Amplify-Forward Relays with Superimposed Pilot Signals for Frequency-Selective Fading Channels

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***Abstract*— In this paper, a new amplify-and-forward (AF) relay systems using superimposed pilot signals is proposed for frequency selective fading channel environments. In the proposed scheme, the relay superimposes an additional pilot sequence on the received signal from the source, and forwards the combined signal to the destination. The superimposed pilot signal in this way enables the destination node to estimate the channel between the relay and destination (R-D), and this channel information is used to process the received signal at the destination optimally; the noise correlation due to the frequency selective fading in the R-D channel is whitened. The optimal power ratio of the superimposed pilot to the incoming signal at the relay is obtained via simulations, and it is shown under frequency selective channel environments that the proposed scheme yields better bit-error-rate (BER) performance than the conventional method where all relay power is allocated only to the incoming signal.**

I. Introduction

Recently, relay systems are employed for improvement of communication reliability and increment of cell coverage. To extend the communication range without relay stations, higher transmission power is required and it causes shortening of battery life and higher interference [1]. By locating a relay between a source and corresponding destination, however, we can reduce the transmission power because communication range per hop decreases [2], [3]. Therefore, we can achieve the higher system capacity in cellular systems, ad-hoc networks and so on [4].

Several kinds of relaying schemes including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF) have been introduced. In the practical point of view, AF and DF methods can be more reasonable approaches than CF scheme. In the DF relay, the received analog signal is decoded into the bit-level source data. After such a decoding process, the recovered data is re-encoded and modulated as symbol-level signal and then transmitted to the destination. For that reason, the DF relay is also referred as a regenerative or digital relay. An AF relay, which is called a non-regenerative relay, only retransmits the amplified version of the received signal in the analog domain, not performing any decoding procedures in the digital domain. As we can expect, DF relays require much higher complexity and more power than AF ones [5]. In [6], it is shown that the AF relay outperforms over the DF relay when the relay is located closer to the destination than the source node. Throughout this paper, we consider an AF relaying scenario under the above condition.

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In the AF relay systems, the general assumption that the noise at a receiver side is modeled as an additive white Gaussian noise (AWGN) is not valid any more. Since the AF relay simply transmits the amplified version of the received signal from the source, AWGN at the relay is also forwarded by the relay. Therefore, the relayed signal including AWGN is passed through frequency selective fading R-D channels and another AWGN is added to that at the destination. Consequently, the effective noise at the destination is the colored noise (i.e., the noise values are correlated by frequency selective fading R-D channels) that degrades receiver performance. To deal with this problem, we propose a new AF relay system employing superimposed pilots. In the proposed system, the relay superimposes predetermined pilot symbols on the received signal and transmits it to the destination simultaneously. Using superimposed pilots, the destination can estimate the R-D channel response and determine the coefficients of the whitening filter which makes the colored noise be white. In this system model, the optimal power ratio between the superimposed pilots and the forwarded symbols is optimized for the performance improvement. The optimal power ratio can be determined through computer simulation. It is shown that the proposed system can achieve the better BER performance than the conventional AF relay system. If the channel coding scheme is employed, furthermore, the performance gap between the proposed and conventional ones becomes larger than that of the uncoded cases.

The organization of this paper is as follows. Section II describes the conventional and proposed AF relay system model, and the proposed receiver architecture at the destination is derived in Section III. In section IV, the advantages of the proposed system over the conventional one are demonstrated through computer simulations. Finally, Section V contains the conclusion.

II. Relay System Model

A. Conventional Relay System

Fig. 1(a), (b) show the the conventional AF relay system and the corresponding frame structure. The considering AF relay

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system consists of single source, single destination, and one AF relay. In the figure, T_p and T_d denote the length of pilot and data symbols, respectively. For coherent communication, the pilots are used to estimate the source to destination (S-D) channels. For simplicity, we assume that there is no direct path between the source and the destination. The S-R, R-D channels are frequency selective fading ones with coefficients denoted by $\mathbf{h}_{SR}$ and $\mathbf{g}_{RD}$ with the number of channel taps, M and N, respectively.

It is assumed that the relay operates in a half-duplex mode, i.e., can not transmit and receive simultaneously, and has fixed relay gain. The overall transmission of one packet requires two time slots. At first, the source transmits its own data to the relay. Next, the relay amplifies and forwards the received signals to the destination.

