

Formulas for McClellan Transform Parameters in Designing 2-D Zero-Phase FIR Fan Filters

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Abstract—New formulas for McClellan transform parameters for the design of 2-D zero-phase FIR fan filters are optimally derived under the integral squared error (ISE) criterion. By imposing the constraint that $F(0,0) = \cos w_c$, where $F(\cdot)$ is the McClellan transform and w_c is the cutoff frequency of the one-dimensional (1-D) prototype filter, the ISE is directly minimized without modifying it and, as a consequence, closed-form formulas for the McClellan transform parameters are obtained. It is shown that these formulas lead to a very efficient design for 2-D zero-phase FIR fan filters.

I. INTRODUCTION

THE MCCLELLAN transform [1] has been recognized as a powerful tool in designing two-dimensional (2-D) zero-phase FIR filters [2]–[6]. This transform, which maps the one-dimensional (1-D) frequency points of a 1-D prototype filter onto 2-D frequency contours, is easy to perform and leads to a 2-D filter with an efficient structure. The parameters of this transform are so determined that a 1-D cutoff frequency is mapped onto a contour that best approximates the desired 2-D cutoff contour.

The 2-D zero-phase FIR fan filters were designed using the McClellan transform in [4] and [5]. Simple formulas for the transform parameters were derived by approximating an optimality criterion [4], and in [5] the parameters were determined by applying the eigenfilter method. It has been shown that the eigenfilter approach results in a cutoff contour having smaller absolute deviation from the ideal as compared to the approximation approach. The former, however, requires some numerical search and is computationally more expensive than the latter.

In this letter, we present a new approach to the McClellan transform parameters for the design of 2-D zero-phase FIR fan filters. In particular, new closed-form formulas for the transform parameters are derived. It will be shown that the proposed method is as simple as the approximation method in [4], and that its performance is comparable to that of the eigenfilter approach in [5].

II. THE MCCLELLAN TRANSFORM

Consider a 1-D zero-phase FIR filter with frequency response

$$H(w) = \sum_{n=0}^N a(n)T_n(\cos w) \quad (1)$$

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where $T_n(\cdot)$ is the n -th Chebyshev polynomial. The 2-D frequency response $H(w_1, w_2)$ is obtained by substituting $F(w_1, w_2)$ for $\cos w$. Specifically

$$H(w_1, w_2) = H(w)|_{\cos w = F(w_1, w_2)} \\ = \sum_{n=0}^N a(n)T_n(F(w_1, w_2)). \quad (2)$$

Here

$$F(w_1, w_2) = t_{00} + t_{10} \cos w_1 + t_{01} \cos w_2 \\ + t_{11} \cos w_1 \cos w_2 \quad (3)$$

in the original McClellan transform [1]. Since $|\cos w| \leq 1$, $F(w_1, w_2)$ should satisfy $|F(w_1, w_2)| \leq 1$. The transform $F(w_1, w_2)$ in (3), which is quadrantly symmetric, has been used for the design of 2-D zero-phase fan filters, which also have quadrantal symmetry. If we map 1-D frequency points $w = 0$ and $w = \pi$, respectively, onto 2-D points $(0, \pi)$ and $(\pi, 0)$ as in [4] and [5], then the parameters of $F(w_1, w_2)$ are related as $t_{00} = t_{11}$ and $t_{10} = 1 + t_{01}$. Now $F(w_1, w_2)$ is rewritten as

$$F(w_1, w_2) = t_{01}(\cos w_1 + \cos w_2) \\ + t_{11}(1 + \cos w_1 \cos w_2) + \cos w_1. \quad (4)$$

It was shown in [4] that this $F(w_1, w_2)$ satisfies the inequality $|F(w_1, w_2)| \leq 1$ if and only if

$$|t_{11}| \leq \min\{1 + t_{01}, -t_{01}\}. \quad (5)$$

Note that this inequality holds only for $-1 \leq t_{01} \leq 0$. When (5) is satisfied, $|t_{11}| \leq 0.5$. In the following section, we shall derive simple formulas for fast calculation of t_{01} and t_{11} .

