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Transmit Optimization for Relay-based Cellular OFDMA Systems

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***Abstract*—This paper considers a broadband cellular orthogonal frequency-division multiple-access (OFDMA) system with relay nodes operating in decode-and-forward and half-duplex mode. Two transmit resource allocation problems for sum-rate maximization are formulated for such a system. The first one is the optimization of subcarrier allocation with predetermined power assignment for each subcarrier, and the other is the joint optimization of power and subcarrier allocation. Since these problems can't be solved easily in direct forms, we make continuous relaxation and solve the dual problems using a subgradient method. Numerical results show that a remarkable increment in sum-rate is achieved, with the aid of relay nodes and sophisticated resource allocation, compared to a system without relay nodes.**

I. Introduction

Currently, due to its suitability for efficient and high data-rate transmission, orthogonal frequency-division multiple-access (OFDMA) is considered as a promising technology for packet-based cellular communications [13] and next generation cellular systems. In designing such cellular systems, a formidable performance issue is the channel impairment at the edge of a cell induced by a bad propagation environment, such as path-loss and shadowing, and co-channel interference from adjacent cells. In many previous works [1]-[4], it is shown that these channel impairments can be mitigated through the use of cooperative transmission by wireless relay nodes. That is, by an efficient design of relaying strategy and resource allocation, the reliability and transmission rate of a wireless network can be improved.

In this paper, we consider the use of wireless relays for the sum-rate maximization of a broadband cellular OFDMA system. We assume the decode-and-forward strategy and the half-duplex operation of relays, which is considered to be practical [4]. We further assume that the channel is fixed or slowly-varying, and thus the channel can be estimated and fed-back from mobile stations (MSs) and relay stations (RSs) to the base station (BS) for centralized processing. Then, we investigate the downlink resource allocation for such a system.

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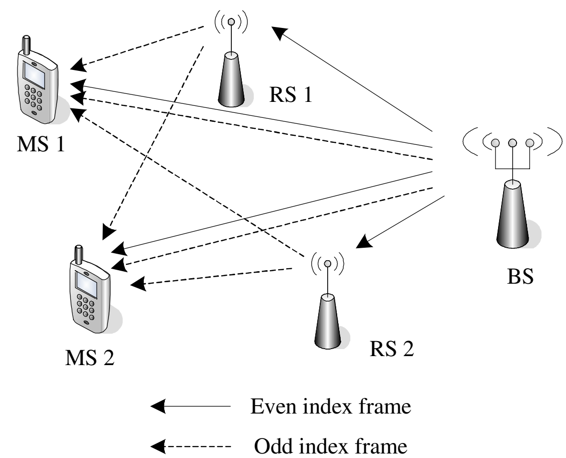

Fig. 1. System model

As an optimization problem, we first formulate the optimal subcarrier allocation problem with predetermined power assignment for each subcarrier. After that, we also formulate the optimal joint power and subcarrier allocation problem, which is more complex than the previous one. Then, it is observed that these problems belong to the class of integer programming problems, and thus, it is hard to find the optimal solution in a direct approach. Instead of solving these problems directly, we use the continuous relaxation, as in [5] and [6], to recast them as linear and convex optimization problems, respectively, and solve the dual problems using the subgradient method [7]-[9]. As the result of optimization, we show numerically that the sum-rate of a cellular OFDMA system can be markedly increased with the aid of RSs, compared to a system without RSs.

II. System Description

We consider a cellular system, as shown in Fig. 1; this system is composed of a single BS, M RSs, and K MSs sharing the same frequency band. We assume that the wireless channel between each pair of transmitting and receiving nodes is frequency selective, and OFDMA is employed for data communication to covert the channel into a set of N orthogonal subcarriers with flat channel responses and additive white Gaussian noises (AWGN). It is also assumed that the channel

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remains unchanged for at least two frames, and all channel information is fed-back to BS for centralized processing.

Many previous works on this kind of relay-based network assume that the RS can transmit and receive simultaneously in the same frequency band [1]-[3], i.e., full-duplex. However, since many limitations in radio implementation preclude the terminals from full-duplex operation, we assume the half-duplex operation of RSs. Due to the half-duplex characteristics, each RS operates in two phases; it receives data from BS at, say, an even index frame, and then relays the data to MSs at the following odd index frame. We further assume the decode-and-forward operation of RSs.

From the next section, we will focus on the downlink resource allocation for the system, which maximizes the sum-rate.

