

Double Smoothing of Images Using Median and Wiener Filters

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Abstract—In this correspondence, we propose double smoothers, consisting of the median and Wiener filters, which we will call the median and Wiener (MW) double smoothers. It is shown that the MW double smoother is suitable for smoothing a covariance stationary process which is a useful model of images. The statistical analysis and experimental results indicate that the MW double smoother is an efficient edge-preserving smoother that can outperform the individual median and Wiener filters.

I. INTRODUCTION

Fig. 1 illustrates the double smoothing (or reroughing) algorithm which employs two smoothers denoted by F1 and F2. The signal $Z(i, j)$, smoothed by F1, is subtracted from the corresponding input signal $X(i, j)$, and the residual signal $X(i, j) - Z(i, j)$ is fed into the smoother F2. The output is obtained by adding the smoothed input $Z(i, j)$ and the smoothed residual. Double smoothing was proposed by Tukey [1] to correct the smoothed input whenever it is so smooth as to lose the details of the original, and it was applied to speech processing by Rabiner *et al.* [2].

In this correspondence, the double smoothing algorithm is used to suppress noise components in images. The proposed double smoother employs the median [3] and Wiener [4] filters and is called the median and Wiener (MW) double smoother. Specifically, again referring to Fig. 1, F1 is a median filter and F2 is a Wiener filter. It is shown that the MW double smoother is an effective edge-preserving smoother which can outperform individual median and Wiener filters.

Two types of median filters called the standard median (SM) and recursive median (RM) filters are employed in MW double smoothing. These filters are defined as follows. Let $X(i, j)$ and $Y(i, j)$ be the input and output of a median filter, respectively. The output of the SM filter is given by $Y(i, j) = \text{Median} \{X(m, n) | (m, n) \in W(i, j)\}$ where $W(i, j)$ is a 2-D window centered at image coordinates (i, j) . The output of the RM filter is given by $Y(i, j) = \text{Median} \{Y(m_1, n_1), X(m_2, n_2) | (m_1, n_1) \in W_1(i, j), (m_2, n_2) \in W_2(i, j)\}$, where $W_1(i, j) \cup W_2(i, j) = W(i, j)$, $W_1(i, j) \cap W_2(i, j) = \emptyset$, and $W_1(i, j)$ encompasses all the points in $W(i, j)$ at which outputs have been calculated. For example, when the square window of span $2N + 1$ passes along the horizontal direction, $W(i, j) = \{(m, n) | i - N \leq m \leq i + N, j - N \leq n \leq j + N\}$ and $W_1(i, j) = \{(m_1, n_1) | i - N \leq m_1 \leq i - 1, j - N \leq n_1 \leq j + N\} \cup \{(m_1, n_1) | m_1 = i, j - N \leq n_1 \leq j - 1\}$. Throughout this correspondence, only the square window of span $2N + 1$ passing along the horizontal direction is considered. The double smoothers employing the SM and RM filters are called the standard median and Wiener (SMW) and the recursive median and Wiener (RMW) double smoothers, respectively.

In Section II, we shall show that the MW double smoother is suitable for smoothing a covariance stationary process which is a useful model of images [5]. The statistical properties of the MW double smoother are studied through computer simulation in Section III, and the MW double smoothers are applied to real images in Section IV.

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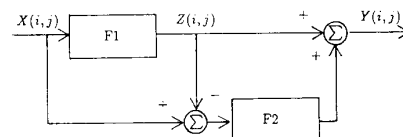


Fig. 1. The double smoothing algorithm.

