

# Design of Efficient FIR Filters with Cyclotomic Polynomial Prefilters Using Mixed Integer Linear Programming

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**Abstract**—The cyclotomic polynomial (CP) prefilter design problem is formulated as an optimization problem with linear objective functions by applying logarithms to the transfer function of the CP prefilter. This problem is then solved by mixed integer linear programming (MILP). Design examples demonstrate that this method leads to more efficient cascaded finite impulse response (FIR) prefilter-equalizers than existing methods.

## I. INTRODUCTION

ONE approach to the design of efficient finite impulse response (FIR) filters requiring fewer arithmetic operations than conventional ones is based on a cascade structure composed of a prefilter, which is often multiplierless, followed by an FIR equalizer [1]–[4]. The prefilter provides reasonable stopband attenuation and the equalizer makes the overall filter meet passband and stopband specifications. Most prefilters introduced so far are based on the use of the recursive running sum (RRS). The equalizer is usually designed via a modified Parks–McClellan algorithm.

In a recent paper [4], Hartnett and Boudreaux-Bartels proposed the use of cyclotomic polynomials (CP's) [5] to form multiplierless prefilters. Their prefilter consists of cascaded subfilters whose system functions are represented as CP's. This class of CP prefilters includes the RRS as a special case. It has been shown that the CP prefilter-equalizer method performs better and can be applied to a wider range of filter types as compared with other prefilter-equalizer methods. The CP prefilter design method, however, is *ad hoc* and is not able to design the “best” prefilter for the specifications at hand. The objective of this letter is to develop an optimal procedure for designing CP prefilters with minimal complexity.

In what follows, we first show that the CP prefilter design problem can be formulated as an optimization problem with linear objective functions by applying logarithms to the transfer function of the CP prefilter. The design problem is then solved by mixed integer linear programming (MILP) [6]. Through design examples, we demonstrate that the proposed method leads to more efficient cascaded FIR prefilter-equalizers as compared with existing methods.

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## II. THE CP PREFILTER DESIGN METHOD

In this section, we will briefly review the CP prefilter-equalizer design method proposed in [4], and then describe a new technique for designing prefilters with minimal complexity. Both the prefilters and equalizers considered in this section are linear-phase FIR filters.

### A. Review of the CP Prefilter-Equalizer Design

The system function of the prefilter  $P(z)$  is represented as

$$P(z) = \prod_{q=1}^Q F_q(z)^{m_q} \quad (1)$$

where  $F_q(z)$  are CP's in  $z^{-1}$ ,  $m_q$  are nonnegative integers and  $Q$  is a positive integer. In order to obtain multiplierless CP prefilters, only the first 104 CP's that contain only the coefficients  $\{0, 1, -1\}$  are considered. The CP prefilter-equalizer is designed as follows.

- Step 1: Choose the maximum length of the prefilter and the equalizer.
- Step 2: Determine the set of eligible CP's whose zeros locate within the stopband or within some intrusion into the transient bands. In (1),  $F_q(z)$  denotes the eligible CP's and  $Q$  is the number of such CP's.
- Step 3: Determine the order  $m_q$  of each eligible CP selected in Step 2. The orders are so determined that  $P(z)$  meets the prefilter specifications of the maximum length, passband deviation and stopband attenuation. If such  $m_q$  are found, conclude the prefilter design and proceed to the next step. Otherwise, this step is repeated with an increased value for the maximum length of the prefilter.
- Step 4: Design the equalizer using either the modified Parks–McClellan algorithm or the subset selection method.

In [4], an iterative algorithm for finding the order  $m_q$  in Step 3 is provided. However, this algorithm is *ad hoc* and, in general, cannot provide a CP prefilter with minimum complexity. In implementing the prefilter, some prefilter subsections are combined into one efficient multiplierless structure and some are implemented recursively. This approach provides savings in the number of additions at the expense of delays. In an effort to exploit this fact in a systematic manner, we shall obtain efficient structures of eligible CP's by either combining or

recursively implementing them, and then add these structures to the set of eligible CP's. Now the number of eligible CP's is denoted as  $Q'$ ,  $Q' > Q$  and  $P(z) = \prod_{q=1}^{Q'} F_q(z)^{m_q}$ .