III. Optimization with Fixed Power

In this section, we formulate the sum-rate maximizing downlink subcarrier allocation problem while assuming that the average power for each subcarrier is predetermined.

A. Fixed Power Subcarrier Allocation Formulation

For the ease of exposition, we assume that the average transmit power, denoted as P, for each subcarrier of RSs and BS is equal. For each subcarrier of index n, we define variables $\rho_{m,n}$ indicating whether the subcarrier is allocated to the BS to mth RS ($m = 1, \ldots, M$) or MS ($m = M+1, \ldots, M+K$) link, which has signal-to-interference-plus-noise ratio (SINR) $\alpha_{m,n}$, in an even index frame. We also define $\sigma_{m,k,n}$ indicating whether the subcarrier is allocated to the BS ($m = 0$) or mth RS ($m = 1, \ldots, M$) to kth MS link, which has SINR $\beta_{m,k,n}$, in an odd index frame. If the channel remains unchanged, $\alpha_{M+j,n} = \beta_{0,j,n}$ ($j = 1, \ldots, K$) holds. Then, using these variables, the subcarrier allocation problem is formulated as

$$\max_{\rho,\sigma} \frac{1}{2}\sum_{m=1}^{M} \min\{A_m, B_m\} + \frac{1}{2}\left(\sum_{m=M+1}^{M+K} A_m + B_0\right) \quad (1)$$

$$\text{s.t.} \quad \textstyle\sum_{m=1}^{M+K} \rho_{m,n} = 1, \forall n \quad (2a)$$

$$\textstyle\sum_{m=0}^{M}\sum_{k=1}^{K} \sigma_{m,k,n} = 1,\ \forall n \quad (2b)$$

$$\rho_{m,n} \in \{0,1\},\ \forall m,n \quad (2c)$$

$$\sigma_{m,k,n} \in \{0,1\},\ \forall m,k,n \quad (2d)$$

where

$$A_m \triangleq \sum_{n=1}^{N} \rho_{m,n} \log_2(1 + \alpha_{m,n}P) \quad (3)$$

and

$$B_m \triangleq \sum_{n=1}^{N}\sum_{k=1}^{K} \sigma_{m,k,n} \log_2(1 + \beta_{m,k,n}P). \quad (4)$$

The OFDMA constraint (2) guarantees that each subcarrier is occupied by only one transmitter-receiver pair, that is, there is no subcarrier reuse in the system. In (1), the first term represents the rate of data flow from BS to MSs via RSs operating in decode-and-forward strategy, and the second term represents the data rate of direct channels from BS to MSs.

Unfortunately, due to the OFDMA constraint, the above problem belongs to the class of integer programming problems [11], for which an exact solution usually involves an exhaustive search, which is computationally extensive when the number of variables is large. Therefore, for an easier formulation, we relax the integer constraints (2c) and (2d) as $\rho_{m,n} \geq 0$ and $\sigma_{m,k,n} \geq 0$ respectively. This continuous relaxation corresponds to permitting time sharing of each subcarrier and was shown to be optimal in the limiting case ($N \to \infty$) in [5] and [6]. Accordingly, the following lemma holds.

Lemma 1: The optimal solution to (1)-(2) with continuous relaxation satisfies $A_m = B_m$ for $m = 1, \ldots, M$.

Proof: Assume that the optimum is such that $A_m \neq B_m$ for some $m \in \{1, \ldots, M\}$. We first consider the case that $A_m > B_m$. Then, for some $\rho_{m,n} \neq 0$, we can decrease $\rho_{m,n}$ and increase $\rho_{m',n}$ for some $m' \in \{M+1, \ldots, M+K\}$ while maintaining $A_m > B_m$ and satisfying (2a). Then, we obtain a new feasible solution with a larger objective value, which is a contradiction. Similarly, we can show the contradiction for the case that $A_m < B_m$. □

Physically, the above lemma implies that when the subcarriers are allocated optimally, the rates of receiving and transmitting data of each RS are equal. Using Lemma 1, the subcarrier allocation problem with continuous relaxation can be rewritten as

$$\max_{\rho,\sigma} \frac{1}{4}\sum_{m=1}^{M} (A_m + B_m) + \frac{1}{2}\left(\sum_{m=M+1}^{M+K} A_m + B_0\right) \quad (5)$$

$$\text{s.t.} \quad A_m = B_m,\ \text{for } m = 1, \ldots, M \quad (6a)$$

$$\textstyle\sum_{m=1}^{M+K} \rho_{m,n} = 1,\ \forall n \quad (6b)$$

$$\textstyle\sum_{m=0}^{M}\sum_{k=1}^{K} \sigma_{m,k,n} = 1,\ \forall n \quad (6c)$$

$$\rho_{m,n} \geq 0,\ \forall m,n \quad (6d)$$

$$\sigma_{m,k,n} \geq 0,\ \forall m,k,n \quad (6e)$$

This is a linear optimization problem which can be efficiently solved using the simplex [11] or interior-point method [10].

