathbf{w}(k,n) = [w_1(k,n), w_2(k,n), \cdots, w_{M_t}(k,n)]^T$. At the receiver, after processing with the combining vector $\mathbf{z}(k,n) = [z_1(k,n), z_2(k,n), \cdots, z_{M_r}(k,n)]^T$, the combined signal is written as

$$y(k,n) = \mathbf{z}^H(k,n)\{\mathbf{H}(k,n)\mathbf{w}(k,n)s(k,n) + \mathbf{n}(k,n)\},$$

for $1 \le k \le N$ and all time index $n$, where a $\mathbf{H}(k,n)$ is the $M_r$-by-$M_t$ channel matrix for the $k$-th subcarrier at the time index $n$, and $\mathbf{n}(k,n)$ is the $M_r$-dimensional noise vector with entries that have an independent and identically distributed (i.i.d.) complex Gaussian distribution with a zero mean and a variance of $N_0$. For the channel matrix $\mathbf{H}(k,n)$, it is assumed that all entries are i.i.d. wide-sense stationary (WSS) random processes whose time-domain autocorrelation is given as $r(m) = E\left\{[\mathbf{H}(k,n)]_{p,q}[\mathbf{H}(k,n-m)]^*_{p,q}\right\}$, for all $k$, $p$, and $q$ (where $[\mathbf{A}]_{p,q}$ denotes the $(p,q)$-th element of a matrix $\mathbf{A}$).[1] It is assumed that the power is allocated equally across all tones and across time; thus, $E[|s(k,n)|^2] = \mathcal{E}_s$ is a constant and $\|\mathbf{w}(k,n)\| = 1$ (where $\|(\cdot)\|$ represents the 2-norm of $(\cdot)$) to maintain the overall power constraint. In the MIMO-OFDM system under consideration, $\mathbf{w}(k,n)$ and $\mathbf{z}(k,n)$ are designed based on $\mathbf{H}(k,n)$ to maximize the signal-to-noise ratio (SNR) for each subcarrier $k$ and time index $n$. Given $\mathbf{w}(k,n)$ and an assumption of full CSI at the receiver, the maximum SNR for subcarrier $k$ and time index $n$ with the optimal choice of $\mathbf{z}(k,n)$ is given by $\frac{\mathcal{E}_s}{N_0}\|\mathbf{H}(k,n)\mathbf{w}(k,n)\|^2$ as in [9].[2] Here, the SNR is determined only by the transmit beamforming vector $\mathbf{w}(k,n)$. Henceforth, beamforming indicates transmit beamforming unless otherwise noted.

In this paper, frame-based communication in which each frame consists of multiple MIMO-OFDM symbols is considered. The channel is assumed to vary continuously, but remains fixed in a frame. Considering the beamforming vectors, they do not need to vary in a frame. Thus the time index $n$ represents the frame index. It is assumed that full CSI is not available to the transmitter, but that an error-free feedback channel exists from the receiver to the transmitter; i.e., the receiver informs the transmitter of beamforming vectors, $\mathbf{w}(k,n)$, using limited feedback through the feedback channel. It is also assumed that feedback occurs every $P$ frames, in which $m$-th feedback occurs directly preceding the $n = mP$ frame for some integer $m$.[3] Additionally, the number of feedback bits per single instance of feedback is limited to a constant.

[1] $r(m)$ is assumed to be identical for all $k$. This assumption holds for the wide-sense stationary uncorrelated scattering (WSSUS) wireless channel [14].

[2] For simplicity, full CSI is assumed at the receiver when deriving the transmit beamforming vector. In addition, in the simulations an estimated CSI value is used at the receiver.

