Implementation of Programmable Multiplierless FIR Filters with Powers-of-Two Coefficients

Woo Jin Oh and Yong Hoon Lee

***Abstract*—An observation which is useful for hardware implementation of programmable FIR filters with powers-of-two coefficients (2PFIR filters) is made. Specifically, it is shown that the exponents of filter coefficients representable by the canonical signed digit (CSD) code with M ternary digits can be chosen from some subsets of $\{0, 1, \cdots, M-1\}$. This observation naturally leads to 2PFIR filters with shorter shifters whose length is strictly less than M and, as a consequence, leads to an efficient hardware structure for programmable 2PFIR filtering. In addition, we present some experimental results indicating that the shifters of 2PFIR filters can be shortened further in some cases.**

I. Introduction

Due to their simplicity in implementation FIR filters with powers-of-two coefficients, which are often referred to as 2PFIR filters, have received considerable attention in digital signal processing [1]–[11]. By employing only those coefficients that are sums and differences of signed powers-of-two, each multiplication in 2PFIR filtering can be replaced with simple shift-and-add operations.

Implementation of a 2PFIR filter is particularly efficient when its coefficients are fixed for dedicated applications, since hard-wired shifters can be employed [6]–[9]. On the other hand, realization of a programmable 2PFIR filter is considerably more difficult than that of a fixed filter, because it requires programmable shifters such as barrel shifters, shift registers and preshifters [10], [11], which greatly increase the hardware complexity or slacken the processing speed. In this paper, we observe that the exponents of 2PFIR filter coefficients representable by the *canonical signed digit* (CSD) code with M ternary digits [3], [12] can be chosen from some subsets of $\{0, 1, \cdots, M-1\}$. This observation naturally leads to 2PFIR filters having shifters of shorter length, and to an efficient hardware structure for programmable 2PFIR filtering.

The organization of this paper is as follows. In Section II, we derive a property of the CSD code, and show that the shifter of programmable 2PFIR filter can be shortened by using this property. In Section III, we present some experimental results indicating that the shifters of 2PFIR filters can be shortened further.

II. An Efficient Implementation of Programmable 2PFIR Filters

The impulse response $h(m)$ of a 2PFIR filter is represented as sums and differences of powers-of-two. Specifically, $h(m)$ is given by

$$h(m) = \sum_{k=1}^{L} s_k 2^{-p_k} \tag{1}$$

where $s_k \in \{-1, 0, 1\}$, $p_k \in \{0, 1, \cdots, M-1\}$, M is the number of ternary digits and L is the number of nonzero digits. This representation is known as the *radix-2 signed-digit* code [13]. In general, there are several signed-digit representations for a given $h(m)$.

A signed-digit code that leads to a unique representation of a number is the CSD code, which is specified as follows. For CSD codes, the number of nonzero digits L should be the minimum and in addition, no two nonzero digits are adjacent, i.e., in (1)

$$|p_i - p_j| \geq 2 \tag{2}$$

for any i and j, $i \neq j$. For example, the CSD representation of 0.375 is $2^{-1} - 2^{-3}$, and neither $2^{-1} - 2^{-2} + 2^{-3}$ nor $2^{-2} + 2^{-3}$ are CSD codes.

The product of $h(m)$ in (1) and an input value can be evaluated using adders and shifters, as illustrated in Fig. 1, which shows an overall structure for realizing a programmable 2PFIR filter and details of a tap. In particular, Fig. 1(b) and (c) presents two alternative structures for multiplying $h(m)$ with an input value $x(n-m)$. In Fig. 1(b), each product $x(n-m)2^{-p_k}$ is obtained by using either a barrel shifter or a shift register. Since each p_k in (1) is conventionally selected from the set $\{0, 1, \cdots, M-1\}$, each of the shifters should have $N \times M$ bits where N is the wordlength of the input[1] The structure in Fig. 1(c) employs M hardwired shifters, called preshifters in [11], followed by a programmable multiplexer which selects $x(n-m)2^{-p_k}$ for each k from $\{x(n-m), x(n-m)2^{-1}, x(n-m)2^{-2}, \cdots, x(n-m)2^{-(M-1)}\}$. Next we shall show that the length of the shifters in Fig. 1(b) can be shortened, and the number of hardwired shifters in Fig. 1(c) can be reduced by exploiting a property of the CSD codes.