B. Dual Problem for Subcarrier Allocation

Although the standard optimization tools, such as simplex or interior-point method, can solve (5)-(6) efficiently, they still have large complexity when the number of variables is large. Furthermore, unless the number of subcarriers is sufficiently large, the continuous relaxation generally yields a fractional-valued solution, and we should round it to 0 or 1 to get a integer-valued solution.

As an indirect approach, we can solve the dual problem of (5)-(6). Since (5)-(6) is a linear optimization problem, the duality gap is zero, and thus the solution of the dual problem is equal to that of the primal problem. For the formulation of the dual problem, we first define the Lagrangian as follows:

$$L(\rho,\sigma,\lambda) = \frac{1}{4}\sum_{m=1}^{M}(A_m + B_m) + \frac{1}{2}\left(\sum_{m=M+1}^{M+K} A_m + B_0\right) - \sum_{m=1}^{M}\lambda_m (A_m - B_m), \quad (7)$$

where $\lambda \triangleq [\lambda_1,\ldots,\lambda_M]^T$ is the vector of Lagrangian dual variables for the equality constraint (6a). Then, the dual objective function problem is computed as

$$g(\lambda) = \begin{cases} \max\limits_{\rho,\sigma} & L(\rho,\sigma,\lambda) \\ \text{s.t.} & \text{(6b)-(6e)} \end{cases} \quad (8)$$

and the dual problem is given as

$$\min_{\lambda} g(\lambda). \quad (9)$$

Since the dual problem is always convex, it is guaranteed that the gradient-type algorithms converge to the global optimum. For (9), we can use a subgradient method [7]-[9], since it is hard to differentiate with λ. As the first step of this method, we initialize $\lambda^{(0)}$ and, given $\lambda^{(i)}$, where i is the iteration index, compute the dual objective function (8). From (7) and (8), we observe that the dual objective function can be decoupled into two independent subproblems as $g(\lambda) = g_1(\lambda) + g_2(\lambda)$, where

$$g_1(\lambda) = \begin{cases} \max\limits_{\rho} & \sum_{m=1}^{M}\left(\frac{1}{4}-\lambda_m\right)A_m + \sum_{m=M+1}^{M+K}\frac{1}{2}A_m \\ \text{s.t.} & \text{(6b) and (6d)} \end{cases} \quad (10)$$

and

$$g_2(\lambda) = \begin{cases} \max\limits_{\sigma} & \frac{1}{2}B_0 + \sum_{m=1}^{M}\left(\frac{1}{4}+\lambda_m\right)B_m \\ \text{s.t.} & \text{(6c) and (6e).} \end{cases} \quad (11)$$

$g_1(\lambda)$ can be further decoupled into N independent per-subcarrier problems as $g_1(\lambda) = \sum_{n=1}^{N} g_{1,n}(\lambda)$, where

$$g_{1,n}(\lambda) = \begin{cases} \max\limits_{\rho} & \sum_{m=1}^{M}\left(\frac{1}{4}-\lambda_m\right)\rho_{m,n}\log_2(1+\alpha_{m,n}P) \\ & \quad + \sum_{m=M+1}^{M+K}\frac{1}{2}\rho_{m,n}\log_2(1+\alpha_{m,n}P) \\ \text{s.t.} & \sum_{m=1}^{M+K}\rho_{m,n} = 1 \\ & \rho_{m,n} \geq 0,\ \forall m. \end{cases} \quad (12)$$