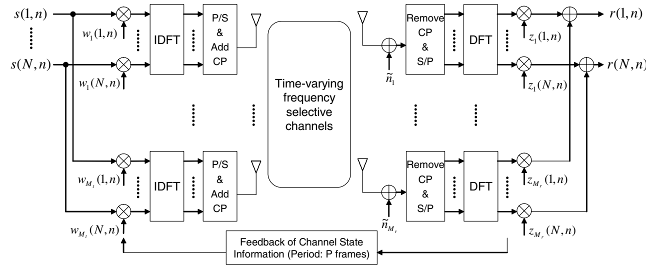


**Figure 1: Block diagram of a MIMO-OFDM system with $M_t$ transmit antennas, $M_r$ receive antennas, and $N$ subcarriers.**

## 3. SNR MAXIMIZING BEAMFORMING VECTOR FOR FUTURE CHANNELS

Given that the beamforming vectors used from the $mP$-th to the $(mP+P-1)$-th frames are developed from the feedback information up to the $(mP-1)$-th frame, the transmitter side, receiver side, or both sides must consider future channels. In this section, a technique that guarantees optimality of the beamforming vector that maximizes the expected effective channel gains for future channels is investigated.

### 3.1 Managing the time-varying channel

First, it is necessary to decide where the time-varying channels are managed. Possible answers for this include the transmitter side, the receiver side, or both sides. For the receiver side, it can be assumed that all information concerning the previous *channels* is given, as the receiver can easily track the channel by utilizing the pilot signals. However, for the transmitter side, it can be assumed that at most only the previous *beamforming vectors* are given. As the transmitter requires only beamforming vectors to operate, it is not necessary to transmit more feedback than that required to inform the transmitter of the beamforming vectors. Therefore, it is easier to consider the time-varying channel at the receiver side than at the transmitter side, as then it is possible to enjoy the stationary property of the channel at the receiver. On the other hand, assuming that the beamforming vectors for time-varying channels are stationary is not feasible. Consequently, it is best to focus on processing by the receiver.

### 3.2 Maximizing the expected effective channel gains

Frequency-domain processing (a subcarrier-wise scheme) is considered for simplicity. Given $M$ channel observations $\mathbf{\Upsilon} = \{\mathbf{H}(k,n')|mP-M \le n' \le mP-1\}$ at the receiver, the optimal beamforming vector that maximizes the effective channel gains for a subcarrier $k$ and a future frame $n$, where

[3] A maximum of $P$ frames can exist for the feedback delay.

$mP \le n \le (mP + P - 1)$, is the solution of the following optimization problem:

$$\mathbf{w}^*(k,n) = \arg\max_{\mathbf{w}} E\left[\|\mathbf{H}(k,n)\mathbf{w}\|^2 | \mathbf{\Upsilon}\right], \text{ s.t. } \|\mathbf{w}\| = 1 \quad (1)$$

with expectation over the distribution of the future channel $\mathbf{H}(k,n)$ given $\mathbf{\Upsilon}$. For the WSS process $\mathbf{H}(k,n)$, a classical statistical modeling technique is applicable. The $M$-th order AR modeling is introduced as

$$\mathbf{H}_{AR}(k, mP-1+d) = \\ -\sum_{p=1}^{M} a_d(p)\mathbf{H}(k, mP-p) + \sum_{q=1}^{d} b_d(q)\mathbf{G}(q) \quad (2)$$

where $d$ $(1 \le d \le P)$ denotes a time offset, $a_d(p)$ $(1 \le p \le P)$ and $b_d(q)$ $(1 \le q \le d)$ are the AR coefficients, and $\mathbf{G}(q)$ $(1 \le q \le d)$ is an $M_r$-by-$M_t$ matrix whose entries are i.i.d. unit variance white noise representing the innovations (obtaining AR coefficients is considered in the next subsection). By defining $\hat{\mathbf{H}}(k,d) = -\sum_{p=1}^{M} a_d(p)\mathbf{H}(k, mP-p)$, which is the $d$-step linear minimum mean-square error (LMMSE) predictor, and substituting (2) into (1), it is possible to obtain the modified cost function after some derivation:

$$\begin{aligned} &E\left[\|\mathbf{H}(k, mP-1+d)\mathbf{w}\|^2 | \mathbf{\Upsilon}\right] \\ &= E\left[\|\mathbf{H}_{AR}(k, mP-1+d)\mathbf{w}\|^2\right] \quad (3a) \\ &= \mathbf{w}^H \hat{\mathbf{H}}^H(k,d)\hat{\mathbf{H}}(k,d)\mathbf{w} + \sum_{q=1}^{d} |b_d(q)|^2, \quad (3b) \end{aligned}$$

where the expectation in (3a) is taken with respect to $\mathbf{G}(q)$, which satisfies $E[\mathbf{G}(q)] = \mathbf{0}_{M_r \times M_t}$ for all $q$, and $E[\mathbf{G}^H(p) \cdot \mathbf{G}(q)] = \delta(p-q) \cdot \mathbf{I}_{M_r}$ for all $p$ and $q$. Given that the optimal beamformer is independent from coefficients $b_d(q)$, the optimization problem (1) can be rewritten as

$$\mathbf{w}^*(k, mP-1+d) = \arg\max_{\mathbf{w}} \Psi_{\mathbf{w}}(k,d), \text{ s.t. } \|\mathbf{w}\| = 1 \quad (4)$$

where $\Psi_{\mathbf{w}}(k,d) = \mathbf{w}^H \hat{\mathbf{H}}^H(k,d)\hat{\mathbf{H}}(k,d)\mathbf{w}$ is the expected effective channel gain for the subcarrier $k$ and the frame offset $d$, given the beamforming vector $\mathbf{w}$. The optimal solution for (4) is well known as the eigenvector of $\hat{\mathbf{H}}^H(k,d)\hat{\mathbf{H}}(k,d)$ corresponding to the maximum eigenvalue when the unquantized beamforming vector is considered. The optimal quantized vector is obtained as

$$\mathbf{w}^*(k, mP-1+d) = \arg\max_{\mathbf{w} \in \Omega} \Psi_{\mathbf{w}}(k,d), \quad (5)$$

where $\Omega$ denotes the set of $2^B$ quantized beamforming vector, i.e., a beamforming codebook. In this paper, the focus is on a practical case that uses a quantized beamforming vector, and the codebook is designed according to the technique in [7], as proposed for narrowband systems, is employed in the simulations.

## 3.3 Obtaining AR coefficients from noisy channel observations

To obtain the LMMSE predictor, $\{a_d(p)\}$ must be derived. An estimated channel at the receiver is modeled as $[\hat{\mathbf{H}}(k,n)]_{p,q} = [\mathbf{H}(k,n)]_{p,q} + [\mathbf{n}'(k,n)]_{p,q}$, in which $[\mathbf{n}'(k,n)]_{p,q}$ is assumed as the i.i.d Gaussian distribution with a zero mean and a variance of $\sigma_e^2$ for all $k$, $n$, $p$, and $q$.[4] By the Yule-Walker method, $\mathbf{a}_d = [a_d(1), \cdots, a_d(M)]^T$ is obtained via $\mathbf{a}_d = -(\mathbf{R} + \sigma_e^2\mathbf{I}_M)^{-1}\mathbf{r}_d$, in which $\mathbf{R}$ is a $M$-by-$M$ matrix defined as $[\mathbf{R}]_{p,q} = r(p-q)$, $\mathbf{I}_M$ is the $M$-by-$M$ identity matrix, and $\mathbf{r}_d$ is a $M$-dimensional vector with the $p$-th element defined as $r(d+p)$ [15]. In practice, estimates of $\{r(m)\}$, $\{\hat{r}(m)\}$, are needed. This can be performed using a sample averaging method: $\hat{r}(m) = 1/(NM_tM_rN_s) \cdot \sum_k \sum_p \sum_q \sum_n [\hat{\mathbf{H}}(k,n)]_{p,q}[\hat{\mathbf{H}}(k,n-m)]^*_{p,q}$, for which $N_s$ is the number of samples counted in the time-domain. This approaches $\hat{r}(m) = r(m) + \delta(m)\sigma_e^2$ as the number of samples increases under the assumption of WSS channels. It is important to note that $\sigma_e^2$ can be estimated implicitly.

