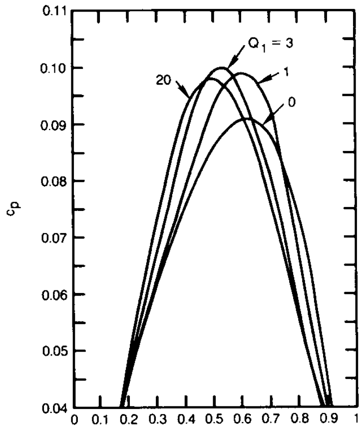

Fig. 1. Power-output capability c_p of the Class E amplifier as a function of D at $Q_1 = 0$, 1, 3, and 20 for optimum operation.

TABLE I
POWER-OUTPUT CAPABILITY OF THE CLASS E AMPLIFIER AS FUNCTIONS OF D AND Q_1 FOR OPTIMUM OPERATION

D	Q_1 = 0	1	3	5	10	20	100	∞
	c_p							
0.2	0.0446	0.0449	0.0456	0.0463	0.0475	0.0483	0.0493	0.0493
0.3	0.0617	0.0627	0.0661	0.0686	0.0713	0.0731	0.0745	0.0749
0.4	0.0755	0.0787	0.0878	0.0907	0.0922	0.0928	0.0931	0.0931
0.5	0.0857	0.0936	0.0997	0.0996	0.0990	0.0986	0.0982	0.0981
0.6	0.0908	0.0994	0.0965	0.0947	0.0932	0.0924	0.0915	0.0914
0.7	0.0883	0.0892	0.0812	0.0792	0.0776	0.0767	0.0759	0.0757
0.8	0.0744	0.0646	0.0565	0.0557	0.0550	0.0545	0.0539	0.0537

any loaded quality factor Q_1 and any ON switch duty cycle D, with the component values chosen for optimum circuit operation. This letter extends that publication giving the power-output capability of the Class E amplifier as a function of D over the entire range of Q_1, from zero to infinity.

Assuming ideal components, the efficiency of the amplifier is 100 percent and the output power P_0 is equal to the dc supply power $P_{CC} = I_{CC}V_{CC}$. Hence, the power-output capability can be expressed as [4]

$$c_p = \frac{P_0}{I_{CM}V_{CEM}} = \frac{I_{CC}V_{CC}}{I_{CM}V_{CEM}} \tag{1}$$

where I_{CM} and V_{CEM} are the peak values of the collector current and the collector-to-emitter voltage, respectively. The parameter c_p was calculated by the method given in [4]. Fig. 1 shows c_p plotted as a function of D at $Q_1 = 0$, 1, 3, and 20. Table I gives the numerical values of c_p. Fig. 2 illustrates the maximum power-output capability $c_{p\max}$ as a function of Q_1.

The conclusions are as follows.

1) The maximum power-output capability $c_{p\max}$ occurs at values of D decreasing from 0.62 to 0.5, over the entire range of Q_1, from zero to infinity.

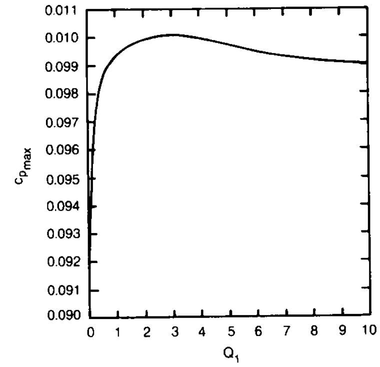

Fig. 2. The maximum power-output capability $c_{p\max}$ of the Class E amplifier as a function of Q_1.

2) The minimum value of $c_{p\max}$ is 0.0909 and occurs for $Q_1 = 0$ at $D = 0.62$. The maximum value of $c_{p\max}$ is 0.1002 and occurs for $Q_1 = 3$ at $D = 0.53$. As $Q_1 \to \infty$, $c_{p\max} = 0.0981$ and occurs at $D_1 = 0.5$.

REFERENCES

[1] N. O. Sokal and A. D. Sokal, "Class E—A new class of high-efficiency tuned single-ended switching power amplifiers," *IEEE J. Solid-State Circuits*, vol. SC-10, pp. 168–176, June 1975.

