Side Information-Free PTS-PAPR Reduction via Pilot Assisted Estimation of Phase Factors in an OFDM Frame with a Preamble

Youngseok Oh*, Eui-Rim Jeong†, Sungho Choi* Wooseok Nam*, and Yong H. Lee*

*Dept. of Electrical Engineering, KAIST, Daejeon, Korea
†Division of Information Communication and Computer Engineering, Hanbat National University, Daejeon, Korea
Email: *{ysoh, shchoi, wsnam}@stein.kaist.ac.kr, †erjeong@hanbat.ac.kr,*yohlee@ee.kaist.ac.kr

***Abstract*— A new partial transmit sequence (PTS) technique for orthogonal frequency division multiplexing (OFDM) signals which does not require side information, is proposed. In the proposed method, the transmitter applies PTS to reduce the peak-to-average power ratio (PAPR) and does not send the side information on the PTS phase factors to the receiver. Based on the OFDM symbol structure having pilot subcarriers, the receiver jointly estimates the channel and PTS phase factors from the pilot subcarriers. The proposed technique inherently causes bit error rate (BER) loss depending on the PTS subgroup size. However, simulation results demonstrate that if the PTS subgroup size is carefully chosen, the proposed method can reduce the PAPR significantly with a negligible BER loss.**

Key Words: OFDM, PAPR reduction, PTS, side information free

I. Introduction

The orthogonal frequency division multiplexing (OFDM) is a popular transmission technique in wireless communications due to its high spectral efficiency and robustness against frequency selective fading channels [1]–[3]. Since OFDM is a multicarrier transmission technique, signals at different frequencies can be added constructively, and this leads to a high peak to average power ratio (PAPR) [4]. The high PAPR limits the average transmission power due to the nonlinearity of the power amplifier. Many techniques have been proposed to reduce the PAPR, and a comprehensive review can be found in [5]. Among the techniques, the partial transmit sequence (PTS) is one of the effective methods [6]–[8], where the OFDM subcarriers are partitioned into several subgroups and each group of subcarriers is multiplied by a phase factor to reduce the PAPR. In order to detect the data symbols properly at the receiver, the phase factors are required. Hence, the additional side information on the phase factors should be transferred from the transmitter to the receiver, which leads to a loss in spectral efficiency and data throughput. To relieve from this loss, several techniques have been proposed [9], [10]. Jaylath and Tellambura [9] proposed a joint ML (maximum likelihood) approach to estimate phase factors and detect transmitted symbols simultaneously without side information at the receiver. This technique performs effectively under perfect channel state information at the receiver (CSIR). However, it is observed from our simulation that this method is sensitive to the channel estimation errors, and it is complex to implement. Garcia *et al.* [10] suggested the use of orthogonal sequences as the pilot to reduce the PAPR. The receiver can easily determine which sequence is used at the transmitter without side information using the orthogonal property. However, the amount of PAPR reduction is limited because this method exploits only pilot subcarriers to reduce the PAPR.

This work was supported by the IT R&D program of MKE/KEIT, [KI001835].

In this paper, we propose a side information-free PTS technique for a frame structure preceded by a known preamble and an OFDM symbol structure where pilot subcarriers are inserted regularly. As in [9], the transmitter applies the PTS to reduce the PAPR and does not send the phase factor information to the receiver. In the proposed scheme, the receiver jointly estimates the channel and phase factors from the pilot subcarriers by joint ML approach. The estimation performance is proportional to the PTS subgroup size, while the amount of PAPR reduction is inversely proportional to that. Therefore, the PTS subgroup size is carefully chosen to ensure negligible bit error rate (BER) loss and sufficient PAPR reduction. The performance of the proposed method is empirically demonstrated through computer simulation. When the number of total subcarriers is 128; each subcarrier is modulated by 16QAM; and 1/8 of total subcarriers are pilots, subgroup size 16 (16 subcarrier in a subgroup) provides 6.2dB PAPR reduction while negligible BER loss. In addition, it is observed that the proposed technique is more robust to the channel estimation error and has smaller complexity at the receiver than the method in [9].

The mathematical expressions used in this paper are defined as follows. $[\mathbf{A}]_{m,n}$ represents the (m,n) th elements of matrix $\mathbf{A}$, and $\lceil a \rceil$ denotes the minimum integer bigger than a. $\mathbb{E}\{\cdot\}$ is the expectation and $diag\{\mathbf{a}\}$ is a diagonal matrix whose (n,n) th diagonal element is n th element of vector $\mathbf{a}$.