Then, (12) is a knapsack problem [11], and given $\lambda^{(i)}$, the solution is given for each n as

$$\rho_{m,n}^{(i)} = \begin{cases} 1, & m = m'^{(i)} \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

where

$$m'^{(i)} \triangleq \arg\max_{m=1,\ldots,M+K} \gamma_{m,n}^{(i)} \quad (14)$$

and

$$\gamma_{m,n}^{(i)} \triangleq \begin{cases} \left(\frac{1}{4}-\lambda_m^{(i)}\right)\log_2(1+\alpha_{m,n}P), m = 1,\ldots,M \\ \frac{1}{2}\log_2(1+\alpha_{m,n}P), m = M+1,\ldots,M+K. \end{cases} \quad (15)$$

Similarly, $g_2(\lambda)$ also decouples into N subproblems as $g_2(\lambda) = \sum_{n=1}^{N} g_{2,n}(\lambda)$. One clear fact is that if the solution of $g_{2,n}(\lambda)$ is $\sigma_{\bar{m},\bar{k},n} = 1$ for a $(\bar{m},\bar{k})$ pair and $\sigma_{m,k,n} = 0$ for $(m,k) \neq (\bar{m},\bar{k})$, then $\bar{k}$ is given as

$$\bar{k} = \arg\max_{k=1,\ldots,K} \beta_{\bar{m},k,n}. \quad (16)$$

This implies that, if the $\bar{m}$th RS uses the nth subcarrier, then it always transmits to the MS with the best channel for sum-rate maximization. Using this property, for each n, the solution of $g_{2,n}\left(\lambda^{(i)}\right)$ is given as

$$\sigma_{m,k,n}^{(i)} = \begin{cases} 1, & m = \bar{m}^{(i)},\ k = \bar{k}^{(i)} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

where

$$\bar{m}^{(i)} \triangleq \arg\max_{m=0,\ldots,M} \delta_{m,n}^{(i)}, \quad (18)$$

$$\bar{k}^{(i)} \triangleq \arg\max_{k=1,\ldots,K} \beta_{\bar{m}^{(i)},k,n}, \quad (19)$$

$$\delta_{m,n}^{(i)} \triangleq \begin{cases} \frac{1}{2}\log_2(1+\beta_{m,n}P), m = 0 \\ \left(\frac{1}{4}+\lambda_m^{(i)}\right)\log_2(1+\beta_{m,n}P), m = 1,\ldots,M \end{cases} \quad (20)$$

and $\beta_{m,n} = \max_{k=1,\ldots,K}\beta_{m,k,n}$. Note that $\rho_{m,n}$ and $\sigma_{m,k,n}$ obtained in computing the dual objective function are always integer-valued, that is, there is no time-sharing. They can have fractional values only when there are more than one equal maxima of $\gamma_{m,n}$ or $\delta_{m,n}$. However, due to the numerical error induced from the finite-precision computation, such a case seldom happens in practice.

After computing $g(\lambda^{(i)})$ as above, $\lambda^{(i)}$ is updated using the subgradient method as

$$\lambda_m^{(i+1)} = \lambda_m^{(i)} + s^{(i)}\left(A_m^{(i)} - B_m^{(i)}\right),\quad m = 1,\ldots,M \quad (21)$$

where $s^{(i)}$ is a sequence of step size. Since the subgradient method is guaranteed to converge to the optimum provided that $s^{(i)}$ is designed properly [9], we can obtain a near-optimal solution for (5)-(6) with a sufficiently large number of iterations. Then, since the solution is integer-valued, it is also a solution of (1)-(2).

IV. Optimal Power and Subcarrier Allocation

Although the fixed-power subcarrier allocation problem considered in the previous section is simple, the available transmission power is inflexibly and inefficiently exploitted in that problem. Therefore, in this section, we consider the joint power and subcarrier allocation problem maximizing the sum-rate of a given relay-based cellular system under the OFDMA and total power constraint.

A. Joint Power and Subcarrier Allocation Formulation

For the formulation of the joint power and subcarrier allocation problem, instead of indication variables, such as ρ and σ, we define positive variables $p_{m,n}$ and $q_{m,k,n}$, which represent the amount of power allocated to the nth subcarrier of BS to mth RS ($m = 1,\ldots,M$) or MS ($m = M+1,\ldots,M+K$) link and BS ($m = 0$) or mth RS ($m = 1,\ldots M$) to kth MS link, respectively. Then, using these variables, the joint power and subcarrier allocation problem is formulated as

$$\max_{\mathbf{p},\mathbf{q}} \frac{1}{2}\sum_{m=1}^{M} \min\left\{\tilde{A}_m, \tilde{B}_m\right\} + \frac{1}{2}\left(\sum_{m=M+1}^{M+K} \tilde{A}_m + \tilde{B}_0\right) \quad (22)$$