At the first time slot, the received signals at the relay is then represented as

$$\mathbf{r} = \mathbf{X}_R \mathbf{h}_{SR} + \mathbf{w}_1,$$

where $\mathbf{h}_{SR} = [h_{SR}(1), h_{SR}(2), ..., h_{SR}(M)]^{\mathrm{T}}$ is the M-tap S-R channel vector, $\mathbf{w}_1$ is a complex valued AWGN vector whose components follow the complex Gaussian distribution with zero mean and variance $\sigma^2_{w_1}$. $\mathbf{X}_R = \underset{M}{\mathrm{Toef}}\,[x(1), \cdots, x(T_p), \cdots, x(T), 0, \cdots, 0]^{\mathrm{T}}$ which denotes $(T + M - 1) \times M$ a convolution matrix of the transmitted data defined as follows:

$$\begin{aligned} \mathbf{X}_R &= \underset{M}{\mathrm{Toef}}\,[x(1), \cdots, x(T_p), \cdots, x(T), 0, \cdots, 0]^{\mathrm{T}} \\ &= \begin{bmatrix} x(1) & 0 & \cdots & 0 \\ x(2) & x(1) & \cdots & \vdots \\ \vdots & \ddots & \cdots & 0 \\ \vdots & \ddots & \cdots & x(1) \\ x(T-1) & \vdots & \cdots & \vdots \\ x(T) & x(T-1) & \vdots & \vdots \\ 0 & x(T) & \cdots & \vdots \\ \vdots & \vdots & \vdots & x(T-1) \\ 0 & 0 & \cdots & x(T) \end{bmatrix} \end{aligned}$$

At the second time slot, the relay amplifies and forwards the signals to the destination. The received signals at the destination is given by

$$\mathbf{y} = \beta \mathbf{X}\mathbf{h} + \beta \mathbf{W}_1 \mathbf{g}_{RD} + \mathbf{w}_2, \qquad (1)$$

where β is a relay gain, $\mathbf{h} = \mathbf{g}_{RD} * \mathbf{h}_{SR} = [h(1), h(2), ..., h(K)]^{\mathrm{T}}$ is the composite channel response vector of S-D with $K = M + N - 1$ channel taps, $\mathbf{g}_{RD} = [g(1), g(2), ..., g(N)]^{\mathrm{T}}$ is the R-D channel vector, $\mathbf{X} = \underset{K}{\mathrm{Toef}}\,[x(1), x(2), \cdots, x(T), 0, \cdots, 0]^{\mathrm{T}}$ which denotes the convolution matrix of the transmit data, $\mathbf{W}_1$ denotes the convolution matrix of complex AWGN at the relay, and $\mathbf{w}_2$ is the complex AWGN vector with zero mean and variance $\sigma^2_{w_2}$ at the destination. From (1), we can find that the effective

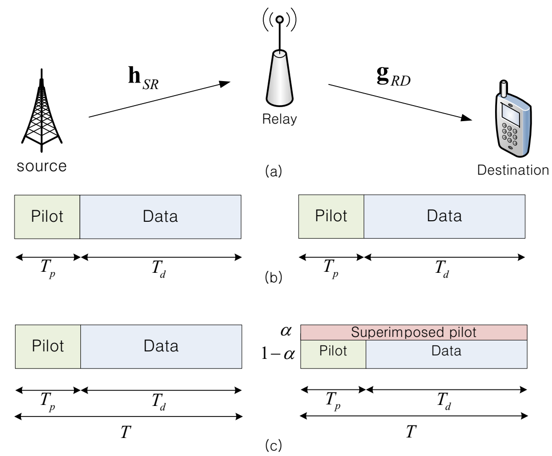

Fig. 1. AF relay system (a) AF relay system consists of a source, destination, and a relay (b) Frame structure of the conventional system (c) Frame structure of the proposed system.

noise at the destination ,$\beta \mathbf{W}_{1T} \mathbf{g}_{RD} + \mathbf{w}_2$, is a colored noise vector, i.e., is correlated due to the frequency selective fading R-D channels. Hence, the performance of the AF relay system can be improved by whitening such colored noise at the destination. In the next section, we propose a AF relay system with superimposed pilots and the optimal receiver structure of the destination considering the colored noise effects.

B. Proposed AF Relay Employing Superimposed Pilot

The proposed AF relay system and frame structure are shown in the Fig. 1(a) and 1(c). The relay superimposes its own pilot sequence on the received signal from the source and forwards it to the destination. The additional superimposed pilots can be used to estimate the R-D channels at the destination. Using the estimates of the R-D channel response, a whitening filter can be designed to whiten the colored noise caused by frequency selective R-D channels.

