Asymmetric Complex Signaling for Full-Duplex Decode-and-Forward Relay Channels

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Abstract—**In this paper, it is proposed to use asymmetric complex signaling in full-duplex decode-and-forward single antenna relay channels in order to eliminate the self-interference signal and increase the throughput. Specifically, the relay system considered in this paper is as follows: the source has a transmit weight; the relay has an Rx processing whose operation is an inner product between the received signal and a weight; the relay has a transmit weight as well; the destination has a receive weight. The objective is to find the optimum weights to increase the smaller SNR between source-relay and relay-destination under the constraint of perfect self-interference nulling. We convert the complex variable into a 2-dimensional real vector. As a consequence, the problem becomes a joint vector optimization problem. The simulation result shows that the proposed signalling achieves higher rate than the conventional full-duplex relay.**

Keywords—**Full-duplex relay, asymmetric complex signaling**

I. Introduction

To increase coverage area and throughput by fully utilizing the time and frequency resources, the use of full-duplex relay is recommended instead of the half-duplex relay in wireless communications. However, the relay should eliminate the self-interference (or echo) signals to avoid oscillation or saturation of the relay and the detection failure of the desired signals [1]. To tackle this problem, it was proposed to use additional antennas at transmit or at receive side of the relay [2–4], which increases the cost and size of the relay.

In this paper, we consider single antenna full-duplex relays. Assuming that the transmit signals are real signals (not complex signals) and the signal processing is done in complex domain (I/Q channels), an asymmetric signaling method is considered for both self-interference cancellation and throughput maximization.[1] Specifically, we adopt the system model similar to the one in [3], and convert the problem into a joint real vector optimization problem. the relay system model considered in this paper is as follows: the source has a transmit weight; the relay has an Rx processing whose operation is an inner product between the received signal and a weight; the relay has a transmit weight as well; the destination has a receive weight. The objective is to find the optimum weights to increase the smaller SNR between source-relay and relay-destination under the constraint of perfect self-interference nulling. The main contribution of this paper is that we show the possibility of both self-interference elimination and throughput maximization using single antenna through the asymmetric complex signaling in full-duplex relay systems. The simulation result demonstrates that the proposed scheme achieves higher average rate than the usual symmetric Gaussian signaling scheme for all SNR region.

[1]The asymmetric signaling approach is widely studied in the interference channel (refer to [5] and related papers).

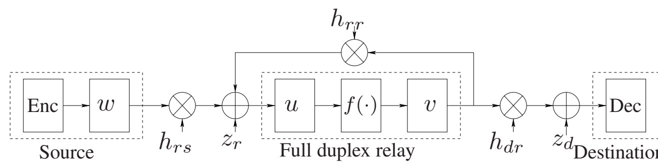

Fig. 1. Block diagram of the considering two-hop decode-and-forward relay system with the self-feedback cancellation.

Notations: In this paper, a complex number $a \in \mathbb{C}$ is written *interchangeably* with the two dimensional vector $\mathbf{a} = [\text{re}(a), \text{imag}(a)]^T \in \mathbb{R}^{2\times 1}$, where re$(a)$ (imag(a)) is the real part (the imaginary part) of a. Let $a = |a|e^{j\phi_a} \in \mathbb{C}$, where $|a|$ is the moduli of a and ϕ_a is the argument of a. $\mathbf{\Theta}_a$ is defined as

$$\mathbf{\Theta}_a := \begin{bmatrix} \cos\phi_a & -\sin\phi_a \\ \sin\phi_a & \cos\phi_a \end{bmatrix}.$$