[4] $\sigma_e^2$ is the mean-square error (MSE) of channel estimates at the receiver.

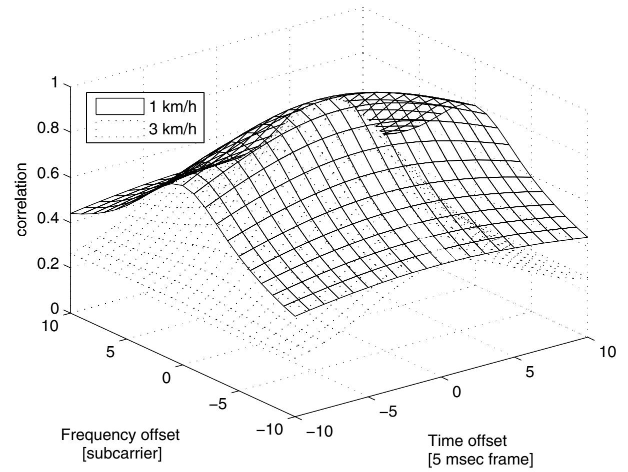


**Figure 2: Frequency- and time-domain correlation of beamforming vectors.**

# 4. SELECTION OF BEAMFORMING VECTORS

As the optimal beamformers are obtained at the receiver side, the receiver must convey information regarding the derived beamforming vectors. However, $BNP$ bits per instance of feedback is required to inform the transmitter of the beamforming vectors for all subcarriers ($N$) of all future frames ($P$) when the beamforming codebook with $|\Omega| = 2^B$ is employed: for example, with $N = 64$, $B = 4$, and $P = 4$, 1024 bits of feedback are required for every 4 frames. This amount is excessive in practical situations. Accordingly, two-dimensional (2D) clustering is introduced to reduce the amount of feedback information, and several schemes for the selection of beamforming vectors are proposed.

## 4.1 Correlation between beamforming vectors

Given that channels have correlation within the coherence time, beamforming vectors have correlation over time, which is similar to the case of the frequency-domain correlation of beamformers in [9]. Fig. 2 exhibits the computer simulated time- and frequency-domain correlation of beamforming vectors defined as $\eta(l,d) = E[|\mathbf{w}^H(k,n)\mathbf{w}(k-l, n-d)|^2]$.[5] Here, beamforming vectors are the optimal vectors given the perfect channel information. The system parameters are: $M_t = 4$, $M_r = 2$, $N = 64$, and the CP length of 16; the

[5] When $\mathbf{w}(k,n)$ is the optimal beamforming vector for the subcarrier $k$ and the frame $n$, $e^{j\phi}\mathbf{w}(k,n)$ is also optimal, where $\phi$ is the arbitrary phase; thus, the correlation is measured using a definition of the beamformer correlation which is independent of $\phi$.

frame duration is 5 msec; the carrier frequency is 2.3 GHz; the power density profile of a time-domain channel impulse response follows ETSI/BRAN Channel Model B in [16]; and each tap of the time-domain impulse response of the channel is generated using Jakes model with mobile speeds 1 and 3 km/h. In the Fig. 2, the correlation decreases as the mobile speed increases. This implies that the amount of beamformer correlation is inversely proportional to the mobile speed, as is the channel. This result shows that a sufficient amount of correlation exists to introduce the 2D clustering technique over time when the channel varies slowly.