Consider the set of all numbers between –1 and 1 that can be generated from (1) with a given M and L. Such a set, which will be denoted by $S_{M,L}$, is the set of all numbers in [–1, 1] representable by the CSD code. For example, $S_{2,1} = \{-1, -0.5, 0, 0.5, 1\}$. $S_{M,L}$ is generated from (1) with p_k's which are selected from $\{0, 1, \cdots, M-1\}$. In the following, we shall show that $S_{M,L}$ can also be generated with p_k's chosen from some proper subsets of $\{0, 1, \cdots, M-1\}$.

Property: Let $Z_{M,L}(k) \subset \{0, 1, \cdots, M-1\}$, $1 \leq k \leq L$, be a set of successive integers given by

$$Z_{M,L}(k) = \{2(k-1), 2(k-1)+1, \cdots, (M-1) - 2(L-k)\}. \tag{3}$$

Then $S_{M,L}$ can be generated from (1) with $p_k \in Z_{M,L}(k)$.

Proof: Suppose without loss of generality that $p_1 < p_2 < \cdots < p_L$, and 0 and 1 are elements of $Z_{M,L}$ (1). Since, $|p_i - p_j| \geq 2$, $p_1 < p_2 < \cdots < p_L$ and $0 \in Z_{M,L}(1)$, then $p_k \geq 2$ for all $k \geq 2$. Thus it is not necessary to include 0 and 1 in all $Z_{M,L}(k)$'s, $k \geq 2$. Let $2 \in Z_{M,L}(2)$, then we can eliminate 3 and 4 from all $Z_{M,L}(k)$'s, $k \geq 3$. In this manner, it can be shown that $\{0, 1, \cdots, 2(k-1)-1\}$ can be excluded from $Z_{M,L}(k)$, $k \geq 2$. Now, consider $Z_{M,L}(L) = \{2(L-1), 2(L-1)+1, \cdots, M-1\}$. Due to (2), $p_1 < p_2 < \cdots < p_L$ and $(M-1) \in Z_{M,L}(L)$, $(M-1)$ and $(M-2)$ are excluded from all $Z_{M,L}(k)$'s, $k \leq L-1$. Similarly, we can show that $\{(M-1) - 2(L-k) + 1, \cdots, M-1\}$ are not necessarily the elements of $Z_{M,L}(k)$, $1 \leq k \leq L-1$. ∎

The number of elements in $Z_{M,L}(k)$ is $M - 2L + 2$. This property indicates that $(M - 2L + 2)$ bit shifters can be used in place of M

[1] The implementation in Fig. 1(b) is rather conservative. By exploiting the fact that $p_i \neq p_j$ in CSD codes, $(M - L + 1)$ bit shifters associated with hardwired input shifters can be used instead of the M bit shifters.

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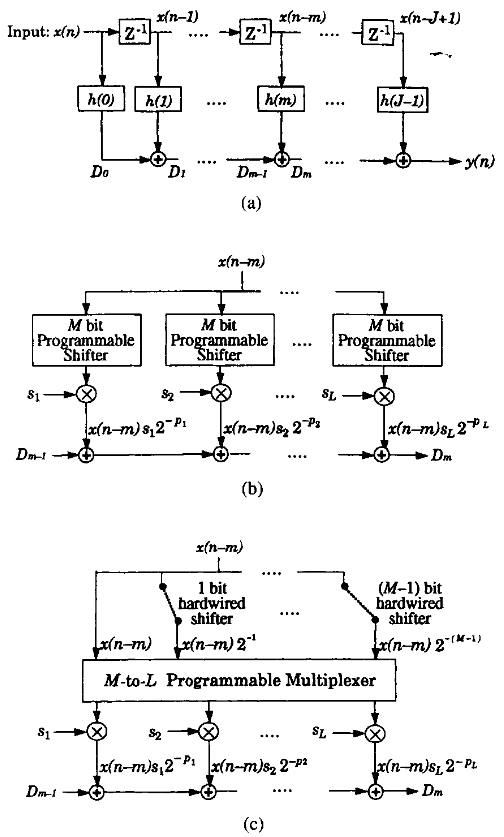

Fig. 1. (a) A structure for realizing 2PFIR filters with J taps. (b) and (c) Two alternative structures of a tap evaluating $D_m = D_{m-1} + h(m)x(n-m)$.