$$\begin{aligned} \text{s.t.} \quad & \textstyle\sum_{n=1}^{N}\sum_{m=1}^{M+K} p_{m,n} \le P_0 & (23a)\\ & \textstyle\sum_{n=1}^{N}\sum_{k=1}^{K} q_{m,k,n} \le P_m,\ \forall m & (23b)\\ & p_{m,n}p_{m',n} = 0,\ \forall m \ne m', n & (23c)\\ & q_{m,k,n}q_{m',k',n} = 0,\ \forall (m,k) \ne (m',k'), n & (23d)\\ & p_{m,n} \ge 0,\ \forall m, n & (23e)\\ & q_{m,k,n} \ge 0,\ \forall m, k, n & (23f) \end{aligned}$$

where

$$\tilde{A}_m \triangleq \sum_{n=1}^{N} \log_2\left(1 + \alpha_{m,n} p_{m,n}\right), \quad (24)$$

$$\tilde{B}_m \triangleq \sum_{n=1}^{N}\sum_{k=1}^{K} \log_2\left(1 + \beta_{m,k,n} q_{m,k,n}\right), \quad (25)$$

and P_m (P_0) is the total transmit power of mth RS (BS). In this problem, the OFDMA constraint is represented by (23c) and (23d). As in (1), the first term of (22) represents the data rate of BS-RS-MS channels, and the second term represents the data rate of direct channels from BS to MSs.

Similarly with the fixed-power case, (22)-(23) is a nonlinear integer programming problem [11], and it is hard to find the optimal solution. For this case also, we can consider a continuous relaxation by allowing the time sharing of each subcarrier. For this purpose, we define additional variables $v_{m,n}$ and $w_{m,k,n}$, which represent the fraction of time that the nth subcarrier is occupied by BS to mth RS ($m = 1, \ldots, M$) or MS ($m = M+1, \ldots, M+K$) link and BS ($m = 0$) or mth RS ($m = 1, \ldots M$) to kth MS link, respectively. Lemma 1 also holds in this case, and thus the relaxed problems are written as

$$\max_{\mathbf{p},\mathbf{q},\mathbf{v},\mathbf{w}} \frac{1}{4}\sum_{m=1}^{M} \left(\hat{A}_m + \hat{B}_m\right) + \frac{1}{2}\left(\sum_{m=M+1}^{M+K} \hat{A}_m + \hat{B}_0\right) \quad (26)$$

$$\begin{aligned} \text{s.t.} \quad & \hat{A}_m = \hat{B}_m,\ \text{for } m = 1, \ldots, M & (27a)\\ & \textstyle\sum_{n=1}^{N}\sum_{m=1}^{M+K} p_{m,n} \le P_0 & (27b)\\ & \textstyle\sum_{n=1}^{N}\sum_{k=1}^{K} q_{m,k,n} \le P_m,\ \forall m & (27c)\\ & \textstyle\sum_{m=1}^{M+k} v_{m,n} = 1,\ \forall n & (27d)\\ & \textstyle\sum_{m=0}^{M}\sum_{k=1}^{K} w_{m,k,n} = 1,\ \forall n & (27e)\\ & v_{m,n} \ge 0,\ \forall m, n & (27f)\\ & w_{m,k,n} \ge 0,\ \forall m, k, n & (27g) \end{aligned}$$

where

$$\hat{A}_m \triangleq \sum_{n=1}^{N} v_{m,n} \log_2\left(1 + \alpha_{m,n}\frac{p_{m,n}}{v_{m,n}}\right) \quad (28)$$

and

$$\hat{B}_m \triangleq \sum_{n=1}^{N}\sum_{k=1}^{K} w_{m,k,n} \log_2\left(1 + \beta_{m,k,n}\frac{q_{m,k,n}}{w_{m,k,n}}\right). \quad (29)$$

Then, this is a convex optimization problem and can be solved efficiently using standard optimization tools [10].