## 4.2 The proposed 2D clustering with beamformer selection schemes

Here, 2D clustering is introduced to reduce the amount of the feedback information and several schemes for the selection of beamforming vectors are proposed. For 2D clustering, the rectangle of the $K$-subcarrier by $P$-frame is considered, and the entire frequency-time domain of $N$ subcarriers and $P$ frames is partitioned into 2D clusters with $N/K$ clusters (where $K$ divides $N$). The same beamforming vector will be used in each 2D cluster at the transmitter. The beamformer selection schemes are combinations of two one-dimensional (1D) schemes comprising clustering [9] and smart clustering [10]. These perform based on maximization and max-min criterions for the effective channel gains, and these criterions tend to decrease the average BER. The clustering uses the beamforming vector that maximizes the effective channel gain of the *center subcarrier* in the cluster; for the smart clustering, the beamformer is chosen by considering all subcarriers in the cluster, as $\mathbf{w}^* = \arg\max_{\mathbf{w}\in\Omega} \min_{k\in\mathbf{S}_k} \|\mathbf{H}(k, n^*)\mathbf{w}\|^2$ where $\mathbf{S}_k$ is the set of subcarrier indices in the cluster and $n^*$ is the index of the present frame. The followings represent the 2D beamforming vector selection methods for the $l$-th cluster denoted as the 'Frequency-domain selection, Time-domain selection':

- 'Clustering, Clustering' ('CC'):
$$\mathbf{w}_l^* = \arg\max_{\mathbf{w}\in\Omega} \Psi_{\mathbf{w}}(lK + K/2, P/2).$$
- 'Smart clustering, Clustering' ('SC'):
$$\mathbf{w}_l^* = \arg\max_{\mathbf{w}\in\Omega} \min_{1\le k\le K} \Psi_{\mathbf{w}}(lK + k, P/2).$$
- 'Clustering, Smart clustering' ('CS'):
$$\mathbf{w}_l^* = \arg\max_{\mathbf{w}\in\Omega} \min_{1\le d\le P} \Psi_{\mathbf{w}}(lK + K/2, d).$$
- 'Smart clustering, Smart clustering' ('SS'):
$$\mathbf{w}_l^* = \arg\max_{\mathbf{w}\in\Omega} \min_{1\le k\le K, 1\le d\le P} \Psi_{\mathbf{w}}(lK + k, d).$$

Here, $\Psi_{\mathbf{w}}(k, d)$ in (4) is exploited as a measure; this represents the expected effective channel gain for a future channel given beamforming vector $\mathbf{w}$. Comparing the computational complexities, 'CC' is the simplest, 'CS' and 'SC' are comparable, and 'SS' is the most complicated.

# 5. SIMULATION RESULTS

To illustrate the performance of the proposed approach, Monte Carlo BER simulations were performed for a system with parameters $M_t = 4$, $M_r = 2$, and $N = 64$, and a CP length of 16. In this system, the frame duration is 5 msec, the carrier frequency is 2.3 GHz, the power density profile of a time-domain channel impulse response follows the ETSI/BRAN Channel Model B in [16] (15 taps), each tap of the time-domain impulse response of the channel is generated by Jakes model with a mobile speed of 4 km/h, and QPSK is used. The receiver uses MRC with estimated channels ($\sigma_e^2 = (2\text{SNR})^{-1}$ is assumed.) The beamforming codebook with $|\Omega| = 2^4$ ($B = 4$) designed according to the technique in [7] is employed and channel coding is not considered. For clustering, $K = 8$ and $P = 6$ are utilized. The four 2D clustering schemes 'CC', 'SC', 'CS', and 'SS' as well as two 1D clustering schemes 'C' (Clustering) and 'S' (Smart clustering) are employed. For the 2D clustering, the 4-th order AR model is employed ($M = 4$) in which $\{r(m)\}$ is estimated using estimated channels at the receiver observing the channels of the 30 frames.[6] For all selection schemes, the feedback requirement is 32 bits per 6 frames. Additionally, for references in comparison, BER performances with all feedback for all the subcarriers and frames are investigated for cases: the optimal quantized beamforming vectors obtained from the exact channels are used (as denoted by 'IQ'); and the quantized beamforming vectors derived as in (5) are employed (as presented by 'IPQ').