978-1-4244-8016-6/10/$26.00 ©2010 IEEE

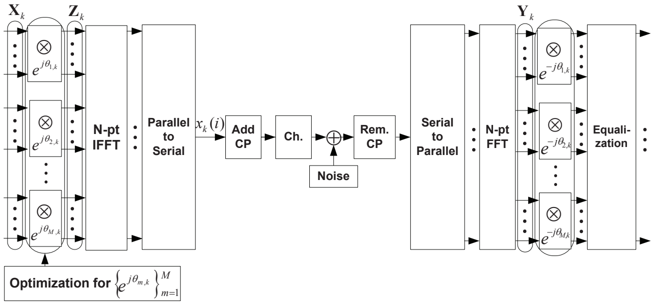

Fig. 1. An OFDM system employing a PTS technique.

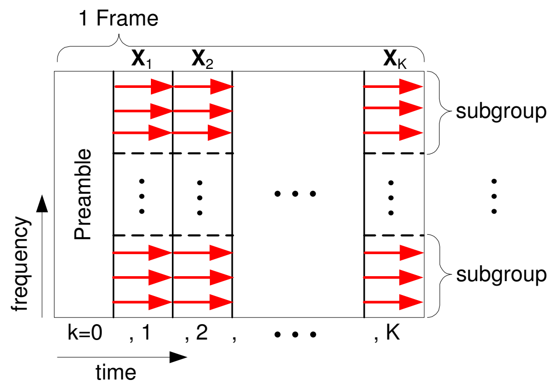

Fig. 2. An OFDM frame consisting of a preamble and K OFDM symbols, where $\mathbf{X}_k$ is an OFDM symbol and each subgroup contains N_P equi-spaced pilots (arrows in the OFDM symbols represent pilot subcarriers).

II. System Model

An OFDM system employing a PTS technique for PAPR reduction is shown in Fig. 1. An OFDM symbol of span N, denoted by $\mathbf{X}_k \equiv [X_{k,0}, X_{k,1}, \cdots, X_{k,N-1}]^T$ at time k, is partitioned into M subgroups $\{\mathbf{X}_{k,1}, \cdots, \mathbf{X}_{k,M}\}$ where M divides N, $\bigcup_{m=1}^{M} \mathbf{X}_{k,m} = \mathbf{X}_k$ and $\mathbf{X}_{k,m} = [X_{k,\frac{(m-1)N}{M}}, \cdots, X_{k,\frac{mN}{M}-1}]^T$. Then a phase factor $e^{j\theta_{k,m}}$, $m \in \{1, 2, \cdots, M\}$, is assigned to each subgroup and multiplied with the elements of $\mathbf{X}_{k,m}$ to yield $\mathbf{Z}_{k,m} = e^{j\theta_{k,m}}\mathbf{X}_{k,m}$. Let $\mathbf{Z}_k \equiv [\mathbf{Z}_{k,1}^T, \mathbf{Z}_{k,2}^T, \cdots, \mathbf{Z}_{k,M}^T]^T$. Then $\mathbf{Z}_k \in \mathbb{C}^{N\times 1}$ can be represented as

$$\mathbf{Z}_k = \sum_{m=1}^{M} \mathbf{D}_m \mathbf{X}_k e^{j\theta_{k,m}}, \tag{1}$$

where $\mathbf{D}_m \in \mathbb{R}^{N\times N}$ is a diagonal matrix whose (n, n) th entry is defined as $[\mathbf{D}_m]_{n,n} = 1$ if $\frac{(m-1)N}{M} \le n \le \frac{mN}{M} - 1$ and $[\mathbf{D}_m]_{n,n} = 0$, otherwise. The inverse fast Fourier transform (IFFT) of $\mathbf{Z}_k$ is denoted by $\mathbf{x}_k = [x_k(0), \cdots, x_k(N-1)]^T$. If $\mathbf{F}^{-1} \in \mathbb{C}^{N\times N}$ is a IFFT matrix whose entry is given by $[\mathbf{F}^{-1}]_{l,n} = e^{j2\pi ln/N}$, $\mathbf{x}_k = \mathbf{F}^{-1}\mathbf{Z}_k = \mathbf{F}^{-1}\sum_{m=1}^{M}\mathbf{D}_m\mathbf{X}_k e^{j\theta_{k,m}}$. The phase factors $\{e^{j\theta_{k,m}}\}_{m=1}^{M}$ are selected to minimize the PAPR given by

$$PAPR \equiv \frac{\max |x_k(i)|^2}{\mathbb{E}\{|x_k(i)|^2\}}. \tag{2}$$

In general, $\{e^{j\theta_{k,m}}\}_{m=1}^{M}$ is found through exhaustive search. The received signal after the fast Fourier transform (FFT) is then written as

$$\mathbf{Y}_k = \mathbf{H}_k\mathbf{Z}_k + \mathbf{n}_k, \tag{3}$$

where $\mathbf{H}_k \in \mathbb{C}^{N\times N}$ is a channel matrix which is diagonal and $\mathbf{n}_k \in \mathbb{C}^{N\times 1}$ is a noise vector.