### B. The Proposed Design Method

Let  $A_p(\omega)$  and  $A_{F_q}(\omega)$ , respectively, denote the real parts of the frequency response of the prefilter  $P(z)$  and that of an eligible CP  $F_q(z)$ . Then  $A_p(\omega) = \prod_{q=1}^{Q'} A_{F_q}(\omega)^{m_q}$ . The CP prefilter with minimal complexity may be designed by solving the following optimization problem.

$$\begin{aligned} & \text{Minimize} && \sum_{q=1}^{Q'} m_q (a_q + c \cdot d_q) && \text{(complexity measure)} \\ & \text{Subject to} && |1 - s \cdot A_p(\omega)| \leq r_p && \text{(in passbands)} \\ & && |s \cdot A_p(\omega)| \leq r_s && \text{(in stopbands)} \end{aligned} \quad (2)$$

where  $a_q$  and  $d_q$  are the number of adders and delays required in implementing the  $q$ th eligible polynomial, respectively,  $c$  is a constant that is determined depending on the complexity of the adder and the delay,  $0 < c < 1$ ,  $s$  is a positive scale factor that keeps  $s \cdot A_p(\omega) \leq 1$ , and  $r_p$  and  $r_s$  are the ripples of passband and stopband, respectively. Our objective is to find  $m_q$  and  $s$ , assuming the others are given. It is obvious that this design problem cannot be solved by using conventional filter design methods such as the Remez exchange algorithm. Due to the nonlinearity between  $A_{F_q}(\omega)$  and  $m_q$ , linear programming (LP) cannot be applied directly to this prefilter design, either. However, after using the logarithm on  $|A_p(\omega)|$ , this problem can be formulated as a MILP problem as shown below. Denote  $-20 \log |A_p(\omega)|$  by  $A_{pdB}(\omega)$ . Then  $A_{pdB}(\omega) = \sum_{q=1}^{Q'} m_q A_{F_qdB}(\omega)$  where  $A_{F_qdB}(\omega) = -20 \log |A_{F_q}(\omega)|$ . Now the optimization problem in (2) may be rewritten as

$$\begin{aligned} & \text{Minimize} \\ & \sum_{q=1}^{Q'} m_q (a_q + c \cdot d_q) && \text{(complexity measure)} \\ & \text{Subject to} \\ & \sum_{q=1}^{Q'} m_q A_{F_qdB}(\omega) - s_{dB} \leq r_{pdB} && \text{(in passbands)} \\ & \sum_{q=1}^{Q'} m_q A_{F_qdB}(\omega) - s_{dB} \geq r_{sdB} && \text{(in stopbands)} \\ & \sum_{q=1}^{Q'} m_q A_{F_qdB}(\omega) - s_{dB} \geq 0 && \text{(scaling constraints)} \\ & r_{sdB} = \delta_{sdB} + \alpha r_{pdB} && \text{(ripple relation)} \end{aligned} \quad (3)$$

where  $s_{dB} = 20 \log(s)$ ,  $r_{pdB} = -20 \log|1 - r_p|$ ,  $r_{sdB} = -20 \log(r_s)$ ,  $\delta_{sdB}$  is the stopband attenuation of the overall filter, and  $\alpha$  is a positive constant. In (3), the passband constraint is derived under the assumption that  $s \cdot A_p(\omega) \leq 1$ ; we use the scaling constraint to keep  $s \cdot A_p(\omega) \leq 1$  and the ripple relation to represent  $r_{sdB}$  as a linear function of  $r_{pdB}$ . This ripple relation is derived based on the facts that the prefilter stopband attenuation  $r_{sdB}$  is considerably larger than the stopband attenuation of the overall filter  $\delta_{sdB}$  and that  $r_{sdB}$  is generally proportional to the passband deviation  $r_{pdB}$ . The problem in (3) can be solved by MILP, taking  $m_q$ ,  $s_{dB}$ ,

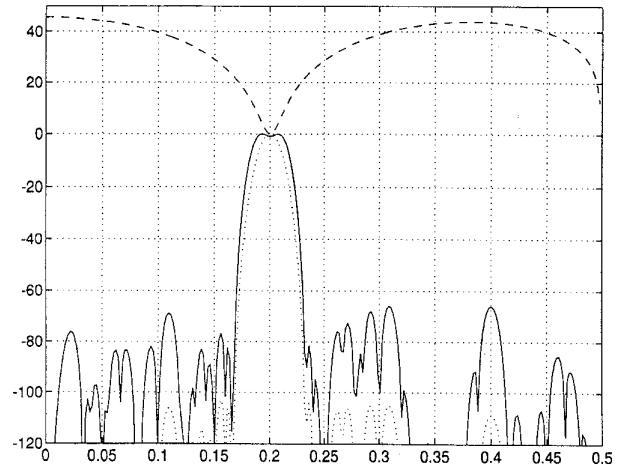


Fig. 1. Frequency responses of the prefilter (dotted line), equalizer (dashed line), and overall cascaded filter (solid line).