In the proposed system, the power ratio between the transmit signal and the superimposed pilot should be determined at the relay. We define the power of the overall symbols in a packet, the superimposed pilot part, the pilot transmitted by the source, and the data part as σ^2_t, σ^2_{sp}, σ^2_p, and σ^2_d, respectively. Then, the following power and time relation should be satisfied

$$\sigma^2_p T_p + \sigma^2_d T_d + \sigma^2_{sp} T = \sigma^2_t T.$$

With assumption of constant modulus packet which means that $\sigma^2_p = \sigma^2_d$, the power ratio between the transmit signal and the superimposed pilot can be defined as follows:

$$\frac{\sigma^2_{sp}}{\sigma^2_t} = \alpha, \frac{\sigma^2_p}{\sigma^2_t} = 1 - \alpha,$$

where α is the power ratio. By optimally selecting this parameter, we can minimize BER.

Similarly to the conventional relay system, the received

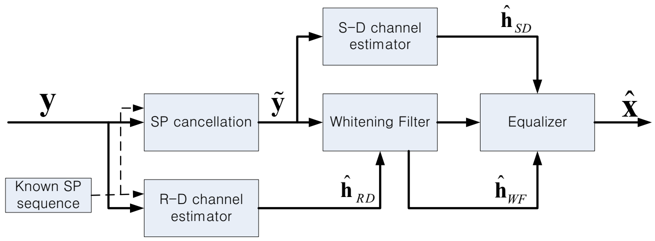

Fig. 2. Proposed receiver architecture .

signal at the destination is represented as follows:

$$\mathbf{y} = \beta\sqrt{\alpha}\mathbf{P}_T\mathbf{g}_{RD} + \beta\sqrt{1-\alpha}\mathbf{X}\mathbf{h} + \beta\sqrt{1-\alpha}\mathbf{W}_1\mathbf{g}_{RD} + \mathbf{w}_2 \tag{2}$$

where $\mathbf{P}_T = \underset{N}{\text{Toef}}\left[p(1), p(2), \cdots, p(T), 0, \cdots, 0\right]^{\mathrm{T}}$ denotes a convolution matrix of the superimposed pilot.

III. Proposed Receiver Algorithm

Fig.2 shows the proposed structure of the receiver at the destination. The algorithm is described as follows: At first, the R-D channel, $\mathbf{g_{RD}}$, is estimated with the superimposed pilots. Assuming the least squares (LS) estimation is used, the estimate of $\mathbf{g_{RD}}$ can be written as follows:

$$\hat{\mathbf{g}}_{LS} = \frac{1}{\beta\sqrt{\alpha}}(\mathbf{P_T^H P_T})^{-1}\mathbf{P_T^H y}. \tag{3}$$

After obtaining the R-D channel estimate, the superimposed pilot sequence can be canceled out by subtracting from the received signal $\mathbf{y}$.

$$\tilde{\mathbf{y}} = \frac{1}{\beta\sqrt{1-\alpha}}(\mathbf{y} - \beta\sqrt{\alpha}\mathbf{P_T}\hat{\mathbf{g}}_\mathbf{LS}) = \mathbf{X_T h} + \tilde{\mathbf{n}} \tag{4}$$

where

$$\tilde{\mathbf{n}} = \mathbf{P}_T\tilde{\mathbf{g}}_{LS} + \mathbf{W}_1\mathbf{g}_{RD} + \frac{1}{\beta\sqrt{1-\alpha}}\mathbf{w}_2. \tag{5}$$

After canceling the superimposed pilots, the S-D channel can be estimated as follows:

$$\hat{\mathbf{h}}_{LS} = (\mathbf{X}_P^H\mathbf{X}_P)^{-1}\mathbf{X}_P^H\tilde{\mathbf{y}}_\mathbf{T_P} \tag{6}$$

where $\tilde{\mathbf{y}}_{T_P} = [\tilde{y}(1), \tilde{y}(2), \cdots, \tilde{y}(T_P)]$ and $[\mathbf{X}_P]_{ij} = [\mathbf{X}]_{ij}$ for $1 \le i \le T_p, 1 \le j \le K$. Now using the estimated channel $\mathbf{g}_{LS}$, the whitening filter that whiten the colored noise of the received signal can be designed. From (3), the power spectrum of the effective noise can be written as

$$P_{\tilde{w}} = \beta^2(1-\alpha)\sigma_{w_1}^2 H_{RD}(z)H_{RD}^*(1/z^*) + \sigma_{w_2}^2. \tag{7}$$

Applying spectral factorization in (7), $P_{\tilde{w}}$ can be represented as [7]

$$P_{\tilde{w}} = \sigma^2 Q(z)Q^*(1/z^*). \tag{8}$$

In (8), the whitening filter can be derived as $1/Q(z)$. However, $1/Q(z)$ is an IIR type filter, we can approximate the filter as a FIR type filter [8]. When the S-D channel and whitening filter is fixed, the optimal detection is done by the minimum mean square error (MMSE) equalizer. The signal $\tilde{\mathbf{y}}$ is passed through the whitening filter and equalizer, the decision is made at the output of the equalizer.