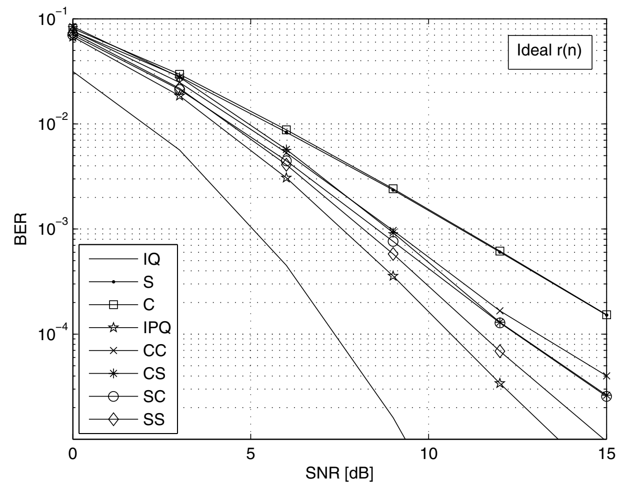


**Figure 3: Uncoded BER performances of 1D and 2D beamformer selection schemes in time-varying channels with ideal $r(n)$.**

Figs. 3 and 4 exhibit BER for the cases of the ideal $\{r(n)\}$ and estimated $\{r(n)\}$, respectively. Comparing Fig. 3 and Fig. 4, the performances of the systems with the ideal $\{r(n)\}$ and estimated $\{r(n)\}$ are comparable. The proposed 2D schemes outperform the conventional 1D schemes; this holds true with the simplest 'CC' as well. 'SS' shows comparable performance to 'IPQ', and this implies that 'SS' is effective in reducing feedback. The results also demonstrate the trade-off between the BER performance and the computational complexity of the beamformer selection schemes: BER increases in the order of 'SS', 'CS'≈'SC', and 'CC' while the complexity increases in order of 'CC', 'CS'≈'SC', and 'SS', as mentioned in the last section.

[6]For comparison, a case using an ideal $\{r(m)\}$ value was also simulated.

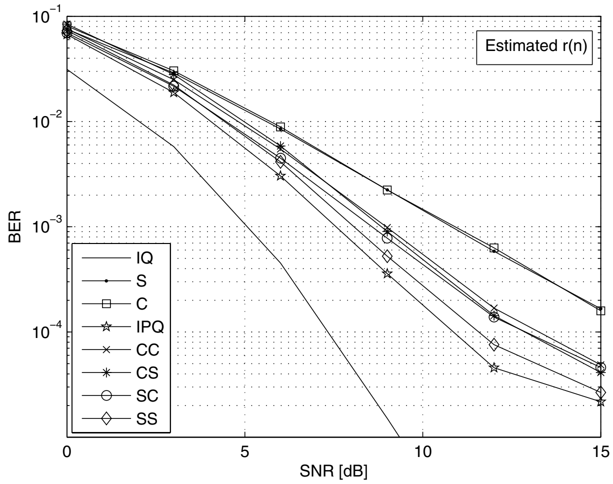


**Figure 4: Uncoded BER performances of 1D and 2D beamformer selection schemes in time-varying channels with estimated $r(n)$.**

# 6. CONCLUSIONS

The performance degradation of transmit beamforming MIMO-OFDM systems caused by feedback delays in time-varying channels is compensated by the proposed technique. With this technique, the expected effective channel gain for a future channel is maximized. 2D beamformer selection schemes are employed to reduce the amount of feedback information. Simulation results utilizing the technique show that impairment caused by channel variations in the time domain are mitigated when utilizing the same amount of feedback.

Further research in this area will include the design of a more sophisticated 2D beamformer interpolator (2D clustering is the 0-th order interpolator), and 2D interpolation schemes for precoded MIMO-OFDM with spatial multiplexing. There remain numerous pristine opportunities for research in this area.

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