A frame, which starts with a preamble, is formed by collecting OFDM symbols$\{\mathbf{X}_k\}$ (Fig. 2). A PAPR of the preamble is assumed to be sufficiently low, and the PAPR reduction is not performed on the preamble[1]. In addition, it is assumed that each subgroup $\mathbf{X}_{k,m}$ of an OFDM symbol $\mathbf{X}_k$ contains N_P equi-spaced pilot subcarriers which are used for both channel tracking and estimating PTS phase factors. Those frame and pilot structures are based on commercial standards [1]–[3].

[1]In 802.16e downlink [1], PAPR of the preamble is less than 5 dB, while that of data OFDM symbols is usually greater than 10 dB.

III. Proposed Phase Factor and Channel Estimator

Based on the system model described in the previous section, this section proposes the joint estimation technique for the PTS phase factors and the channel $\mathbf{H}_k$ without side information.

A. Joint Estimation

Since the phase factors and the channel response are multiplied with each other, joint estimation is rather difficult. However, noting that the PAPR reduction is not performed on the preamble, the initial channel can be estimated during the preamble period. Assuming that the channel is changed slowly, the previous channel estimate can be used at the next OFDM symbol. Further, introducing prediction techniques for estimating the next channel, channel estimation performance can be improved. Using this idea, the joint phase factor and channel estimation problem can be separated into two estimation problems. The detailed procedure for the proposed method is as follows:

Step 1. $k = 0$ (Initial channel estimation)

Estimate the initial frequency-domain channel from the preamble. First, the time-domain channel vector $\mathbf{h}_0 \in \mathbb{C}^{L\times 1}$ is estimated where L is the channel length. Among various existing techniques, the least squares (LS) estimator is considered [11]:

$$\hat{\mathbf{h}}_0 = ((\mathbf{PQ})^H(\mathbf{PQ}))^{-1}(\mathbf{PQ})^H\mathbf{Y}_0,$$

where $\mathbf{P} = diag\{X_{0,0}, X_{0,1}, ..., X_{0,N-1}\}$ is a diagonal matrix whose diagonals are frequency-domain preambles and $\mathbf{Q} \in \mathbb{C}^{N\times N}$ is the N-point discrete Fourier transform (DFT) matrix. The estimate of frequency-domain channel matrix is obtained by

$$\hat{\mathbf{H}}_0 = diag\{\mathbf{Q}_L\hat{\mathbf{h}}_0\}.$$

where $\mathbf{Q}_L \in \mathbb{C}^{N\times L}$ is a submatrix of $\mathbf{Q}$, defined as $[\mathbf{Q}_L]_{p,q} = (1/\sqrt{N})e^{-j\frac{2\pi}{N}(p-1)(q-1)}$ for $1 \le P \le N$, and $1 \le q \le L$).

Step 2. $k = k + 1$ (channel prediction and phase factor estimation)

The frequency-domain channel of the current OFDM symbol is predicted from the past channel estimates. The prediction is performed on the time-domain channel taps using the recursive least squares (RLS) algorithm [12]. If the channel estimates $\{\hat{\mathbf{h}}_i\}_{i=k-V}^{k-1}$ are used for the prediction of $\mathbf{h}_k$, the predicted channel vector $\tilde{\mathbf{h}}_k$ is obtained by

$$\tilde{\mathbf{h}}_k = \sum_{v=1}^{V} \mathbf{w}_v^H \hat{\mathbf{h}}_{k-v}$$

where $\{\mathbf{w}_v\}_{v=1}^{V}$ are the prediction weight vectors and V is the prediction memory length. The weight vectors are designed to minimize the following cost function:

$$\mathcal{E} = \sum_{i=0}^{k} \|\hat{\mathbf{h}}_i - \sum_{v=1}^{V} \mathbf{w}_v^H \hat{\mathbf{h}}_{i-v}\|^2$$

and it is assumed that $\hat{\mathbf{h}}_i = \mathbf{0}$ for $i < 0$. The predicted frequency-domain channel matrix $\tilde{\mathbf{H}}_k$ is given by $\tilde{\mathbf{H}}_k = \mathrm{diag}\{\mathbf{Q}_L \tilde{\mathbf{h}}_k\}$.