[2] F. H. Raab, "Idealized operation of the Class E tuned power amplifier," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 725–735, Dec. 1977.

[3] R. E. Zulinski and J. W. Steadman, "Class E power amplifiers and frequency multipliers with finite dc-feed inductance," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 1074–1087, Sept. 1987.

[4] M. K. Kazimierczuk and K. Puczko, "Exact analysis of Class E tuned power amplifier at any Q and switch duty cycle," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 149–159, Feb. 1987.

Effects of Window Overlapping on Input/Output Interdependencies in Median Filters

ADLY T. FAM AND YOON HOON LEE

Abstract —**Overlapping of the windows in median filtering causes interesting input/output interdependencies, propagated over time. In this paper, we derive the monotone-envelope property of median filters, which provides upper and lower monotone bounds on future outputs in terms of the inputs in the present window. It is shown that the monotone-envelope property can be thought of as an extension of the finite sample breakdown point. The monotone-envelope property is then extended to provide more refined bounds and properties.**

I. INTRODUCTION

The median filter is a simple nonlinear smoother that can suppress noise while retaining sharp sustained changes (edges) in signal values [1]. It is particularly effective in reducing impulsive-type noise. The output of the median filter at a point is the median value of the input data inside the window centered at that point. If we let $\{x(k)\}$ and $\{y(k)\}$ be the input and the

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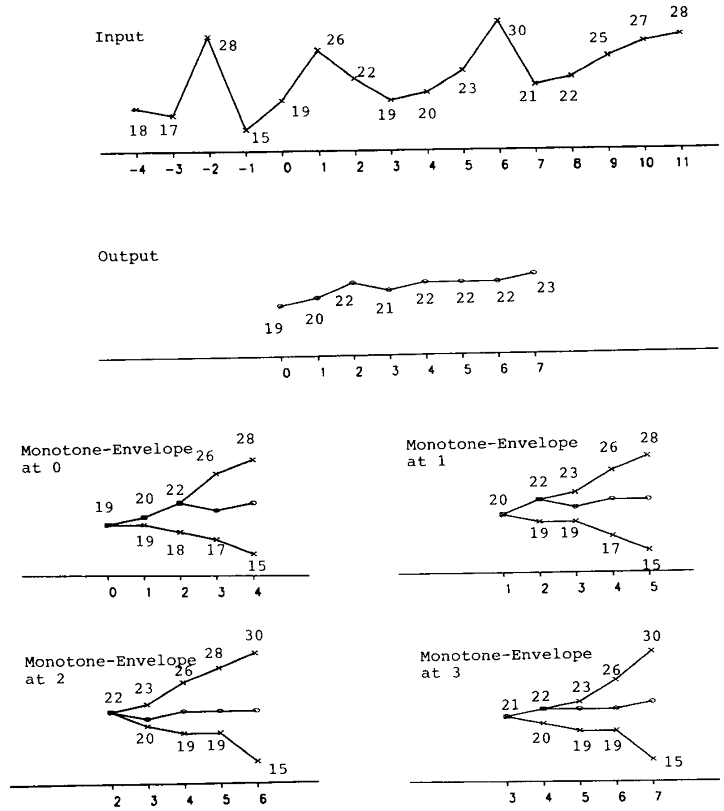

Fig. 1. The input and output of the median filter of window width 9, and the monotone-envelope at each point. (In the monotone-envelope x represents the bounds and o represent the outputs.)

output of the median filter of window size $2N+1$, then $y(k)=\text{med}\{x(k-N),\cdots,x(k),\cdots,x(k+N)\}$.

The edge preserving and impulse suppressing characteristics of median filtering may be explained by the finite-sample breakdown point (FSBP) ϵ_N^* which is a useful measure of robustness of estimators [2]. The FSBP of an estimator $T(.)$ at the set of finite-valued samples $\{x_{k-N},\cdots,x_{k+N}\}$ is given by

$$\epsilon_N^*(x_{k-N},\cdots,x_{k+N}) = \frac{1}{2N+1}\max\left\{m:\ \max_{i_1,\cdots,i_m}\ \sup_{w_1,\cdots,w_m}\ |T(z_1,\cdots,z_{2N+1})|<\infty\right\} \tag{1}$$