B. Dual Problem for Power and Subcarrier Allocation

As we did in Section III, we try to tackle (22)-(23) by solving its dual problem. Since it is a convex problem, the duality gap is zero. The Lagrangian is formed as follows:

$$\begin{aligned} L\left(\mathbf{p},\mathbf{q},\mathbf{v},\mathbf{w},\lambda,\mu,\xi\right) &= \frac{1}{4}\sum_{m=1}^{M}\left(\hat{A}_m + \hat{B}_m\right)\\ &+ \frac{1}{2}\left(\sum_{m=M+1}^{M+K}\hat{A}_m + \hat{B}_0\right) - \sum_{m=1}^{M}\lambda_m\left(A_m - B_m\right)\\ &+ \mu\left(P_0 - \sum_{n=1}^{N}\sum_{m=1}^{M+K} p_{m,n}\right)\\ &+ \sum_{m=1}^{M}\xi_m\left(P_m - \sum_{n=1}^{N}\sum_{k=1}^{K} q_{m,k,n}\right). \end{aligned} \quad (30)$$

where λ, μ, and ξ are the vectors of dual variables. Then, the dual objective function is computed as

$$h(\lambda,\mu,\xi) = \begin{cases} \max\limits_{\mathbf{p},\mathbf{q},\mathbf{v},\mathbf{w}} & L\left(\mathbf{p},\mathbf{q},\mathbf{v},\mathbf{w},\lambda,\mu,\xi\right)\\ \text{s.t.} & \text{(27d)-(27g)} \end{cases} \quad (31)$$

and the dual problem is given as

$$\begin{aligned} &\min_{\lambda,\mu,\xi} h(\lambda,\mu,\xi)\\ &\text{s.t. } \mu \ge 0, \xi \succeq 0 \end{aligned} \quad (32)$$

where "$\succeq$" represents the element-wise inequality. To solve the dual problem with a subgradient method, we first initialize $\lambda^{(0)}$, $\mu^{(0)}$, and $\xi^{(0)}$ and compute the dual objective function given $\lambda^{(i)}$, $\mu^{(i)}$, and $\xi^{(i)}$. We observe that the dual objective function can be decoupled into two subproblems as $h(\lambda,\mu,\xi) = h_1(\lambda,\mu) + h_2(\lambda,\xi)$, where

$$h_1(\lambda,\mu) = \begin{cases} \max\limits_{\mathbf{p},\mathbf{v}} & \sum_{m=1}^{M}\left(\frac{1}{4} - \lambda_m\right)\hat{A}_m + \sum_{m=M+1}^{M+K}\frac{1}{2}\hat{A}_m\\ & +\mu\left(P_0 - \sum_{n=1}^{N}\sum_{m=1}^{M+K} p_{m,n}\right)\\ \text{s.t.} & \text{(27d) and (27f)} \end{cases} \quad (33)$$

and

$$h_2(\lambda,\xi) = \begin{cases} \max\limits_{\mathbf{q},\mathbf{w}} & \frac{1}{2}\hat{B}_0 + \sum_{m=1}^{M}\left(\frac{1}{4} + \lambda_m\right)\hat{B}_m\\ & + \sum_{m=1}^{M}\xi_m\left(P_m - \sum_{n=1}^{N}\sum_{k=1}^{K} q_{m,k,n}\right)\\ \text{s.t.} & \text{(6c) and (6e).} \end{cases} \quad (34)$$

Then, (33) and (34) are equal to the dual objective functions of multi-carrier broadcast and multiple-access channel optimization problems considered in [5] and [6], respectively. Given

$\lambda^{(i)}$, $\mu^{(i)}$, and $\xi^{(i)}$, the solution of (33) is given for each n as

$$v_{m,n}^{(i)} = \begin{cases} 1, & m = \hat{m}^{(i)} \text{ and } X_{\hat{m}^{(i)},n}^{(i)} > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{35}$$

$$p_{m,n}^{(i)} = v_{m,n}^{(i)} \left[\frac{c_m^{(i)}}{\mu^{(i)}} - \frac{1}{\alpha_{m,n}} \right]^+ \tag{36}$$

where

$$\hat{m}^{(i)} \triangleq \arg\max_{m=1,\ldots,M+K} X_{m,n}^{(i)}, \tag{37}$$

$$X_{m,n}^{(i)} \triangleq c_m^{(i)} \ln \left(1 + \alpha_{m,n} \left[\frac{c_m^{(i)}}{\mu^{(i)}} - \frac{1}{\alpha_{m,n}} \right]^+ \right) \tag{38}$$

$$- \mu^{(i)} \left[\frac{c_m^{(i)}}{\mu^{(i)}} - \frac{1}{\alpha_{m,n}} \right]^+, \tag{39}$$

and

$$c_m^{(i)} \triangleq \begin{cases} \left(\frac{1}{4} - \lambda_m^{(i)} \right), & m = 1, \ldots, M \\ \frac{1}{2}, & m = M+1, \ldots, M+K. \end{cases} \tag{40}$$