III. FORMULAS FOR THE MCCLELLAN TRANSFORM PARAMETERS

Fig. 1 shows the ideal specifications of a 2-D fan filter. Let $\theta \in (0, \pi/2)$ be the angle between the cutoff frequency line and the w_1 -axis. The cutoff line is expressed as $w_2 = r w_1$ where $r = \tan \theta$. For a given θ , our goal is to find the values of t_{01} and t_{11} in (4) and the 1-D cutoff frequency w_c such that the resulting contour corresponding to w_c best approximates the cutoff frequency line. We define a deviation function as

$$D(w_1, r w_1, w_c) = F(w_1, r w_1) - \cos w_c. \quad (6)$$

Since it is impossible to find $F(w_1, r w_1)$ and w_c that make $D(w_1, r w_1, w_c) = 0$ for all $w_1 \in [0, \pi]$, the following minimization problem is formulated: Find t_{01} , t_{11} and w_c minimizing the integral squared error (ISE)

$$J = \int_0^u D^2(w_1, r w_1, w_c) dw_1 \quad (7)$$

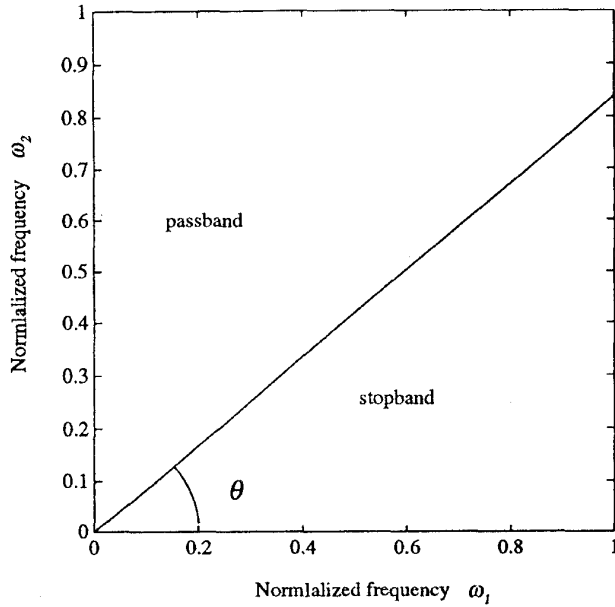
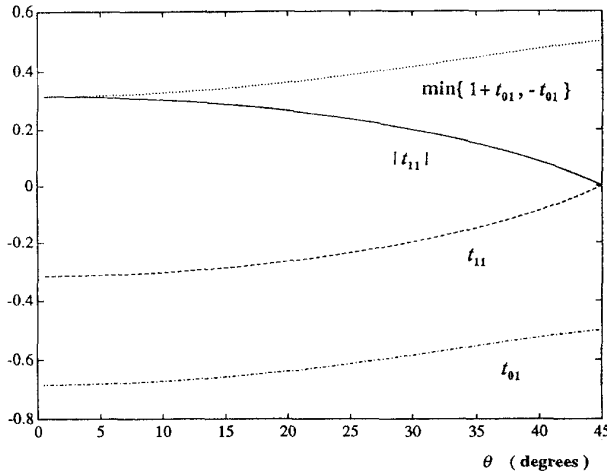


Fig. 1. Specifications of an ideal fan filter within the first quadrant.

Fig. 2. The values of t_{01} and t_{11} obtained using derived formulas.

where $u = \pi$ for $\theta \in (0, \pi/4]$ and $u = \pi/r$ for $\theta \in (\pi/4, \pi/2)$. As shown in [4], it is sufficient to consider only the case where $\theta \in (0, \pi/4]$. This is true because for $\theta \in (\pi/4, \pi/2)$, the parameters are given by $t_{11} = -\hat{t}_{11}$ and $t_{01} = -(1 + \hat{t}_{01})$ where \hat{t}_{01} and \hat{t}_{11} are those computed for $\hat{\theta} = \pi/2 - \theta$. Note that if \hat{t}_{01} and \hat{t}_{11} satisfy the condition in (5), so do t_{01} and t_{11} . The minimization problem in (7) was previously considered in [4] and [5], but they modified the problem either by approximating the deviation function [4] or by employing a modified deviation function [5]. In our work, we minimize J in (7) without any modification except that a constraint is imposed on the minimization. In particular, we impose

$$F(0,0) = \cos w_c. \quad (8)$$

This constraint is justified by the fact that any ideal fan filter cutoff frequency line passes through the origin. Under this