After obtaining $\tilde{\mathbf{H}}_k$, the PTS phase factors $\{e^{j\theta_{k,m}}\}_{m=1}^{M}$ are estimated from the pilot subcarriers. Specifically, the phase factors are estimated based on the ML criterion:

$$e^{j\hat{\theta}_{k,m}} = \arg \min_{\forall e^{j\theta_{k,m}}} \|\mathbf{D}_m^p \mathbf{Y}_k - e^{j\theta_{k,m}} \tilde{\mathbf{H}}_k \mathbf{D}_m^p \mathbf{1}\|^2,$$

where $\mathbf{1} \in \mathbb{C}^{N\times 1}$ is all-one vector denoting pilot signals, $\mathbf{D}_m^p \in \mathbb{R}^{N\times N}$ is a diagonal matrix defined as

$$[\mathbf{D}_m^p]_{n,n} = \begin{cases} 1 & \text{if } n \in \mathbb{A}_m^p \\ 0 & \text{otherwise} \end{cases},$$

and $\mathbb{A}_m^p$ is a set of pilot subcarrier indices in mth PTS subgroup.

Step 3. (Channel estimation update and data detection)

First, derotate the received signal using $\{e^{j\hat{\theta}_{k,m}}\}_{m=1}^{M}$:

$$\hat{\mathbf{Y}}_k = \sum_{m=1}^{M} e^{-j\hat{\theta}_{k,m}} \mathbf{D}_m \mathbf{Y}_k$$

where $\hat{\mathbf{Y}}_k$ is a PTS phase-free frequency-domain received vector. Then, the frequency-domain channel estimate $\hat{\mathbf{H}}_k$ is calculated by the similar approach in **Step 1**, and data is detected from $\hat{\mathbf{H}}_k$ and $\hat{\mathbf{Y}}_k$ by an equalizer. One example is using a zero-forcing equalizer:

$$\hat{X}_{k,i} = \hat{Y}_{k,i}/\hat{H}_{k,i} \quad 0 \le i \le N-1$$

Step 4. (Iteration)

If $k < K$, goto *Step 2* and repeat *Steps 2–4*.

In the proposed scenario, if the number of pilots in a subgroup is big by enlarging the PTS subgroup size, the PTS phase factor estimation performance will be improved, and so will the BER. However, a large PTS subgroup size degrades the PAPR reduction performance. Therefore, the PTS subgroup size leads to a trade-off between the PAPR reduction gain and the BER performance. In general, the BER performance is more important than the PAPR, thus the PTS subgroup size should be determined carefully not to degrade the BER performance significantly.

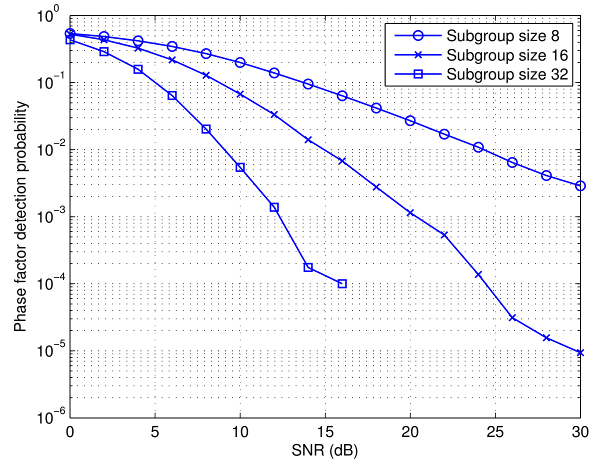

Fig. 3. Phase factor detection error probability under quasi-static fading channels.

B. Complexity comparison at the receiver

The transmitter operation of the proposed method is the same as a conventional PTS method. Hence, we compare the complexities at the receiver between the proposed and conventional methods. Table I summarizes the complexities at the receiver in one subgroup. Here, $N_p \equiv \lfloor \frac{N}{S_p M} \rfloor$, $N_d \equiv \frac{N}{M} - \lfloor \frac{N}{S_p M} \rfloor$, C are the number of pilots in one subgroup, the number of data subcarriers excluding pilots, and the number of data constellations, respectively. S_p is the interval between adjacent pilots. The last column of table I represents the ratio of the complexity when the PTS subgroup size is 16 and the prediction memory length of the proposed method is 2.[2] Note that the prediction is used in the proposed method only, and the method in [9] is not a function of prediction memory length. According to table I, Proposed method has much smaller complexity than the method in [9] at the receiver.

