Design of Digital Predistorters for Wideband Power Amplifiers in Communication Systems with Dynamic Spectrum Allocation

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***Abstract*— A new predistortion technique for dynamic spectrum allocation systems such as cognitive radio (CR) is proposed. The system model considered in this paper occupies a small band at a time, but the center frequency can be changed in the wide range of frequency. In this scenario, the front-end filter may not eliminate the harmonics of the power amplifier (PA) output. The proposed PD reduces the spectral regrowth of the fundamental signal at the carrier frequency (ω_o) and removes the harmonics ($2\omega_o, 3\omega_o, \cdots$) at the same time. The proposed PD structure is composed of multiple predistorters (PDs) centered at integer multiples of ω_o. The PD at ω_o is for removing spectral regrowth of the fundamental signal, and the others are for harmonic reduction. In the proposed PD structure, parameters of PDs are found jointly. Simulation results show that the spectral regrowth can be reduced by 20dB, and the 2nd and 3rd harmonics can be reduced down to -70dB from the power of the fundamental signal.**

***Index Terms*—Cognitive radio (CR), dynamic spectrum allocation, harmonics, power amplifier (PA), predistortion**

I. Introduction

Because the number of wireless systems and services has been increased rapidly, we are short of frequency resources now. To utilize the frequency resource efficiently, new concepts of radio system such as cognitive radio (CR) systems have been developed [1]. CR is a system that uses a dynamic spectrum allocation based on spectrum sensing to find out spectrum holes. Since CR inherently requires multi-mode and multi-band radio systems, software defined radio (SDR) is a good candidate for implementing of the CR [2]. In SDR systems, the radio is implemented with a common hardware which is reconfigurable by a software.

This paper considers a digital predistortion (PD) problem for CR systems. Digital PD is a digital signal processing technique that linearizes the power amplifier (PA) by introducing the inverse characteristic of the PA at the baseband digital stage [3]–[7]. In general, nonlinear PAs generate spectral regrowth at the carrier frequency (ω_o) and harmonics at the integer multiples of ω_o as shown in Fig. 1(a). In typical wireless communication systems, transmission band is pre-designated and limited to a small band compared with the carrier frequency. Thus, the harmonic signals at the PA output can be effectively removed by a front-end filter between the PA and the Tx antenna. Existing PDs did not consider the harmonics except the spectral regrowth of the fundamental signal. However, the SDR system supporting the CR should have much wider bandwidth to support multi-mode and multi-band signals. The front-end filter should also have a wide bandwidth and the harmonics may not be no longer eliminated by the front-end filter as shown in Fig. 1(b). Those harmonics can interfere with other services so that they should be removed.

We propose a new PD structure to eliminate the spectral regrowth of the fundamental signal and harmonics simultaneously. The PD structure is composed of parallel PDs, each of which is up-converted to one of integer multiples of ω_o. The PD at ω_o is for spectral regrowth reduction for the fundamental signal and the other PDs at $2\omega_o$, $3\omega_o$, $\cdots$ are for harmonic reduction. The parameters of parallel PDs are jointly calculated by solving multiple nonlinear equations. The performance of the proposed PD is examined through computer simulation. According to the results, by using the proposed PD, the spectral regrowth of the fundamental signal can be reduced by 20dB, and the 2nd and 3rd harmonic powers can be reduced down to -70dB from the power of the fundamental signal.

The organization of this paper is as follows. Section II describes the spectral regrowth and harmonics by a nonlinear PA. The PD structure for removing the spectral regrowth and harmonics is introduced in Section III. Section IV shows simulation results and Section V concludes the paper.

II. Nonlinear PA Model

Fig. 2 shows the system model considered in this paper, which is a typical wireless transmitter for the CR. $y(n)$ is a baseband digital signal to be transmitted and modulated by

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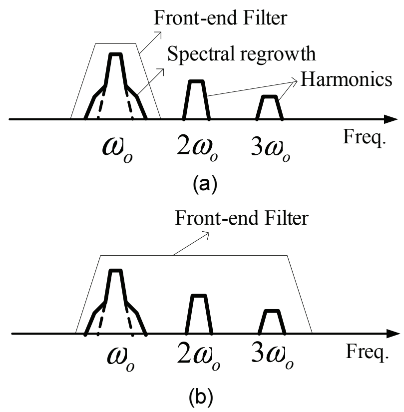

Fig. 1. PA output for (a) single-band system (b) dynamic spectrum allocation system.