$r_{pdB}$  and  $r_{sdB}$  as variables. The proposed prefilter-equalizer design algorithm is summarized as follows.

- Determine the set of eligible CP's. This step is identical to Step 2 in Section II-A.
- Obtain all possible efficient multiplierless building blocks either by combining some eligible CP's or by finding the recursive structures of the CP's. We then add these blocks to the set of eligible CP's.
- Determine the order  $m_q$  by solving the optimization problem in (3). We repeat this for several values of  $\alpha$ . Consequently, several prefilters are obtained. In our examples, we found that  $\alpha$  values in the neighborhood of  $0.3W_s/W_p$  lead to efficient filter design, where  $W_p$  and  $W_s$ , respectively, are the stopband and passband widths.
- Design the equalizer for each prefilter obtained in Step 3. Among the prefilter-equalizers, we select the one with minimal complexity.

### III. FILTER DESIGN EXAMPLES

In order to compare the proposed CP prefilter design method with the previous one, our method is applied to the filter design problems considered in [4]. In the following example, the equalizer was designed by linear programming (LP). Using the commercial package in [7], we were able to solve the MILP and LP problems within a few minutes in Sparc II.

*Example (Bandpass Filter with Center Frequency 0.2):* The desired specifications are

- 1) passband:  $F \in [0.189, 0.211]$ ;
- 2) stopband:  $F \in [0, 0.168] \cup [0.232, 0.5]$ ;
- 3) ripple:  $\delta_{pdB} \leq 0.5$  dB.

In passband,  $\delta_{sdB} \geq 60$  dB in stopband. Conventional linear phase equiripple FIR filters require 111 taps to meet these specifications. For this filter design, 15 eligible CP's ( $Q = 15$ ) were found in [4]. To obtain efficient building blocks, we examined all possible combinations of these eligible CP's and found 62 combinations exhibiting reduced complexity. For

example, one of them is

$$\begin{aligned} C_5(z^{-1})C_{10}(z^{-1}) &= (1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}) \\ &\quad \cdot (1 - z^{-1} + z^{-2} - z^{-3} + z^{-4}) \\ &= (1 - z^{-10}) / (1 - z^{-2}). \end{aligned}$$

Note that implementation of  $C_5(z^{-1})C_{10}(z^{-1})$  originally requires eight additions but after reduction requires only two additions. These efficient building blocks are added to the set of eligible CP's. Now,  $Q' = 77$ . We designed 11 prefilters for  $\alpha = 5.4, 5.5, \dots, 6.3, 6.4$  (here  $0.3W_s/W_p = 5.94$ ), and found that the prefilter-equalizer corresponding to  $\alpha = 5.5$  is the most efficient among the 11 filters. The prefilter for  $\alpha = 5.5$  is represented as

$$\begin{aligned} P(z) &= (1 + z^{-1})^3 (1 - z^{-2})^2 (1 - z^{-3}) (1 + z^{-4}) (1 + z^{-5})^3 \\ &\quad \cdot (1 - z^{-7}) (1 - z^{-8})^2 (1 + z^{-9}) (1 + z^{-10})^2 \\ &\quad \cdot (1 - z^{-15}) (1 - z^{-2} + z^{-4}) (1 - z^{-3} + z^{-6})^2 \\ &\quad \cdot (1 + z^{-10} + z^{-15} + z^{-20} + z^{-30}) \end{aligned}$$

. Here the length of the equalizer is six. The frequency responses of this prefilter, the equalizer, and the overall filter are shown in Fig. 1. Implementation of this prefilter-equalizer requires three multiplications, 32 additions, and 147 delay elements. By comparison with the filter designed in [4], which requires eight multiplications, 39 additions, and 138 delay

elements, our design reduced five multiplications and seven additions at the expense of nine delays.

The proposed method was also applied to the design of the lowpass and the multiband filters considered in [4]. In each instance, we were able to reduce the number of multiplications and additions at the expense of some increase in the number of delays. Details of these results are not presented here because of space limitations.

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