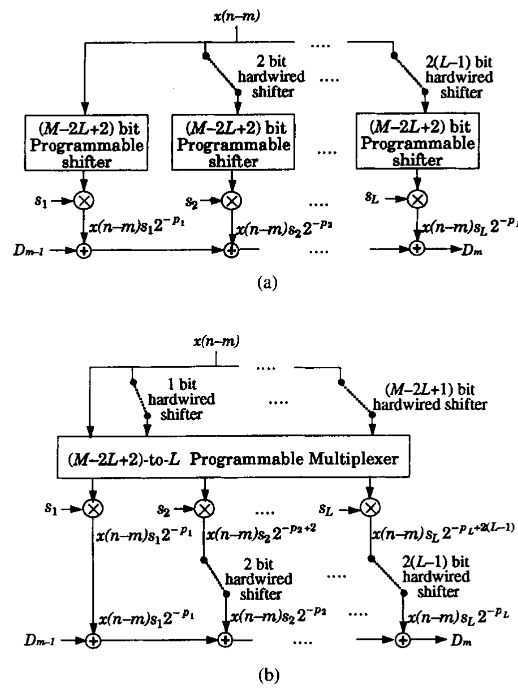

Fig. 2. Modification of the structures in Fig. 1(b) and (c).

bit shifters in Fig. 1(b), and that the number of hardwired shifters in Fig. 1(c) can be reduced by $2(L-1)$. For example, when (M, L) = (12, 2) and (12, 3) the length of shifters is reduced by $2(L-1)$ = 2 and 4, respectively. Fig. 2(a) and (b) illustrate modifications of the structures in Fig. 1(b) and (c), respectively. In Fig. 2(a), the multiplication of $x(n-m)$ with 2^{-p_k} is carried out by using a $2(k-1)$ bit hard-wired shifter followed by an $(M-2L+2)$ bit shifter. Note that the length of each shifter is shortened by $2(L-1)$. In Fig. 2(b), $\{x(n-m)2^{-i} | 0 \leq i \leq M-2L+1\}$ are evaluated by $M-2L+2$ hard-wired shifters, $x(n-m)2^{-p_k+2(k-1)}$ is selected for each k by the programmable multiplexer, and $x(n-m)2^{-p_k}$ is obtained through the $2(k-1)$ bit hard-wired shifter. Obviously, the structure in Fig. 2(b) is simpler to implement than that in Fig. 1(c).

III. Further Shortening of Shifters

The length of shifters can be reduced to $P \leq M-2L+2$ by selecting the value of each p_k from a subset of $Z_{M,L}(k)$, denoted by $Z'_{M,L}(k)$, consisting of P successive integers. Let $Z'_{M,L}(1) = \{0, 1, \cdots, P-1\}$ and $Z'_{M,L}(L) = \{M-P, \cdots, M-1\}$. We denote the set of all possible fractional numbers generated from (1) with $p_k \in Z'_{M,L}(k)$ by $S'_{M,L}$. Of course, $S'_{M,L}$ is a subset of $S_{M,L}$. In what follows, we shall show through some experiments that a 2PFIR filter with coefficients from $S'_{M,L}$ can be simpler to implement and can perform better than a 2PFIR filter with coefficients from $S_{(M-i),L}$, $i \geq 1$, in some cases.

We design four types of 2PFIR filters that have been considered in [2]: a low-pass filter with f_p = 0.15 and f_s = 0.25, a high-pass filter with f_p = 0.25 and f_s = 0.15, a band-pass filter with f_p = (0.25, 0.35) and f_s = (0.15, 0.45), and a bandstop filter with f_p = (0.15, 0.45) and f_s = (0.25, 0.35) where f_p and f_s are normalized passband and stopband frequencies, respectively, and the ripple weighting factor is equal to one. The local search method developed in [2] has been applied to design these filters with 31 and 37 taps, for various values of (M, L). Table I lists the peak stopband ripple of the designed 2PFIR filters. The sets $Z'_{M,L}(k)$ associated with $S'_{M,L}$ in the table are specified as follows: for (M, L) = (12, 3)

$$\begin{aligned} Z'_{12,3}(1) &= \{0, 1, \cdots, 4\}, \\ Z'_{12,3}(2) &= \{4, 5, \cdots, 8\}, \\ Z'_{12,3}(3) &= \{7, 8, \cdots, 11\} \end{aligned} \tag{4}$$

and for (M, L) = (12, 2)

$$\begin{aligned} Z'_{12,2}(1) &= \{0, 1, \cdots, 7\}, \\ Z'_{12,2}(2) &= \{4, 5, \cdots, 11\}. \end{aligned} \tag{5}$$