Similarly, for each n, the solution of (34) is given as

$$w_{m,k,n}^{(i)} = \begin{cases} 1, & m = \check{m}^{(i)},\ k = \check{k}^{(i)}, \text{ and } Y_{\check{m}^{(i)},n}^{(i)} > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{41}$$

$$q_{m,k,n}^{(i)} = w_{m,k,n}^{(i)} \left[\frac{d_m^{(i)}}{\xi_m^{(i)}} - \frac{1}{\beta_{m,k,n}} \right]^+ \tag{42}$$

where

$$\check{m}^{(i)} \triangleq \arg\max_{m=0,\ldots,M} Y_{m,n}^{(i)}, \tag{43}$$

$$\check{k}^{(i)} \triangleq \arg\max_{k=1,\ldots,K} \beta_{\check{m}^{(i)},k,n}, \tag{44}$$

$$Y_{m,n}^{(i)} \triangleq d_m^{(i)} \ln \left(1 + \beta_{m,n} \left[\frac{d_m^{(i)}}{\xi_m^{(i)}} - \frac{1}{\beta_{m,n}} \right]^+ \right) \tag{45}$$

$$- \xi_m^{(i)} \left[\frac{d_m^{(i)}}{\xi_m^{(i)}} - \frac{1}{\beta_{m,n}} \right]^+, \tag{46}$$

$$d_m^{(i)} \triangleq \begin{cases} \frac{1}{2}, & m = 0 \\ \left(\frac{1}{4} + \lambda_m^{(i)} \right), & m = 1, \ldots, M, \end{cases} \tag{47}$$

and $\beta_{m,n} \triangleq \max_{k=1,\ldots,K} \beta_{m,k,n}$.

Note that the time sharing parameters $v_{m,n}$ and $w_{m,k,n}$ obtained in computing the dual objective function are always integer-valued, that is, there is no time-sharing. Time-sharing is possible only when there are more than one equal maxima of $X_{m,n}$ or $Y_{m,n}$. However, as mentioned previously, this case seldom happens due to the numerical error.

After computing $h(\lambda^{(i)}, \mu^{(i)}, \xi^{(i)})$ as above, it is minimized using a subgradient method [9]. That is, $\lambda^{(i)}$, $\mu^{(i)}$, and $\xi^{(i)}$ are updated as follows:

$$\lambda_m^{(i+1)} = \lambda_m^{(i)} + s^{(i)} \left(\hat{A}_m^{(i)} - \hat{B}_m^{(i)} \right),\ m = 1, \ldots, M, \tag{48}$$

$$\mu^{(i+1)} = \mu^{(i)} + t^{(i)} \left(P_0 - \sum_{n=1}^{N} \sum_{m=1}^{M+K} p_{m,n}^{(i)} \right), \tag{49}$$

$$\xi_m^{(i+1)} = \xi_m^{(i)} + u^{(i)} \left(P_m - \sum_{n=1}^{N} \sum_{k=1}^{K} q_{m,k,n}^{(i)} \right),\ m = 0, \ldots, M. \tag{50}$$

where $s^{(i)}$, $t^{(i)}$, and $u^{(i)}$ are sequences of step size designed properly [9]. Then, with this subgradient method, we can obtain a near-optimal solution of (26)-(27) with a sufficiently large number of iterations. Then, since the solution is integer-valued, it is also a solution of (22)-(23).

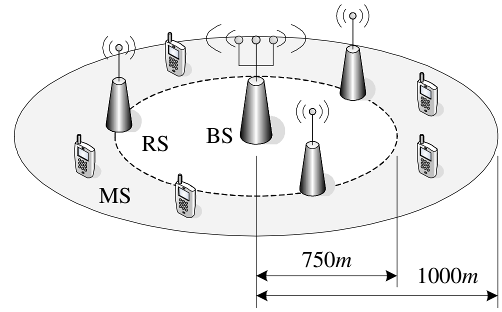

Fig. 2. Topology of a relay-based cellular system.

V. SIMULATIONS

This section compares the performance of two optimization schemes for the relay-based network treated in previous sections through computer simulation. We consider a single cell model with a ring-shaped boundary region. The radius of the inner and outer bound of this region is $750m$ and $1000m$, respectively. MSs are uniformly distributed in the boundary region, and RSs are evenly located on the inner bound of this region. We also assume that all MSs remain stationary and the

TABLE I
SIMULATION PARAMETERS.