where the sample $(z_1,\cdots,z_{2N+1})$ is obtained by replacing the m data points $\{x_{i_1},\cdots,x_{i_m}\}$ by the arbitrary values, $w_1,\cdots,w_m$, and $k-N\leqslant i_j\leqslant k+N$, for $1\leqslant j\leqslant m$. Loosely speaking, the FSBP gives the limiting fraction of bad outliners the estimator can cope with. The FSBP of median filtering, which is obtained by replacing $T(.)$ with median (.) in (1), is $N/(2N+1)$. This elucidates the well-known facts that the median filter of span $2N+1$ can suppress up to N impulses and preserve edges.

From the FSBP we can also explain naturally, the obvious and yet interesting observation, that in median filtering, N future outputs $y(k+1),\cdots,y(k+N)$ are finite if the input samples $\{x(k-N),\cdots,x(k+N)\}$ within the current window are finite. This is so because the FSBP of median filtering is $N/(2N+1)$, and the set of samples inside the window at $k+i$, $1\leqslant i\leqslant N$ are obtained by replacing the leftmost i data points $x(k-N),\cdots,x(k-N+i-1)$ of the current window by the new values $x(k+N+1),\cdots,x(k+N+i)$. In this paper, this observation is extended to the monotone-envelope property which provides upper and lower monotone bounds on the outputs in terms of the inputs in the present window. Throughout, we consider the one-dimensional (1-D) median filter with window size $2N+1$. The input and output sequences are assumed to be two-sided infinite and denoted by $\{x(k)\}$ and $\{y(k)\}$, respectively. The ith smallest sample inside the window centered at k is denoted by $x(i;k)$. Thus the output $y(k)$ of the median filter is $x(N+1;k)$.

II. Input/Output Interdependencies in Median Filters

Lemma 1 below helps to establish Properties 1 and 2 that present the output ranges of the median filter.

Lemma 1: For any i, $2\leqslant i\leqslant 2N$

$$x(i-1;k+1)\leqslant x(i;k)\leqslant x(i+1;k+1). \tag{2}$$

In addition, $x(1;k)\leqslant x(2;k+1)$, and $x(2N+1;k)\geqslant x(2N;k+1)$.

Proof: Suppose that $x(i;k)$ for some i, $1\leqslant i\leqslant 2N+1$, is not $x(k-N)$, which is the sample to be discarded from the window at $k+1$. In the window at $k+1$, $x(i;k)$ becomes either $x(i-1;k+1)$ or $x(i;k+1)$ or $x(i+1;k+1)$ if $2\leqslant i\leqslant 2N$, $x(1;k)$ becomes either $x(1;k+1)$ or $x(2;k+1)$, and $x(2N+1;k)$ becomes either $x(2N;k+1)$ or $x(2N+1;k+1)$. Thus in this case, the above inequalities hold. Now assume that $x(i;k)$ is $x(k-N)$.

If the new sample $x(k+N+1)$ is greater than $x(i+1;k)$, then $x(i+1;k)$ becomes $x(i;k+1)$ and $x(i-1;k)$ becomes $x(i-1;k+1)$ for $2 \leqslant i \leqslant 2N+1$. Thus (2) holds. The remainder of the proof follows from similar arguments.

Next the bounds for future output values are given in terms of input values in the present window, establishing the monotone-envelope property of median filters.

Property 1 The Monotone-Envelope Property: The output $y(k+i)$ of the median filter for every i, $1 \leqslant i \leqslant N$, always lies in the range

$$x(N+1-i;k) \leqslant y(k+i) \leqslant x(N+1+i;k). \quad (3)$$

Proof: Consider a sample $x(j)$ inside the window centered at $k+i, (k+i-N \leqslant j \leqslant k+i+N)$. Suppose that $x(j)$ is greater than $x(N+1+i;k)$. Then $x(j) > y(k+i)$, since $y(k+i) = x(N+1;k+i) \leqslant x(N+2;k+i-1) \leqslant \cdots \leqslant x(N+1+i;k)$ follows from Lemma 1. Similarly, we can show that $x(j) < y(k+i)$ if $x(j)$ is less than $x(N+1-i;k)$. Thus $x(j)$ can be $y(k+i)$ only if it is in the range (3). This concludes the proof.