II. MEDIAN AND WIENER DOUBLE SMOOTHING

Let us consider the problem of estimating the image $S(i, j)$ from the observation $X(i, j)$ described by $X(i, j) = S(i, j) + N(i, j)$, where the image $S(i, j)$ is assumed to be a covariance stationary process having space-varying, piecewise constant means but space-invariant covariances and $N(i, j)$ is assumed to be a zero mean independently and identically distributed (i.i.d.) random noise. In addition, we assume that the covariance of $S(i, j)$ and the variance of $N(i, j)$ are known. Under these assumptions, $X(i, j)$ can be converted into a zero mean WSS process by subtracting the corresponding mean at each image coordinate. The resulting image denoted by $X_0(i, j)$ is given by $X_0(i, j) = X(i, j) - E[X(i, j)] = S_0(i, j) + N(i, j)$, where $S_0(i, j) = S(i, j) - E[S(i, j)]$. Since $S_0(i, j)$ is a WSS process, the Wiener filter is effective in estimating $S_0(i, j)$ from $X_0(i, j)$. The output $\hat{S}_0(i, j)$ of Wiener filtering is given by

$$\hat{S}_0(i, j) = \sum_{m=-L}^L \sum_{n=-L}^L h(m, n) X_0(i - m, j - n), \quad (1)$$

where L is a positive integer and $\{h(m, n)\}$ is a $(2L + 1) \times (2L + 1)$ array satisfying the orthogonality principle, which is $E\{[S_0(i, j) - \hat{S}_0(i, j)]X_0(i - p, j - q)\} = 0$ for all integers p and q that range from $-L$ to L . The original image $S(i, j)$ is estimated by $\hat{S}(i, j) = \hat{S}_0(i, j) + E[S(i, j)]$. In practical situations, $E[S(i, j)]$ is not known and should be estimated. By replacing $E[S(i, j)]$ with its estimate, we get a realizable estimator given by $\hat{S}(i, j) = \hat{S}_0(i, j) + \hat{\mu}_s(i, j)$, where $\hat{\mu}_s(i, j)$ is an estimate of $E[S(i, j)]$. Note that this estimator for $S(i, j)$ is a double smoother shown in Fig. 1, where $Z(i, j) = \hat{\mu}_s(i, j)$ and F2 is the Wiener filter.

Since the mean is piecewise constant, the mean $E[S(i, j)]$ can be estimated from its neighboring samples having the same mean. If the outcomes of random variables having the same mean are given, a variety of estimation techniques can be applied. For example, the maximum likelihood estimator (MLE) can be used under the joint Gaussian assumption (here the MLE results in the weighted average of the samples where the weights are represented by a function of covariances). Since the location of those regions having constant mean is not known *a priori*, an estimator such as the MLE is inadequate for our purpose. In particular, in the neighborhood of a point where the mean undergoes a significant and sustained change in value, the performance of such an estimator is generally poor (regions where such changes occur are called edges). We note that some edge-preserving filters in [3], [6], such as median, and selective average filters, tend to estimate the mean value of the majority of random variables with the same mean within the neighborhood of each point, and thus they seem to be suitable for our purpose. Among them we apply median filters due to their simplicity in implementation and fine performance. This results in the MW double smoother. The output $\hat{S}(i, j)$ of the MW double smoother is given by

$$\hat{S}(i, j) = \sum_{m=-L}^L \sum_{n=-L}^L h(m, n) \{X(i - m, j - n) - M(i - m, j - n)\} + M(i, j), \quad (2)$$

where $h(m, n)$ is the point spread function of the Wiener filter in (1) and $M(i, j)$ is the output of either the SM or the RM filter.

The discussion presented above justifies the use of MW double smoothers for smoothing a covariance stationary process which is a useful model of images. Next, the behavior of the MW double smoother is studied statistically when the input is a covariance stationary process.

III. STATISTICAL PROPERTIES

In this section, we gain some insight into the statistical properties of the MW double smoother through computer simulation, and the results are compared to the properties of the median and Wiener filters.

A. Noise Suppression

We assume that the autocovariance function of $\{S_0(i, j)\}$ is given by $C_s(m, n) = \sigma_s^2 \rho^{(|m|+|n|)}$, where σ_s^2 is the signal variance and ρ , $0 < \rho < 1$, is the correlation coefficient [6]. As has been shown in [6], discrete random processes with the autocovariance function presented above can be generated by the equation

$$S_0(i, j) = \rho S_0(i-1, j) + \rho S_0(i, j-1) - \rho^2 S_0(i-1, j-1) + U(i, j), \quad (3)$$

where $U(i, j)$ is a set of uncorrelated random variables having zero mean and variance $\sigma_u^2 = (1 - \rho^2)^2 \sigma_s^2$. In this section, it is assumed that $U(i, j)$ is zero-mean i.i.d. Gaussian and $\sigma_u^2 = 1$.