These sets were chosen so that $Z'_{M,L}(k) \subset Z_{M,L}(k)$, $\bigcup_{k=1}^{L} Z'_{M,L}(k) = \{0, 1, \cdots, M-1\}$, and the number of elements in $Z'_{M,L}(k)$ remains the same for all k. In Table I(a), where $L = 3$, it is seen that the set $S'_{12,3}$ has more elements than $S_{10,3}$, and that the maximum stopband attenuation of each filter associated with $S'_{12,3}$ is greater than that of the corresponding filter associated with $S_{10,3}$. Since $Z'_{12,3}$ and $Z_{10,3}$ have five and six elements, respectively, implementing the filters with parameters from $S'_{12,3}$ is simpler than the implementation with $S_{10,3}$. Therefore, $S'_{12,3}$ would be preferred to $S_{10,3}$ in most practical applications: the former requires shorter shifters and generally results in better 2PFIR filters.

In Table I(b), where $L = 2$, the set $S'_{12,2}$ has more elements than $S_{10,2}$ but the maximum stopband attenuation of each filter associated

TABLE I
NUMBER OF ELEMENTS IN $S_{M,L}$ AND $S'_{M,L}$, AND THE MAXIMUM STOPBAND ATTENUATION OF VARIOUS 2PFIR FILTERS WITH J TAPS (a) $L = 3$, (b) $L = 2$.

	Number of elements in $S_{M,L}$ and $S'_{M,L}$	Maximum Stopband Attenuation (dB)							
		LPF		HPF		BPF		BSF	
		J=31	J=37	J=31	J=37	J=31	J=37	J=31	J=37
$S_{10,3}$	513	48.80	49.16	47.84	48.43	47.79	48.83	49.17	48.63
$S_{12,3}$	1041	52.02	54.75	51.66	57.04	52.47	58.96	52.01	58.89
$S'_{12,3}$	777	49.16	50.40	50.40	52.21	52.32	55.36	49.38	52.14

(a)

	Number of elements in $S_{M,L}$ and $S'_{M,L}$	Maximum Stopband Attenuation (dB)							
		LPF		HPF		BPF		BSF	
		J=31	J=37	J=31	J=37	J=31	J=37	J=31	J=37
$S_{10,2}$	149	41.59	41.65	43.63	44.20	45.76	46.38	45.09	45.09
$S_{12,2}$	225	42.36	43.80	45.55	46.24	46.48	48.68	48.29	50.31
$S_{16,2}$	425	42.36	43.80	47.13	47.56	47.85	48.68	49.31	50.31
$S'_{12,2}$	205	39.46	43.10	45.37	43.97	44.35	50.35	45.08	48.09

(b)

with $S'_{12,2}$ is often smaller than that of the corresponding filter associated with $S_{10,2}$. Therefore, $S'_{12,2}$, is not always preferred to $S_{10,2}$ and reducing the length of shifters by using $S'_{M,L}$ may not be recommended when L = 2.

Now note that $S'_{12,3}$ requires considerably shorter shifters than $S_{16,2}$. This indicates the possibility that a programmable 2PFIR filter associated with $S'_{12,3}$, although it requires an additional adder, can be simpler than that with $S_{16,2}$. Because of this possibility and the fact that the filters associated with $S'_{12,3}$ outperform those associated with $S_{16,2}$ in Table I, $S'_{12,3}$ appears to be a useful alternative to $S_{16,2}$.

Finally, it should be pointed out that the results presented in this section would be dependent upon the filter optimization method used. Further work in this direction will concentrate on studying the effect of the filter optimization method, and on developing a procedure for obtaining $Z'_{M,L}(k)$.

IV. CONCLUSION

We have derived a property of the CSD code indicating that the exponents of the CSD code with M ternary digits can be chosen from proper subsets of $\{0, 1, \cdots, M-1\}$, consisting of $M - 2L + 2$ successive integers where L is the number of nonzero digits. By exploiting this property, we were able to shorten the length of shifters of programmable 2PFIR filters having CSD coefficients with M ternary digits by $2(L-1)$, without degrading their performance. In addition, it is shown through some experiments that the length of shifters can be shortened further in some cases.