In (3), the upper and the lower bounds, respectively, are nondecreasing and nonincreasing functions of i. Thus monotone bounds are provided on the outputs justifying the name given to this property. As we mentioned before, both the monotone-envelope property and the FSBP indicate that N future outputs $y(k+i)$, $1 \leqslant i \leqslant N$, are finite if the inputs inside the present window are finite, but the former gives exact bounds of the outputs while the latter cannot. In Fig. 1 we present an example where a median filter of window size 9, $N=4$, is applied to an input signal. The output of the median filter, and the monotone-envelope at each point are illustrated. From Property 1, it is obvious that the input values in the present window limits the previous outputs as well. This is stated below.

Corollary 1: The output $y(k-i)$ of the median filter for any i, $1 \leqslant i \leqslant N$, always lies in the range $x(N+1-i;k) \leqslant y(k-i) \leqslant x(N+1+i;k)$.

When $i=1$, the monotone-envelope property is reduced as follows.

Corollary 2: The output $y(k+1)$ of a median filter of span $2N+1>3$ satisfies $y(k+1) \in \{y(k), x(N+2;k), x(N;k), x(k+N+1)\}$. The output $y(k+1)$ is equal to $x(k+N+1)$ only if $x(N;k) \leqslant x(k+N+1) \leqslant x(N+2;k)$.

This corollary has been observed in [3]. For i.i.d. inputs with continuous input distribution, the probability that $y(k+1)$ equals each candidate can be obtained. In [4], it is shown that $\Pr[y(k+1)=y(k)] = N/(2N+1)$ when the input is i.i.d., and continuous. Now it is straightforward to see that $\Pr[y(k+1)=x(k+N+1)] = 1/(2N+1)$ and $\Pr[y(k+1)=x(N;k)] = \Pr[y(k+1)=x(N+2;k)] = N/2(2N+1)$. Thus for i.i.d. continuous inputs, approximately 50 percent of $y(k+1)$ equals the previous output $y(k)$. This explains in part the reason why streaking occurs in median filtered sequences.

Another consequence of Property 1 is that the inputs inside the windows located in the neighborhood of a point k give a number of bounds for the output $y(k)$.

Corollary 3: The output $y(k)$ always lies in the range

$$x(N-i;k-1-i) \leqslant y(k) \leqslant x(N+2+i;k-1-i) \quad (4)$$

for $i=0,1,\cdots,N-1$.

Consider the above N bounds for $y(k)$ as a function of i. Lemma 1 indicates that the bounds become tighter as i decreases. In other words, the input values within the window which is located closer to the point k can give tighter bounds for $y(k)$ (see Fig. 1). It is natural to compare these bounds with the trivial bounds given by

$$x(N;k) \leqslant y(k) \leqslant x(N+2;k). \quad (5)$$

We can observe that the bounds in (5) are either tighter or looser than the bounds in (4) depending on the input values. For example, if the ordered data at $k-N$ and at k, for $N=3$, are given by $\{-3,-2,-1,0,1,1,1\}$ and $\{-4,-4,-4,0,1,1,1\}$, respectively, the bounds given by (4) for $i=N-1$ is tighter than those given by (5). In some other cases, the bounds in (5) are tighter than the ones in (4).

Finally, Property 1 can be modified to give tighter bounds by considering time indexes of ordered data. Specifically, we shall show that if $x(N+1-i;k)$ and/or $x(N+1+i;k)$ in (3) are not within the window centered at $k+i$, then the bounds can be tightened.

Property 2: Let S_{k+i} be the set of samples inside the window centered at $k+i$, $1 \leqslant i \leqslant N$. If $x(N+1-i+j;k) \notin S_{k+i}$ for all j, $0 \leqslant j \leqslant n-1$, and $x(N+1+i-h;k) \notin S_{k+i}$ for all h, $0 \leqslant h \leqslant m-1$, but $x(N+1-i+n;k) \in S_{k+i}$ and $x(N+1+i-m;k) \in S_{k+i}$, then

$$x(N+1-i+n;k) \leqslant y(k+i) \leqslant x(N+1+i-m;k) \quad (6)$$

where n and m are non-negative integers satisfying $n+m \leqslant i$.