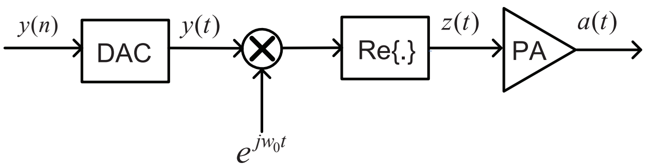

Fig. 2. System model.

a carrier frequency ω_o. The resulting RF (radio frequency) signal is amplified by a power amplifier (PA). The PA is a nonlinear device so that it tends to distorts the signal. This nonlinear distortion causes intermodulation distortion (IMD) and harmonics. As a consequence, the PA output exhibits the spectral regrowth in the fundamental signal at ω_o and harmonic spectrums at integer multiples of ω_o. To linearize the PA, analog or digital techniques have been proposed, and digital predistortion (DPD) is one of the effective techniques. The purpose of conventional DPD techniques is to linearize the PA to mitigate the spectral regrowth of the fundamental signal (ω_o). The existing techniques ignore the harmonics. Although those literatures on DPD do not describe explicitly, they assume the usage of a front-end filter in between the PA and antenna which removes the harmonics. This is effective for single band systems. However, in SDR systems supporting the CR operated in a wide range of frequency band, the front-end filter should also have a wide bandwidth. Consequently, it is possible that the harmonics are no longer removed by the front-end filter. The purpose of this paper is to develop a DPD structure and algorithm to remove the harmonics as well as linearize the fundamental signal. Assuming that the PA is modeled by a 3rd order polynomial, the PA output in Fig. 2 can be represented as

$$a(t) = \alpha_1 z(t) + \alpha_2 z^2(t) + \alpha_3 z^3(t) \qquad (1)$$

where $\{\alpha_i\}$ are real constants characterizing the PA. In (1), $\alpha_1 z(t)$ represents the linear component while $\alpha_2 z^2(t)$ and $\alpha_3 z^3(t)$ are the second and third order nonlinear components, respectively. Substituting $z(t) = \mathrm{Re}\{y(t)e^{j\omega_o t}\}$ in (1) and arranging yield

$$\begin{aligned} a(t) =& \frac{\alpha_2}{2}|y(t)|^2 + \mathrm{Re}\left\{\left(\alpha_1 y(t) + \frac{3\alpha_3}{4} y(t)|y(t)|^2\right) e^{j\omega_o t}\right\} \\ &+ \mathrm{Re}\left\{\frac{\alpha_2}{2} y^2(t) e^{j2\omega_o t}\right\} + \mathrm{Re}\left\{\alpha_3 y^3(t) e^{j3\omega_o t}\right\}. \end{aligned}$$

This indicates that the fundamental signal at ω_o is nonlinearly distorted and the unwanted harmonic signals appear at DC, $2\omega_o$, and $3\omega_o$. Generalizing the expression to higher order PA models, The PA output can be expressed as

$$a(t) = \sum_{k=0}^{K} \mathrm{Re}\left\{G_k(y(t))e^{jk\omega_o t}\right\}$$

where $G_k(y(t))$ is a nonlinear function of $y(t)$ associated with the PA output at frequency $k\omega_o$, and K is the maximum order of the PA nonlinearity. While the conventional DPD methods consider linearization of $G_1(\cdot)$ only, we also consider reduction of harmonics such as $G_2(\cdot)$ and $G_3(\cdot)$. The objective of this paper is development of predistortion structure and algorithm to remove the harmonic signals as well as linearize the fundamental signal at the PA output.