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TABLE I
FILTER MASKS FOR EXPERIMENT

Number	1	2	3	4	5	6
Mask	010 111 010	111 111 111	00100 01110 11111 01110 00100	11111 11111 11111 11111 11111	0011100 0111110 1111111 1111111 1111111 0111110 0011100	1111111 1111111 1111111 1111111 1111111 1111111 1111111

On the Selection of Median Structure for Image Filtering

Stanley J. Reeves

Abstract—**Median filtering is a powerful tool for reducing noise in images, particular for long-tailed noise distributions. However, the choice of filter mask is critical. The proper choice depends on the image and noise statistics, which are often unknown. We propose cross-validation as a method for selecting a median filter structure directly from the corrupted image data. This method requires no knowledge of the statistics of the noise or image. We demonstrate the value of this method with several examples.**

I. Introduction

Median filters provide a powerful method for filtering signals and images [1]. In particular, the median filter performs well at filtering outlier points while leaving edges intact. In fact, the median filter has been shown to be the maximum likelihood estimate for the Laplacian distribution, which is a long-tailed distribution [2]. The median filter is defined as follows:

$$y(m,n;W) = \text{med}\{x(m-k,n-l),\ (k,l) \in W\}, \tag{1}$$

where W is a suitably chosen window, or filter mask.

One must be careful that the filter mask W is chosen appropriately. Too small a mask may fail to filter noise adequately, while too large a mask may cause unnecessary distortions in the image. The optimal mask depends on the statistics of the noise as well as the image. We know of no algorithms for evaluating filter masks based on specified noise and image statistics. Moreover, the noise and image statistics may be unknown or may change either spatially or temporally. Therefore, in this paper we propose to use cross-validation to select the optimal filter mask using only the noisy image data.

II. Cross-Validation

Cross-validation (CV) has been used extensively in data analysis [3]. Recently, CV has been applied to a variety of problems in image restoration [4]. The basic idea of CV is to form a prediction for each data point from all the other data points. The model—or in this case the filter mask—that minimizes the prediction error is considered optimal. The CV-predicted image is defined by

$$y_c(m,n;W) = y(m,n;W_c) \tag{2}$$

where $W_c = W$ but excluding the point $(0,0)$. Note that if W contains an odd number of elements then W_c will contain an even number. In this case we must use the traditional definition of the median, which is the mean of the two middle values when arranged in order of increasing value.

The CV criterion can be expressed as

$$V(x,W;d(\cdot)) = \sum d(x(m,n) - y_c(m,n;W)). \tag{3}$$

The distance function $d(\cdot)$ can be chosen to correspond to the figure of merit desired. For example, if one desires to minimize mean square error (MSE) in the filtered image, then one should choose $d(\cdot) = [\cdot]^2$. If one desires to minimize mean absolute error (MAE), then one should use $d(\cdot) = |\cdot|$.

The formal properties of CV are difficult to analyze. However, a heuristic explanation suggests that CV will perform well as an estimator of the corresponding figure of merit for various filter masks. Consider an image $s(m,n)$ corrupted by independently distributed additive noise $u(m,n)$:

$$x(m,n) = s(m,n) + u(m,n). \tag{4}$$

For a given figure of merit, a median-filtered version of x will have two sources of error—ε_s representing the distortion of the signal and ε_u representing the error introduced by the filtered noise. As the filter mask becomes larger, ε_s will grow larger while ε_u will grow smaller. For the sake of simplicity, we consider only $d(\cdot) = [\cdot]^2$ in the discussion, although the argument can be extended to other distance measures. Thus, the filtered image will have a total error

$$\begin{aligned} \varepsilon &= \frac{1}{MN}\sum [s(m,n) - y(m,n;W)]^2 \\ &= \varepsilon_s + \varepsilon_u, \end{aligned} \tag{5}$$

where M is the number of rows and N the number of columns in the image. The CV criterion can be approximated by rewriting (3) as

$$\begin{aligned} V(x,W;d(\cdot)) &= \frac{1}{MN}\sum [x(m,n) - y_c(m,n;W)]^2 \\ &= \frac{1}{MN}\sum [s(m,n) + u(m,n) - y_c(m,n;W)]^2 \\ &= \frac{1}{MN}\sum \{[s(m,n) - y_c(m,n;W)]^2 + [u(m,n)]^2 \\ &\quad - 2(y_c(m,n;W) - s(m,n))u(m,n)\} \\ &\approx \varepsilon_{s\backslash 0} + \varepsilon_{u\backslash 0} + \varepsilon_0 - \varepsilon_x, \end{aligned} \tag{6}$$

where $\varepsilon_{s\backslash 0} + \varepsilon_{u\backslash 0}$ is equivalent to (5) with the mask W_c, ε_0 is from the uncorrelated (and hence unpredictable) noise corrupting

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