IV. Computer Simulation

In this section, the BER and PAPR reduction performances are empirically evaluated through computer simulations. The system model is as follows. The total number of subcarriers in one OFDM symbol is 128 ($N = 128$), and data subcarrier is modulated by 16QAM. The pilot subcarriers occupy 1/8 of total subcarriers, and every 8th subcarrier is the pilot. The phase factors are chosen in $\theta_{m,k} \in \{0, \pi/2, \pi, 3\pi/2\}$. The channel is assumed to be 16-tap Rayleigh fading channel with equal power profile. In the simulation, one frame is composed

[2]It was observed that the memory length 2 provides a sufficient prediction performance in most cases.

TABLE I
COMPLEXITY COMPARISON

	Proposed scheme	Joint ML in [9]	Complexity ratio in the example $(\frac{\text{Proposed scheme}}{\text{Joint ML in [9]}} \times 100)$
Complex Multiplication	$9N_p + L(\frac{3V^2+3V}{M})$	$3N_d(2C+1)$	3.9%
Complex Addition	$8N_p + L(\frac{3V^2}{M}) - 4$	$4N_d(C+1) - 4$	4.0%

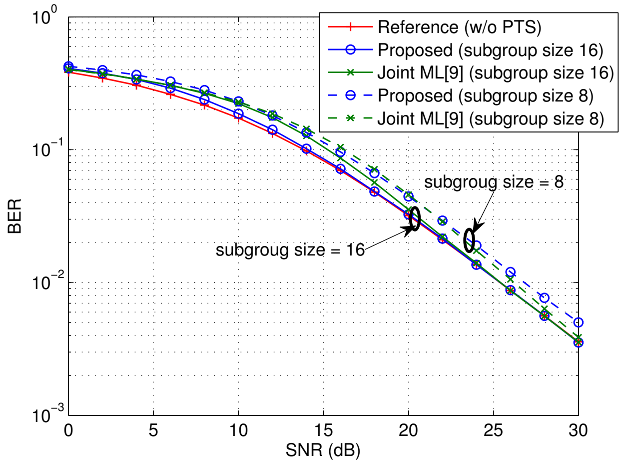

Fig. 4. Comparison of BER performances in quasi-static channels.

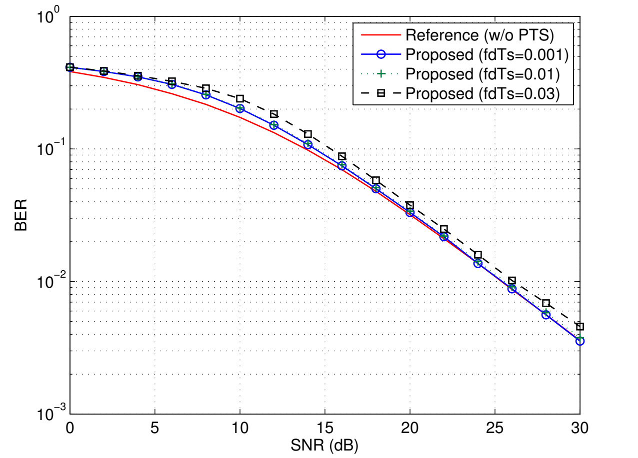

Fig. 5. Comparison of BER performances for various Doppler frequencies.

of 10 OFDM symbols ($K = 10$) and memory length (V) of the predictor is 2.

First, the phase factor detection error probability (PFDEP) is examined. Since one phase factor detection error results in a group of data errors, the PFDEP is important. Fig. 3 shows the simulation result under the quasi-static fading channels. As expected, the PFDEP is improved as the PTS group size increases. Fig. 4 compares the BER performances of the proposed method and the joint ML method in [9] under imperfect channel estimation in quasi-static channels. For the simulation of the method in [9], the LS channel estimator (similar to **Step 1** in Section III) is used. Two subgroup sizes are compared in Fig. 4, and it is observed that subgroup size 16 provides a negligible BER loss for the proposed method in the medium to high SNR range. The joint ML technique in [9] exhibits significant BER loss in the low to medium SNRs. For the PTS subgroup size 8, however, both methods exhibits noticeable BER losses for the entire SNR range of interest. This result indicates that if we choose the subgroup size appropriately, the proposed method outperforms the method in [9] and negligible BER loss while requiring smaller computational complexity at the receiver. Fig. 5 shows BER performances for various normalized Doppler frequency, f_dT_s. As f_dT_s increases, channel prediction performance is degraded and so is the BER performance. However, the degradation is negligible when $f_dT_s \leq 0.01$. Fig. 6 shows the amount of PAPR reduction when the PTS subgroup size is 16. The proposed method can reduce the PAPR by 6.2dB at complementary cumulative