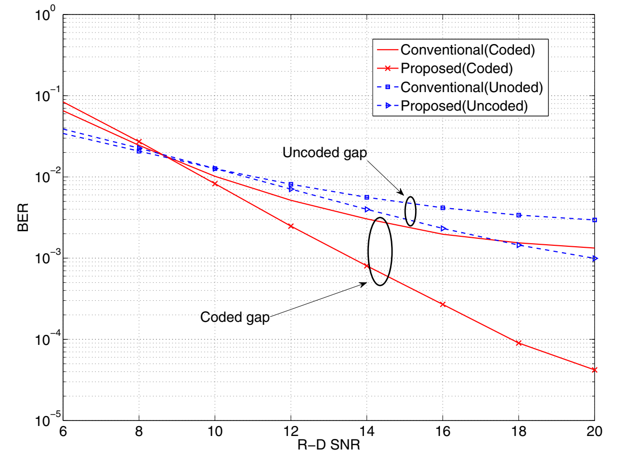

Fig. 3. BER performance versus R-D SNR of the AF relay system (S-R SNR = 8 dB, perfect channel information).

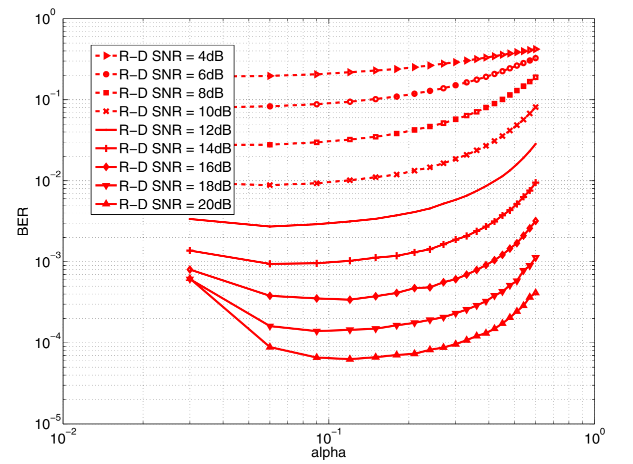

Fig. 4. Coded BER versus α (R-D SNR = 8 dB).

IV. Simulation Results

In this section, the performance of the proposed system is analyzed and compared with the conventional relay system through computer simulations. In the simulation, the following parameters are assumed: The total length of transmission symbols in a packet at the source is 1000 and that of pilots is 100. BPSK modulation is used and a 1/2 convolutional code is used for forward error correction. The channels are quasi-static Rayleigh fading ones and the number of taps of S-R, R-D channels are one and three, respectively. The delay profile is a exponentially decaying profile.

At first, we examine the effect of whitening the colored noise at the destination based on the perfect S-D and R-D

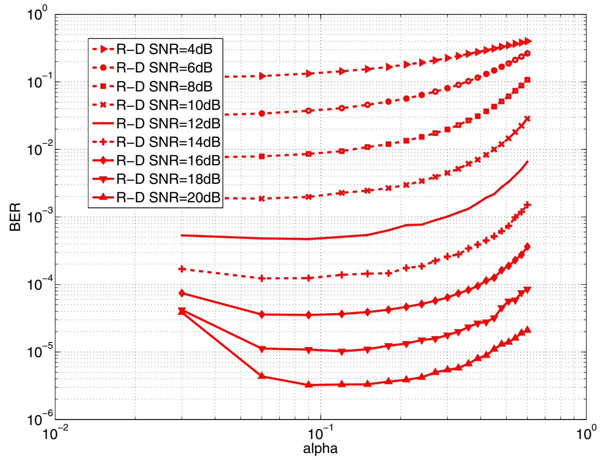

Fig. 5. Coded BER versus α (R-D SNR = 12 dB).

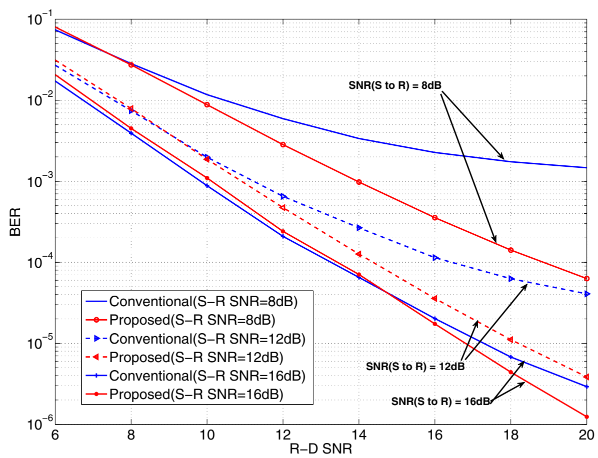

Fig. 6. Coded BER performance (S-R SNR = 8, 12, 16 dB).