II. System Model

Consider a relay system of three nodes, all with single antenna. Due to the absence of direct link, messages $(m_1, m_2, \cdots)$ of rate R are transferred via the relay from the source to the destination. Each message, $m_k \in \{1, \cdots, 2^{nR}\}$, $k \in \mathbb{N}$, occupies n channel uses. It is assumed that the nodes share all channel coefficients accurately. The source transmits the Gaussian coded symbols of average power P_s based on the kth source message m_k at time t. The transmitted signal by the source can be written as

$$\mathbf{x}_s[t] = \sqrt{P_s}\mathbf{w}\widetilde{x}_s[t] \in \mathbb{R}^{2\times 1}, \;\; t \in \{1, \cdots, n\}$$

where $\widetilde{x}_s[t] \sim \mathcal{N}(0,1)$ and is the real-valued transmit signal, $w \in \mathbb{C}$ is the unit-moduli precoding weight of the source, and n represents the time length of the signal. The relay receives the signal and performs a receive processing, which is inner product between the received vector and a weight $u \in \mathbb{C}$. Then, the resulting signals are decoded and re-encoded to form the transmit symbols. Finally, the transmit is multiplied with a weight $v \in \mathbb{C}$.

When the relay receives all n symbols, the relay starts to decode the message m_k after cancelling the self-interference. Since the decoding delay is n (one code length), at time t, the relay transmits the Gaussian coded symbol of average power P_r generated from m_{k-1} with the unit-moduli transmit weight v.

$$\mathbf{x}_r[t] = \sqrt{P_r}\mathbf{v}\widetilde{x}_r[t] \in \mathbb{R}^{2\times 1}, \;\; t \in \{1, \cdots, n\},$$

where $\widetilde{x}_r[t] \sim \mathcal{N}(0,1)$ and is real. The received signal of the relay at time t is

$$\mathbf{y}_r[t] = |h_{rs}|\mathbf{\Theta}_{h_{rs}}\sqrt{P_s}\mathbf{w}\widetilde{x}_s[t] + |h_{rr}|\mathbf{\Theta}_{h_{rr}}\mathbf{v}\widetilde{x}_r[t] + \mathbf{z}_r[t] \in \mathbb{R}^{2\times 1},$$

where $h_{rs} \in \mathbb{C}$ is the channel between the source and the relay, $h_{rr} \in \mathbb{C}$ is the feedback channel of the relay, and $z_r \in \mathbb{C}$ is $\mathcal{CN}(0, N_0)$. The relay decoder admits the received signal after the receive processing as

$$\mathbf{u}^T\mathbf{y}_r[t] = \sqrt{P_s}|h_{rs}|\mathbf{u}^T\mathbf{\Theta}_{rs}\mathbf{w}\widetilde{x}_s[t] + \mathbf{u}^T\mathbf{z}_r[t] \in \mathbb{R},$$

assuming that the self-interference signal $\sqrt{P_r}|h_{rr}|\mathbf{u}^T\mathbf{\Theta}_{rr}\mathbf{v}\widetilde{x}_r[t]$ is eliminated perfectly. As seen in the equation, the receive processing is an inner product between the received vector and a real weight vector $\mathbf{u}$. Obviously for the self-interference signal elimination, it is required to satisfy

$$\mathbf{u}^T\mathbf{\Theta}_{rr}\mathbf{v} = 0.$$

The received signal at the destination at time t is

$$\mathbf{y}_d[t] = \sqrt{P_r}|h_{dr}|\mathbf{\Theta}_{dr}\mathbf{v}\widetilde{x}_r[t] + \mathbf{z}_d[t] \in \mathbb{R}^{2\times 1},$$

where $z_d \in \mathbb{C}$ is $\mathcal{CN}(0, N_0)$. The destination detects the message m_{k-1} after receiving n code symbols. The performance metric is the maximum rate R of the reliable communication for a sufficiently large n. This metric is determined by the minimum rate of each hop, which is maximized by the transmit weights w and v, and by the receive weight u.