TABLE I
MCCLELLAN TRANSFORM PARAMETERS AND THE
CORRESPONDING NISE FOR $\theta = 5, 10, \dots, 45$ DEGREES

θ	Proposed		NISE (J/π)		
	t_{01}	t_{11}	Proposed	[5]	[4]
5	-0.6807778	-0.3131697	2.0263×10^{-7}	2.1032×10^{-7}	6.5695×10^{-7}
10	-0.6719625	-0.3038498	3.1494×10^{-6}	3.1499×10^{-6}	1.0081×10^{-5}
15	-0.6574929	-0.2881636	1.5122×10^{-5}	1.5125×10^{-5}	4.7360×10^{-5}
20	-0.6377381	-0.2658235	4.3885×10^{-5}	4.3896×10^{-5}	1.3313×10^{-4}
25	-0.6132962	-0.2362771	9.3627×10^{-5}	9.3649×10^{-5}	2.7190×10^{-4}
30	-0.5851066	-0.1984593	1.5573×10^{-4}	1.5575×10^{-4}	4.2653×10^{-4}
35	-0.5546773	-0.1502503	1.9405×10^{-4}	1.9409×10^{-4}	4.9107×10^{-4}
40	-0.524608	-0.0872481	1.3621×10^{-4}	1.3623×10^{-4}	3.0876×10^{-4}
45	-0.5	0.0	0	0	0

constraint, $\cos w_c$ in (6) is replaced by $F(0,0) = t_{00} + t_{10} + t_{01} + t_{11}$ and J is rewritten as

$$J = \int_0^\pi [1 - \cos w_1 - t_{01}(\cos w_1 + \cos rw_1) - t_{11}(\cos w_1 \cos rw_1 - 1)]^2 dw_1. \quad (9)$$

Using the matrix-vector notation

$$J = \mathbf{T}^t \mathbf{C} \mathbf{T} - 2\mathbf{T}^t \mathbf{D} + s \quad (10)$$

where t means transposition, $\mathbf{T} = [t_{01}, t_{11}]^t$, \mathbf{C} is a 2×2 symmetric matrix whose elements are denoted by c_{ij} with $c_{11} = 5\pi + [1/(1-r) - 1/(1+r) - 4/r] \sin r\pi + (1/(4r)) \sin 2r\pi$, $c_{22} = 5\pi/4 + [1/(1+r) - 1/(1-r)] \sin r\pi + [1/(16(1+r)) - 1/(16(1-r)) + 1/(8r)] \sin 2r\pi$, $c_{12} = c_{21} = 2\pi + [1/(4(2+r)) - 1/(4(2-r)) + 1/(1+r) - 1/(1-r) - 1/(2r)] \sin r\pi + [1/(4(1-2r)) - 1/(4(1+2r))] \sin 2r\pi$, $\mathbf{D} = [d_1, d_2]^t$ with $d_1 = -5\pi/2 + [1/(2(1+r)) - 1/(2(1-r)) + 1/r] \sin r\pi$, $d_2 = -\pi + [1/(4(2-r)) - 1/(4(2+r)) + 1/(2(1-r)) - 1/(2(1+r)) - 1/(2r)] \sin r\pi$, and $s = 3\pi/2$. Since J is a quadratic function of \mathbf{T} , its minimum occurs at $\mathbf{T} = \mathbf{C}^{-1}\mathbf{D}$. This result leads to the following closed-form formulas for the parameters:

$$t_{01} = \frac{c_{22}d_1 - c_{12}d_2}{c_{11}c_{22} - c_{12}^2}$$

$$t_{11} = \frac{-c_{12}d_1 + c_{11}d_2}{c_{11}c_{22} - c_{12}^2}. \quad (11)$$

From (8)

$$w_c = \cos^{-1}(t_{00} + t_{10} + t_{01} + t_{11}). \quad (12)$$

Using these formulas, we evaluate t_{01} and t_{11} for $\theta = k/2$ degree, $k = 1, 2, \dots, 90$. The results are shown in Fig. 2. This figure also shows $|t_{11}|$ and $\min\{1 + t_{01}, -t_{01}\}$. Note that $|t_{11}| \leq \min\{1 + t_{01}, -t_{01}\}$ for all θ , as desired. Table I lists t_{01}, t_{11} for $\theta = 5, 10, \dots, 45$ degrees, and the corresponding normalized integral squared error (NISE) J/π . For comparison, NISE values corresponding to the parameter values obtained in [4] and [5] are also shown. It is seen that the derived formulas result in smaller NISE than the others. However, the difference between our NISE values and those obtained from [5] is almost negligible. In fact, t_{01} and t_{11}

values in Table I are almost identical to those in [5]. In summary, the proposed method outperforms the approximation method in [4]; it performs like the eigenfilter approach but requires much less computation.

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