Parameter	Assumption
Cell radius	$1000m$
Propagation model (path-loss, in dB)	BS-MS, RS-MS: $128.1 + 37.6 \log_{10}(R)$ BS-RS: $128.1 + 28.8 \log_{10}(R)$ (R in kilometers)
RS antenna gain	10 dB
MS antenna gain	0 dB
MS noise figure	9 dB
Thermal noise density	-174 dBm/Hz
BS and RS Tx power	24 dBm
FFT size	128
# of used subcarriers	96
Channel bandwidth	1.25 MHz
Small-scale fading model	Exponential power delay profile with decaying rate 2 and 10 μs delay spread
# of MSs	8

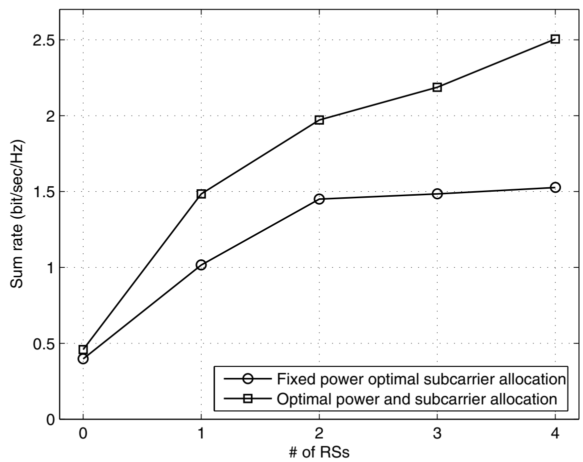

Fig. 3. Sum rate of relay-based cellular system.

channel does not change. A simplified view of this topology is shown in Fig. 2. The simulation parameters are listed in Table I, which refers mainly to [13] and [14].

For performance comparison, we evaluate the sum-rate of each optimization scheme using a different number of RSs, and the results are shown in Fig. 3. We can observe that, as the number of RSs increases, the sum-rate also increases. When the number of RSs is zero, i.e., when there is no RS, the fixed power subcarrier allocation problem is reduced to a knapsack problem [11], and the solution is simply to choose the best receiver (MS) for each subcarrier. Without RSs, the joint power and subcarrier allocation problem is reduced to the sum-rate maximization problem for broadcast channels [5], and the solution is to choose the best receiver for each subcarrier and to perform waterfilling over all subcarriers. For the considered system, the power level is sufficiently high, and thus there is a little difference between the fixed-power allocation and waterfilling [12]. When there are more than one RS, the sum-rate is evaluated using the subgradient method with 2000 iterations. The result shows that the joint power and subcarrier allocation scheme has a much higher sum rate than the fixed power subcarrier allocation scheme, which is because the former exploits the available power more efficiently than the latter.

VI. Conclusion

In this paper, we showed that the sum-rate of a broadband cellular OFDMA system can be remarkably increased by using relay nodes and sophisticated resource allocations. In particular, we formulated two different resource allocation problems for sum-rate maximization. The first one is the optimization of subcarrier allocation with a predetermined power assignment for each subcarrier, and the other is the joint optimization of power and subcarrier allocation. Due to the OFDMA constraint which forces the exclusive use of each subcarrier by transmitter-receiver pairs, these problems belong to the class of integer programming problems, and solving them is computationally prohibitive when the number of variables is large.

Therefore, for easier formulations, we relaxed the problems as linear and convex optimization problems, respectively, by allowing time-sharing usage of each subcarrier by multiple transmitter-receiver pairs. Then, we formulated the dual problem of each relaxed problem and solved it using a subgradient method. From the result of the subgradient method, we always obtained integer-valued solutions, which are the near-optimal solutions to the original unrelaxed problems.

Throughout this paper, we have assumed that all channel information is perfectly known to the BS, which is almost impossible in practice because of the imperfectness of channel estimation and limited bandwidth of the feedback channel. Therefore, our future work will include a study on the effects of nonideal and limited channel information on the performance of the relay-based system. Another topic of future work is solving the uplink resource allocation problem for the relay-based system. Since there is a larger number of constraints on the transmission power of each MS, this problem is more complex than its downlink counterpart. However, we believe that it also can be easily tackled with approaches similar to those treated in this paper.

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