Proof: We first show that $x(N+1;k+i) \leqslant x(N+1+i-m;k)$. Let $x(N+1+i-m;k)$ be $x(r;k+i)$ for some r, $1 \leqslant r \leqslant 2N+1$, and q be the number of samples greater than or equal to $x(N+1+i-m;k)$ among i samples in the set $\{x(k+N+j) | 1 \leqslant j \leqslant i\}$. We can consider r as a decreasing function of q: r decreases as q increases. Suppose that $q=i$. Since m samples greater than or equal to $x(N+1+i-m;k)$ have been discarded from the window centered at k, and i samples greater than or equal to $x(N+1+i-m;k)$ have been included in the window centered at $k+i$, then $x(r;k+i) = x(N+1+i-m;k) \geqslant x(N+1;k+i) = y(k+i)$. Since r is a decreasing function of q, the fact that $x(r;k+i) \geqslant y(k+1)$ for $q=i$ indicates that the inequality holds for $q \leqslant i$. In a similar manner, $x(N+1-i+n;k) \leqslant y(k+i)$ can be proved.

For example, again referring to Fig. 1, consider the monotone-envelope at time one. This envelope indicates that $17 \leqslant y(4) \leqslant 26$. Since 17 is not a sample within the window centered at time four, we can get tighter bounds, $19 \leqslant y(4) \leqslant 26$, by applying (6).

III. Conclusion

In this paper a number of properties expressing bounds on future median filter outputs, in terms of the inputs in present window, were introduced.

Notice that Lemma 1, on which the monotone-envelope property is based, is not limited to median filtering but shows a general relationship among ordered data within neighboring windows. Thus we may conjecture that other median-type (order-statistic) filters have properties similar to the monotone-envelope property. It can be seen that this is true for many median-type filters. For example, let us consider the α-trimmed mean ($\alpha-TM$) filter [6] which is defined as follows:

$$y(k) = \frac{1}{2(N-T)+1} \sum_{j=T+1}^{2N+1-T} x(j;k)$$

where T is an integer, $0 \leqslant T \leqslant N$. (When $T=0$, the $\alpha-TM$ filter becomes the average filter, and when $T=N$, it becomes the median filter.) The FSBP of the $\alpha-TM$ filter is $T/(2N+1)$. Following from Lemma 1, the proof of Property 1, and the definition of the $\alpha-TM$ filter, it is straightforward to show that the output $y(k+i), 1 \leqslant i \leqslant T$ of the $\alpha - TM$ filter with $1 \leqslant T \leqslant N$ lies in the range $x(T+1-i;k) \leqslant y(k+i) \leqslant x(2N+1-T+i;k)$. Note that, in $\alpha - TM$ filtering, T future output values are limited by the input values inside the current window, which may be expected from the fact that the FSBP of $\alpha - TM$ filtering is $T/(2N+1)$. This, again, shows a relationship between the monotone-envelope type property and the FSBP. The investigation of the monotone-envelope type properties for other order statistic filters, as well as extending and finding useful applications of these properties, are topics for further research.

References

[1] T. S. Huang Ed., *Two Dimensional Digital Signal Processing II: Transforms and Median Filters*. Berlin, Germany: Springer-Verlag, 1981.

[2] F. R. Hampel, P. J. Rousseeuw, E. M. Ronchetti, and W. A. Stahel, *Robust Statistics*. New York: Wiley, 1986.

[3] Y. H. Lee, S.-J. Ko, and A. T. Fam, "Efficient impulsive noise suppression via nonlinear recursive filtering," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 303–305.

[4] A. C. Bovik, "Streaking in median filtered images," *IEEE Trans. Acous. Speech Signal Processing*, vol. ASSP-35, pp. 493–503, April 1987.

[5] S. G. Tyan, "Median filtering: Deterministic properties," in *Two Dimensional Digital Signal Processing II: Transforms and Median Filters*. T. S. Huang, Ed. Berlin, Germany: Springer-Verlag, 1981.

[6] Y. H. Lee and S. A. Kassam, "Generalized median filtering and related nonlinear filtering techniques," *IEEE Trans. Acoust., Speech, Signal Processing.*, vol. ASSP-33, pp. 672–683, June 1985.