III. Max-Min Optimization

The problem can be formulated as the following constrained max-min optimization problem:

$$\max_{\mathbf{v},\mathbf{u},\mathbf{w}} \ \min \ \{\text{SNR}_{rs}, \text{SNR}_{dr}\}, \quad \text{s.t. } \mathbf{u}^T\mathbf{\Theta}_{rr}\mathbf{v} = 0 \tag{0}$$

where $\text{SNR}_{rs} = P_s|h_{rs}|^2 \cdot \mathbf{u}^T\mathbf{\Theta}_{rs}\mathbf{w}\mathbf{w}^T\mathbf{\Theta}_{rs}^T\mathbf{u}/(N_0/2)$ and $\text{SNR}_{dr} = P_r|h_{dr}|^2 \cdot \mathbf{v}^T\mathbf{\Theta}_{dr}^T\mathbf{\Theta}_{dr}\mathbf{v}/(N_0/2)$. The constraints are the unit-moduli condition $\mathbf{v}^T\mathbf{v} = \mathbf{u}^T\mathbf{u} = \mathbf{w}^T\mathbf{w} = 1$ and the perfect self-loop elimination $\mathbf{u}^T\mathbf{\Theta}_{rr}\mathbf{v} = 0$. The achievable rate is defined as $\frac{1}{2}\log_2\left(1 + \max_{\mathbf{v},\mathbf{u},\mathbf{w}} \ \min \ \{\text{SNR}_{rs}, \text{SNR}_{dr}\}\right)$.

We determine w first and solve for u and v jointly. Set $\mathbf{w}$ by the dominant right singular vector of $\mathbf{\Theta}_{rs}$.[2] Since $\mathbf{\Theta}_{rs}$ is already orthogonal, the singular values coincide and we can choose any right singular vector. Let SVD of $\mathbf{\Theta}_{rs}$ be $\mathbf{U}_{rs}\mathbf{D}_{rs}\mathbf{V}_{rs}^T$ and let $\mathbf{w} = \mathbf{V}(:,1)$. Then, $\overline{\boldsymbol{\theta}}_{rs} =: \mathbf{\Theta}_{rs}\mathbf{w} = \mathbf{U}_{rs}\mathbf{D}_{rs}\mathbf{V}_{rs}^T\mathbf{V}(:,1) \in \mathbb{R}^{2\times 1}$. We simplify the self-loop elimination condition. Let the half rotation of $\mathbf{\Theta}_{rr}$ be $\overline{\mathbf{\Theta}}_{rr} \in \mathbb{R}^{2\times 2}$, which is invertible, *i.e.*, $\mathbf{\Theta}_{rr} = \overline{\mathbf{\Theta}}_{rr}\overline{\mathbf{\Theta}}_{rr}$, and let $\overline{\mathbf{u}}^T =: \mathbf{u}^T\overline{\mathbf{\Theta}}_{rr} \in \mathbb{R}^{2\times 1}$ and $\overline{\mathbf{v}} =: \overline{\mathbf{\Theta}}_{rr}\mathbf{v} \in \mathbb{R}^{2\times 1}$, where each vector has the unit length. The constraints are expressed in terms of $\overline{\mathbf{u}}$ and $\overline{\mathbf{v}}$,

$$\overline{\mathbf{u}}^T\overline{\mathbf{v}} = 0, \ \text{and} \ \ ||\overline{\mathbf{u}}||_2 = ||\overline{\mathbf{v}}||_2 = 1. \tag{1}$$

In order to convert the objective function into a function of $\overline{\mathbf{u}}$ and $\overline{\mathbf{v}}$, we introduce name several variables for the compact representation. The SNR of the first hop is $\text{SNR}_{rs} = \left|\overline{\mathbf{u}}^T\boldsymbol{\psi}_1\right|^2$, where $\boldsymbol{\psi}_1 = \sqrt{P_s}|h_{rs}|\left(\overline{\mathbf{\Theta}}_{rr}\right)^{-T}\overline{\boldsymbol{\theta}}_{rs}/(\sqrt{N_0/2}) \in \mathbb{R}^{2\times 1}$. For the second hop SNR, $\text{SNR}_{dr} = ||\mathbf{\Psi}_2\overline{\mathbf{v}}||_2^2$, where $\mathbf{\Psi}_2 = \sqrt{P_r}|h_{dr}|\mathbf{\Theta}_{dr}\left(\overline{\mathbf{\Theta}}_{rr}\right)^{-1}/(\sqrt{N_0/2})$. Since the optimization variables are unit real vector of dimension 2, let us define $x \in [0, 2\pi] \cap \mathbb{R}$ to yield