III. Development of The Proposed PD

It is natural to consider incorporation of multiple predistorters at harmonic frequencies to combat the harmonics. The structure for both linearization of the fundamental signal and removal of the harmonics is shown in Fig. 3. $F_1(\cdot)$ is a predistortion function to linearize the fundamental signal and $F_2(\cdot)$ to $F_N(\cdot)$ are for elimination of the harmonics where N is the number of PDs. However, when such multiple-band signals are fed to the PA, the PA output becomes much more complex due to the intermodulation between the multi-band input signals. For example, when $N = 3$ and $K = 3$, the PA output can be represented as

$$\begin{aligned} a(t) =& \mathrm{Re}\{G_0(y_1(t), y_2(t), y_3(t))\} \\ &+ \mathrm{Re}\{G_1(y_1(t), y_2(t), y_3(t))e^{j\omega_o t}\} + \cdots \end{aligned} \qquad (2)$$

where $\{G_k(y_1(t), y_2(t), y_3(t))\}$ are nonlinear functions, summarized in Table I after omitting the time index t. It is observed that the PA output spans to $9\omega_o$ compared to $3\omega_o$ for single band input in Fig. 2 and $\{G_k(y_1(t), y_2(t), y_3(t))\}$ for $k = 0, ..., N \times K$ is composed of the self- and cross- product terms of $y_1(t)e^{j\omega_o t}$, $y_2(t)e^{j2\omega_o t}$, and $y_3(t)e^{j3\omega_o t}$. For example, $[y_1, y_1^* y_2, y_2^* y_3, y_1|y_1|^2, y_1|y_2|^2, y_1|y_3|^2, y_2^2 y_3^*, y_1^{*2} y_3]$ are terms that contribute to ω_o when $K = 3$ and contribute to $G_1(y_1(n), y_2(n), y_3(n))$ in (2). For higher orders of N and K, the $\{G_k(\cdot)\}$ can be easily extended.

Now, assuming $N = 3$, up to third harmonics can be removed. In this scenario, the PA and PD model can be described as a nonlinear 3×3 multiple input multiple output (MIMO) system and 1×3 single input multiple output (SIMO) system, respectively, as in Fig. 3. We define nonlinear functions $F_1(\cdot), F_2(\cdot)$, and $F_3(\cdot)$ where $F_1(\cdot)$ linearizes the fundamental signal at ω_o and $F_2(\cdot)$ and $F_3(\cdot)$ eliminate the second and third harmonics, respectively. To this end, we

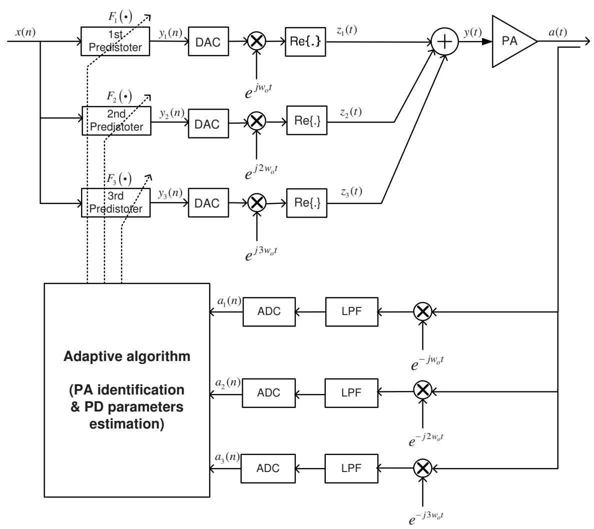

Fig. 3. Proposed PD structure (N=3).

should find digital predistortion functions $F_i(\cdot)$ for $i = 1, 2, 3$ that satisfy the following three nonlinear equations:

$$a_1(n) \triangleq G_1(y_1(n), y_2(n), y_3(n)) = \gamma x(n)$$
$$a_2(n) \triangleq G_2(y_1(n), y_2(n), y_3(n)) = 0$$
$$a_3(n) \triangleq G_3(y_1(n), y_2(n), y_3(n)) = 0,$$

where $y_1(n) = F_1(x(n)), y_2(n) = F_2(x(n)), y_3(n) = F_3(x(n))$ and γ is the desired gain of the PA. Since $\{G_i(\cdot)\}$ are nonlinear functions, solving this problem to get $\{F_i(\cdot)\}$ exactly is very difficult in general. Alternatively, we approximate $\{F_i(\cdot)\}$ as polynomials and solve the following least squares problem:

PD Problem) Assuming $\{G_i(\cdot)\}$ are known, find $\{F_i(\cdot)\}$ minimizing the following cost function

$$\mathcal{E} = \sum_{k=1}^{N=3} ||e_k||^2 = \sum_{k=1}^{N=3} \sum_{n=1}^{M} |e_k(n)|^2$$

where $e_1(n) = \gamma x(n) - a_1(n)$, $e_2(n) = 0 - a_2(n)$, $e_3(n) = 0 - a_3(n)$, and M is the number of observation data. The polynomial model of $\{F_i(\cdot)\}$ can be modeled from the following Observation 1.