Realization of Separable First- and Second-Order 2-D All-Pass Digital Filters

P. S. REDDY, P. SATYANARAYANA, AND M. N. S. SWAMY

Abstract **—We give in this paper structures realizing first- and second-order separable 2-D all-pass digital filters. The method employs signal flow graph technique and Mason's gain formula to obtain the transfer function. The realizations obtained are minimal delay and minimal multiplier structures.**

I. Introduction

Recently there has been some work reported in the literature [1]–[4] on the realization of first-order 2-D all-pass digital filters. The realizations in [2] and [3] employ signal flow graph technique. The structure given in [3] is an improved one since it contains one less multiplier when compared to the number of multipliers required in the realization presented in [2]. Both structures use minimal delay elements. The realization presented in [4] employs 3 delay elements and 3 multipliers. In the opinion of the authors it is not possible to obtain a realization, for a first-order 2-D all-pass digital filter, containing 2 delay elements and 3 multipliers.

In this paper we deal with the realization of separable first- and second-order 2-D all-pass digital filters and obtain minimal delay and minimal multiplier structures.

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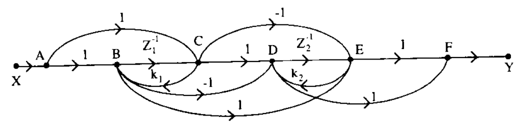

Fig. 1. Signal flow graph for 2-D first-order all-pass filter.

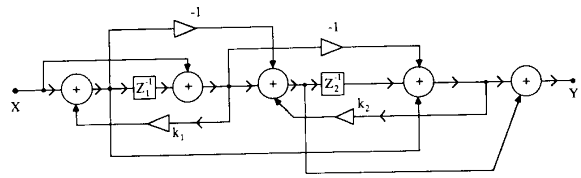

Fig. 2. Network structure realizing the 2-D first-order all-pass filter.

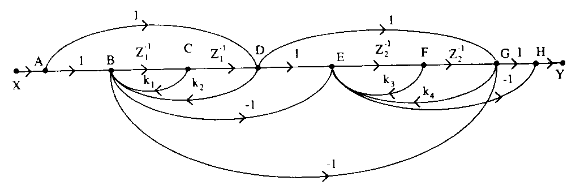

Fig. 3. Signal flow graph for 2-D second-order all-pass filter.

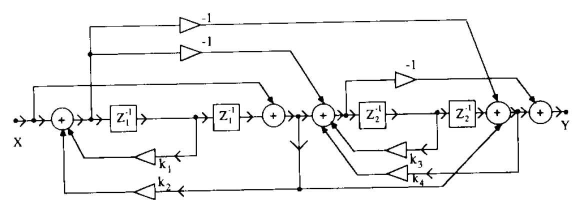

Fig. 4. Network structure realizing the 2-D second-order all-pass filter.

II. First-Order Separable 2-D All-Pass Digital Filters

Let the transfer function of a first-order separable 2-D all-pass digital filter be

$$H(Z_1^{-1}, Z_2^{-1}) = \frac{a_{10}a_{01} + a_{01}Z_1^{-1} + a_{10}Z_2^{-1} + Z_1^{-1}Z_2^{-1}}{1 + a_{10}Z_1^{-1} + a_{01}Z_2^{-1} + a_{10}a_{01}Z_1^{-1}Z_2^{-1}}. \quad (1)$$

We choose a signal flow graph not containing any delay free loops as given in Fig. 1. Using Mason's gain formula [5] we obtain the transfer function of Fig. 1 as

$$H(Z_1^{-1}, Z_2^{-1}) = \frac{k_1k_2 - k_2Z_1^{-1} - k_1Z_2^{-1} + Z_1^{-1}Z_2^{-1}}{1 - k_1Z_1^{-1} - k_2Z_2^{-1} + k_1k_2Z_1^{-1}Z_2^{-1}}. \quad (2)$$

Equating the corresponding coefficients in (1) and (2) we get

$$k_1 = -a_{10}$$
$$k_2 = -a_{01}.$$

Thus we have a network realization as shown in Fig. 2. The number of multipliers is two and it contains only two delays. It

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