$$\overline{\mathbf{v}} = \begin{pmatrix} \cos x \\ \sin x \end{pmatrix}, \ \text{and} \ \ \overline{\mathbf{u}} = s\begin{pmatrix} -\sin x \\ \cos x \end{pmatrix}, \tag{2}$$

[2]The single antenna complex baseband equivalent channel is interpreted as the two antenna real baseband model. In this case, we decompose the effective matrix channel by the singular value decomposition (SVD). The $\mathbf{w}$ plays the same role of precoder that is the dominant singular vector.

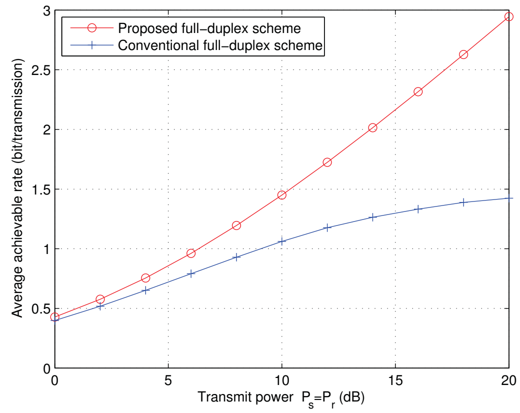

Fig. 2. Average achievable rate of the proposed and the conventional scheme with i.i.d. $\mathcal{CN}(0,1)$ channel and $P_s = P_r$. Averaged 100 times.

where $s \in \{1, -1\}$. Note that this parameterization satisfies (1). Plugging (2) into (0) results in the following simplified optimization problem; let $g_1(x) = \psi_{11}\sin^2 x - s(\psi_{12} + \psi_{12})\cos x \sin x + \psi_{22}\cos^2 x$ and let $g_2(x) = \Psi_{11}\cos^2 x + (\Psi_{12} + \Psi_{12})\cos x \sin x + \Psi_{22}\sin^2 x$. Then, $\max_{s\in\{-1,1\}, 0\le x\le 2\pi} \min\{g_1(x), g_2(x)\}$, where $\{\psi_{ij}\}$ ($\{\Psi_{ij}\}$) is the (i,j)th element of $\boldsymbol{\psi}_1\boldsymbol{\psi}_1^T$ ($\mathbf{\Psi}_2^T\mathbf{\Psi}_2$). The problem can be solved by the line-search with given s. [3]

IV. Simulation Results

We compare the proposed asymmetric signaling scheme with the self-interference elimination with the conventional symmetric signaling scheme without self-interference elimination under the Rayleigh fading channel, whose distributions are i.i.d. $\mathcal{CN}(0,1)$. The conventional full-duplex scheme has the following achievable rate, $\log_2\left(1 + \min\left\{\frac{|h_{rs}|^2 P_s}{|h_{rr}|^2 P_r + N_0}, \frac{|h_{dr}|^2 P_r}{N_0}\right\}\right)$. Fig. 2 indicates that, for all SNR regime, the proposed scheme achieves higher average rate than the conventional scheme. Since the proposed scheme ignores one real dimension, the proposed scheme achieves the one degree-of-freedom(d.o.f), whereas the conventional full-duplex scheme loses d.o.f.

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[3]Although the objective function may have numerous optimal points because of its periodic nature, the optimal values are identical. In fact, we can further simplify the equations be the form of $c + d\sin(2x + \phi)$, a single translated sinusoid, where c, d and ϕ are appropriate constants by applying a series of trigonometric identities.