Observation 1. Since $F_i(\cdot)$ should contribute to the frequency of $i\omega_o$, $F_i(\cdot)$ can be expressed as follows:

$$F_i(x(n)) = \sum_{l=1}^{L_i} p_{i,l} x(n)^i |x(n)|^{2l} = \mathbf{p}_i^T \mathbf{v}_i \quad \text{for} \quad i = 1, 2, ..., N$$

where $2L_i + 1$ is the maximum polynomial order, $\{p_{i,l}\}$ are the coefficients of i'th PD, $\mathbf{p}_i = [p_{i,1}, ..., p_{i,L_i}]^T$, and $\mathbf{v}_i = [x(n)^i, ..., x(n)^i |x(n)|^{2L_i}]^T$.

Now, the cost function of the PD problem can be rewritten

TABLE I
INTERMODULATION TERMS (K=3).

Function	Expression
$G_0(\cdot)$	$\frac{\alpha_1}{2^2}\left(2\lvert y_1\rvert^2 + 2\lvert y_2\rvert^2 + 2\lvert y_3\rvert^2\right)$ $+\frac{\alpha_3}{2^3}\left(3y_1^2 y_2^* + 3y_1^{*2} y_2 + 6y_1 y_2 y_3^* + 6y_1^* y_2^* y_3\right)$
$G_1(\cdot)$	$\frac{\alpha_1}{2} y_1 + \frac{\alpha_2}{2^2}\left(2y_1^* y_2 + 2y_2^* y_3\right)$ $+\frac{\alpha_3}{2^3}(3y_1\lvert y_1\rvert^2 + 3y_2^2 y_3^* + 3y_1^{*2} y_3$ $+6y_1\lvert y_2\rvert^2 + 6y_1\lvert y_3\rvert^2)$
$G_2(\cdot)$	$\frac{\alpha_1}{2} y_2 + \frac{\alpha_2}{2^2}(y_1^2 + 2y_1^* y_3)$ $+\frac{\alpha_3}{2^3}(3y_2\lvert y_2\rvert^2 + 6y_2\lvert y_1\rvert^2 + 6y_2\lvert y_3\rvert^2 + 6y_1 y_2^* y_3)$
$G_3(\cdot)$	$\frac{\alpha_1}{2} y_3 + \frac{\alpha_2}{2^2} 2y_1 y_2$ $+\frac{\alpha_3}{2^3}(y_1^3 + 3y_1^* y_2^2 + 3y_3\lvert y_3\rvert^2 + 6y_3\lvert y_1\rvert^2 + 6y_3\lvert y_2\rvert^2)$
$G_4(\cdot)$	$\frac{\alpha_2}{2^2}(2y_1 y_3 + y_2^2)$ $+\frac{\alpha_3}{2^3}(3y_1^2 y_2 + 3y_2^* y_3^2 + 6y_1^* y_2 y_3)$
$G_5(\cdot)$	$\frac{\alpha_2}{2^2} 2y_2 y_3 + \frac{\alpha_3}{2^3}(3y_1^2 y_3 + 3y_1 y_2^2 + 3y_1^* y_3^2)$
$G_6(\cdot)$	$\frac{\alpha_2}{2^2} y_3^2 + \frac{\alpha_3}{2^3}(y_2^3 + 6y_1 y_2 y_3)$
$G_7(\cdot)$	$\frac{\alpha_3}{2^3} y_1 y_3^2$
$G_8(\cdot)$	$\frac{\alpha_3}{2^3} y_2 y_3^2$
$G_9(\cdot)$	$\frac{\alpha_3}{2^3} y_3^3$

as follows:

$$\mathcal{E} = \sum_{k=1}^{3} ||e_k||^2 = ||\gamma \mathbf{x} - \mathbf{a}_1(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)||^2 + \sum_{k=2}^{3} ||0 - \mathbf{a}_k(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)||^2$$

where $\mathbf{x} = [x(1), ..., x(M)]^T$ and $\mathbf{a}_k(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = [a_k(1), ..., a_k(M)]^T$ where $a_k(n) = G_k(\mathbf{p}_1^T \mathbf{v}_1, \mathbf{p}_2^T \mathbf{v}_2, \mathbf{p}_3^T \mathbf{v}_3)$, $k = 1, 2, 3$. Let $\mathbf{p} = [\mathbf{p}_1^T, \mathbf{p}_2^T, \mathbf{p}_3^T]^T$ be the augmented vector of $\mathbf{p}_1$,$\mathbf{p}_2$, and $\mathbf{p}_3$. Then, based on the gradient descent method, the adaptive algorithm can be derived as follows [8]:

$$\begin{aligned}\hat{\mathbf{p}}(n+1) =& \hat{\mathbf{p}}(n) - \frac{1}{2}\mu_p \nabla_{\mathbf{p}} \mathcal{E} \\ =& \hat{\mathbf{p}}(n) + \mu_p \left[\mathbf{J}_{11}^T(\gamma\mathbf{x} - \mathbf{a}_1(\mathbf{p})) + \mathbf{J}_{12}^T(\gamma\mathbf{x} - \mathbf{a}_1(\mathbf{p}))^*\right] \\ &+ \mu_p \left[\sum_{k=2}^{3} \left\{\mathbf{J}_{k1}^T \mathbf{a}_k(\mathbf{p}) + \mathbf{J}_{k2}^T \mathbf{a}_k(\mathbf{p})^*\right\}\right] \end{aligned} \quad (3)$$

where μ_p is step size, $\{\mathbf{J}_k\}$ are the $M \times (L_1 + \cdots + L_N)$ Jacobian matrix that are defined as $\mathbf{J}_{k1} = [\nabla_{\mathbf{p}} (\mathbf{a}_k(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)^*)]$ and $\mathbf{J}_{k2} = [\nabla_{\mathbf{p}} (\mathbf{a}_k(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3))]$. When $M = 1$, (3) is reduced to the traditional least mean squares (LMS) algorithm as follows:

$$\begin{aligned}\hat{\mathbf{p}}(n+1) =& \hat{\mathbf{p}}(n) + \mu_p \left[\nabla_{\mathbf{p}} (a_1^*(\mathbf{p}))\, e_1(n) + \nabla_{\mathbf{p}} (a_1(\mathbf{p}))\, e_1^*(n)\right] \\ &+ \mu_p \left[\sum_{k=2}^{3} \left\{\nabla_{\mathbf{p}} (a_k^*(\mathbf{p}))\, a_k(\mathbf{p}) + \nabla_{\mathbf{p}} (a_1(\mathbf{p}))\, a_k^*(\mathbf{p})\right\}\right]\end{aligned}$$

where $e_1(n) = \gamma x(n) - a_1(\mathbf{p})$.

IV. SIMULATION RESULTS

The performance of the proposed PD is examined through computer simulation. Simulation environments are as follows. The transmitted data are modulated by 16 quadrature amplitude modulation (16-QAM) and pulse-shaped by a square root raised cosine filter with roll-off 0.25. The sampling clock of pulse shaping filter (PSF) output is 10 times the symbol rate.

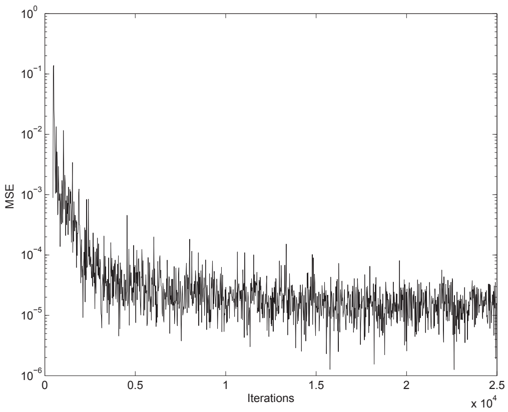

Fig. 4. Learning curve for E[$\mathcal{E}$].