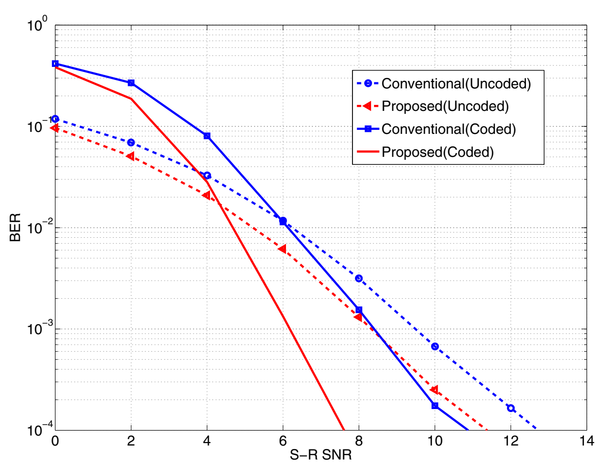

Fig. 7. BER performance versus S-R SNR of the AF relay system (R-D SNR = 20 dB).

channel information. The results are shown in Fig. 3 when the S-R SNR is set to 8dB. Uncoded and coded cases are simulated. From the results, we can see that the proposed relay system outperforms the conventional relay system when the R-D SNR is larger than the 12 dB in uncoded cases and 9 dB in coded cases. The larger R-D SNR is, the better performance gain is achieved and the gain of coded cases is larger than that of uncoded cases.

In the proposed AF relay system, the optimal power ratio α for the superimposed pilot at the relay node should be determined. The α that minimizes the BER is obtained by computer simulations. Fig. 4 and Fig. 5 show the coded BER performances with different values of α when the S-R SNRs are 8 and 12 dBs, respectively. At the region where the R-D SNR is lower than 10 dB, the lower α value shows the better BER performance. Due to the degradation of channel estimation accuracy of the R-D channel, the performance of the resulting whitening filter will be degraded at this R-D SNR region. For the region where the R-D SNR is higher than 12 dB, the BER performance becomes better with we increase the α until about 0.1$\sim$0.2. The performance degradation, however, is shown when α is greater than 0.2. Although increasing α improves the performance of the whitening filter, it decreases the effective SNR at the destination. For the above α values of 0.2, the decrease of the effective SNR is dominant and the overall performance is degraded.

Fig. 6 shows the coded BER performance with various R-D SNR values when S-R SNR values are 8, 12, and 16 dB. In this simulation, the optimal power ratio α for the superimposed pilot is chosen from the above results and the proposed system shows better performance. When the R-D SNR increases, the gain is getting larger at S-R SNR of 8, 10, and 16 dB, respectively. At the BER value of 10^{-4}, the proposed system outperforms the conventional system more than 3 dB when the S-R SNR is 8 dB. However, when the S-R SNR is increased, the gain is reduced. At the BER of 10^{-5}, the gain is less than 1 dB at S-R SNR of 16 dB. This shows that the proposed system does not improve the performance when the S-R SNR is high. That can be explained as follows. At the high S-R SNR, the amplified noise is negligible and the effective SNR of the received signal is reduced because the some portion of power is allocated to the superimposed pilots.

Coded BER performance with respect to S-R SNR of the proposed and the conventional AF relay system is compared in Fig. 7. The R-D SNR is set to 20 dB. The proposed system always have better performance than the conventional one. At the BER value of 10^{-4}, the proposed system has a 3 dB gain. From the results, we can see that the proposed system shows good performance when the S-R SNR is relatively low and R-D SNR is high.

V. Conclusion

In this paper, we have proposed an AF relay system employing the superimposed pilots. In this relay system, the relay superimposes the additional pilot signals over the received signal from the source. Using the channel estimation results with this superimposed pilot, the destination can determine the whitening filter weights to whiten the colored noise caused by frequency selective channels. This approach is the optimal detection procedure in the colored noise environments. By the appropriate selection of the power ratio between the superimposed pilot and forwarded data, we can optimize the BER performance. In frequency selective channels, therefore, the proposed system outperforms the conventional one at low S-R SNR and high R-D SNR.

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