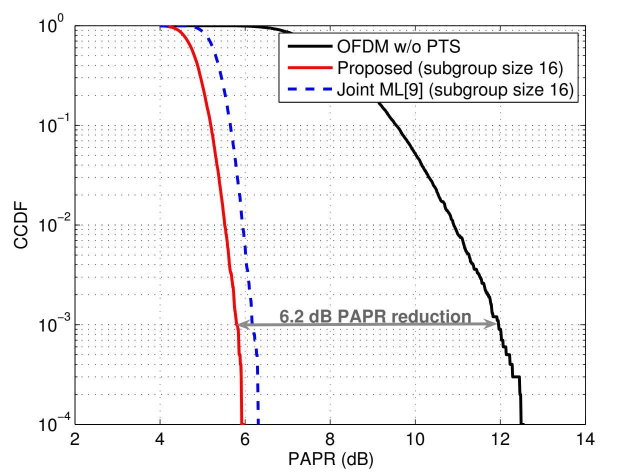

Fig. 6. Comparison of PAPR performances.

density function (CCDF) 10^{-3} whereas the method in [9] exhibits a slightly inferior performance because the latter does not use the pilot subcarriers for PAPR reduction. Note that the proposed method uses both the pilot subcarriers and data subcarriers to reduce the PAPR. These results indicate that the proposed method can perform better than the joint ML method in [9] in imperfect channel estimation scenario.

V. CONCLUSION

A PTS phase estimation method for side information free PTS technique was proposed. In the proposed scheme, the phase factors and channel are jointly estimated from the pilot subcarriers. Through computer simulation, it was verified that the proposed scheme substantially reduces the PAPR with negligible BER loss.

REFERENCES

[1] IEEE 802.16e, *Air interface for fixed and mobile broadband wireless access systems*, Feb. 2006.

[2] IEEE 802.11a, *Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications: High-speed Physical Layer in the 5GHz Band*, Sep. 1999.

[3] 3GPP TR 25.892 V6.0.0, *Feasibility study for orthogonal frequency division multiplexing (OFDM) for UTRAN enhancement*, June. 2004.

[4] P. Van Nee and R. Prasad, *OFDM for Wireless Multimedia Communication*, Artech House Publishers, 2000.

[5] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Commun Magazine.*, vol. 12, pp. 56–65, Apr. 2005.

[6] S. H. Müller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Elect. Lett.*, vol. 33, no. 5, pp. 368–369, Feb. 1997.

[7] Seung Hee Han, Jae Hong Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," *Sig. Lett.*, vol. 11, no. 11, pp. 887–890, Nov. 2004.

[8] Leonard J. Cimini,Jr., Nelson R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *Comm. Lett.*, vol. 4, no. 3, pp. 86–88, Mar. 2000.

[9] A. D. S. Jayalath and C. Tellambura, "SLM and PTS peak-power reduction of OFDM signals without side information," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2006–2013, Sept. 2005.

[10] M. Julia Fernández-Getino García, O. Edfors, and J. M. Páez-Borrallo, "Peak power reduction for OFDM systems with orthogonal pilot sequences," *IEEE Trans. Wireless Commun.*, vol. 5, no. 1, pp. 47–51, Jan. 2006.

[11] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Sig. Proc.*, vol. 49, no. 12, pp. 3065–3073, Dec. 2001.

[12] Dieter Schafhuber and G. Matz, "MMSE and adaptive prediction of time-varying channels for OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 593–602, Mar. 2005.

[13] Ian C. Wong, Antonio Forenza, Robert W. Heath, and Brian L. Evans, "Long range channel prediction for adaptive OFDM systems," *IEEE Asilomar Conf. on Signals, Syst., Comp.*, vol. 1, pp. 732–736, Nov. 2004.

[14] John G. Proakis, *Digital Communications, 4th ed.*, New York: McGraw-Hill, 1995.

[15] David Tse and Pramod Viswanath, *Fundamentals of Wireless Communication*, Cambridge university press, 2005.