The proposed PD is done with PSF output. We assume $N = 3$ so that up to 3rd harmonic can be removed. For modeling the analog up/down-converter and analog PA in digital domain, we further interpolate the PD output by a factor of 20 and the analog parts are modeled in the digital domain. The carrier frequency is $\omega_o = 2\pi \times 10F_s$ where F_s is the symbol rate. The PA model is $a(n) = z(n) - 0.8z^2(n) + 0.7z^3(n)$. $[1, 0, 0, \cdots, 0]^T$ are used for the initial condition for $\{\mathbf{p}_i\}$, and the step size is 1 for μ_p. Fig. 4 shows the learning curve for the mean square errors (MSE), E[$\mathcal{E}$]. The learning curves are obtained by averaging 50 independent trials. The adaptive algorithms converge in 5,000 iterations, and steady state MSE is around 10^{-5}.

Fig. 5 shows the PA output after all PDs are found. Without PD, the harmonics are noticeable. 2nd and 3rd harmonics are 30dB and 50dB, respectively, below the fundamental spectrum. However, by applying the proposed PD, the harmonics are significantly reduced and the harmonic powers are 70dB lower than that of the fundamental signal. Figs. 6 shows the detailed spectrum around ω_o, $2\omega_o$, and $3\omega_o$, respectively. It is confirmed that the spectral regrowth for the fundamental signal as well as 2nd and 3rd harmonics are reduced considerably. These results indicate that the proposed method is very effective for linearizing the fundamental signal and eliminating the harmonics at the same time.

V. Conclusion

A new predistortion structure and algorithm for dynamic spectrum allocation systems were proposed. The proposed PD technique reduces the spectral regrowth and removes harmonics at the same time based on multi-band PD structure. Computer simulation showed that the proposed method can reduce the spectral regrowth of the fundamental signal and harmonics significantly.

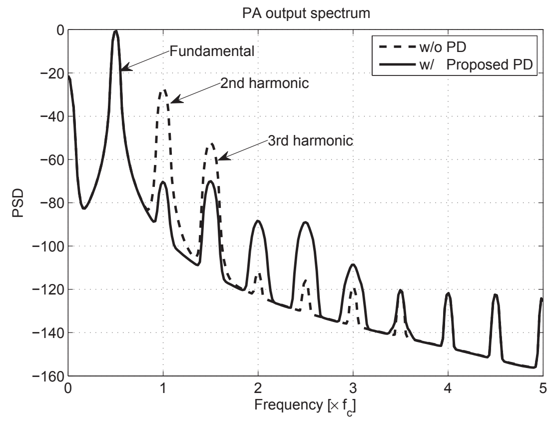

Fig. 5. Power spectral density at PA output.

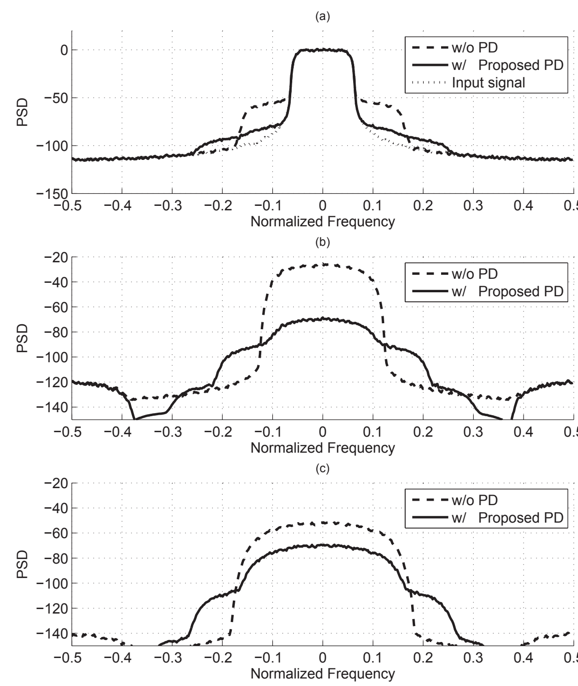

Fig. 6. Power spectral density at PA output. (a) zoom in around ω_o, (b) zoom in around $2\omega_o$, (c) zoom in around $3\omega_o$

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