Two sets of images, each of which encompasses 2000 Gaussian random fields of size 30×50 , $\{S_0(i, j) | -19 \leq i \leq 10, -19 \leq j \leq 30\}$, were generated by using (3). One set is associated with $\rho = 0.9$ and the other with $\rho = 0.5$. The noisy image $X(i, j)$ was generated by adding zero-mean i.i.d. Gaussian noise $N(i, j)$ to each $S_0(i, j)$. (We considered two different noise variances $\sigma_n^2 = 0.5$ and 1.)

A Wiener filter of size 5×5 was designed based on the statistics of $X(i, j)$. For this input, the point spread function of the Wiener filter depends only on ρ and σ_s^2/σ_n^2 , the latter of which is the signal-to-noise ratio (SNR). The designed Wiener filter, MW double smoothers employing the Wiener filter, and median filters of various size were applied to the noisy images. The normalized mean square errors (NMSE) at the pixel location $(0, 0)$, $E[Y(0, 0) - S_0(0, 0)]^2/\sigma_n^2$, where $Y(i, j)$ is the output of a filter, were empirically estimated from each set of images.

Table I summarizes the results. In each case, the NMSE's of SMW and RMW double smoothers approach the NMSE of the Wiener filter as the sizes of the employed median filters increase. In particular, the NMSE of the RMW double smoother is identical to that of the Wiener filter whenever the window is greater than or equal to 7×7 . On the other hand, the NMSE's of the SM and RM filters tend to increase as their window sizes increase. This happens because the SM and RM filters tend to estimate the mean value of the input signal instead of estimating the input signal value.

B. Edge Preservation

Images with noisy step edges were generated by adding a constant h to $X(i, j)$, whenever $j \geq 13$. We are mainly interested in edges with large h , say $h \geq 3(\sigma_s^2 + \sigma_n^2)$, because for such edges edge-preserving properties of filters are more noticeable. Throughout this subsection, h is set at six. The generated images were passed through the filters, and the expected values and NMSE's of the output along the horizontal path, $\{(0, j) | 8 \leq j \leq 17\}$ were estimated empirically. The results are shown in Fig. 2. It is seen that the SMW and RMW double smoothers preserved the edge better than did the other filters. The Wiener filter performed the worst when $\rho = 0.9$, but preserved the edge better than the SM and RM filters when $\rho = 0.5$, because the Wiener filter approaches an identity filter (no filtering) as ρ decreases.

The results in this section indicate that the MW double smoothers are excellent edge-preserving filters that can act like the Wiener filter in gradually varying portions of an image, and can preserve edges better than median filters.

TABLE I
(a) THE NMSE'S OF THE 5×5 WIENER FILTER. (b) THE NMSE'S OF SM AND RM FILTERS OF VARIOUS SIZE, AND THOSE OF SMW AND RMW DOUBLE SMOOTHERS WITH 5×5 WIENER FILTERING AND MEDIAN FILTERING OF VARIOUS SIZE

		SNR ρ	
		0.5	0.9
1		0.38	0.15
2		0.52	0.20

(a)

ρ	SNR	Median size					
		Filter	3X3	5X5	7X7	9X9	11X11
0.5	1	SM	0.67	0.76	0.84	0.89	0.92
		RM	0.68	0.90	0.96	0.95	0.95
		SMW	0.44	0.39	0.39	0.38	0.38
		RMW	0.40	0.39	0.38	0.38	0.38
	2	SM	1.16	1.46	1.64	1.78	1.80
		RM	1.24	1.74	1.90	1.90	1.90
		SMW	0.56	0.53	0.52	0.52	0.52
		RMW	0.54	0.52	0.52	0.52	0.52
0.9	1	SM	0.25	0.23	0.26	0.30	0.34
		RM	0.24	0.41	0.55	0.57	0.57
		SMW	0.23	0.19	0.16	0.15	0.15
		RMW	0.18	0.16	0.15	0.15	0.15
	2	SM	0.34	0.38	0.48	0.58	0.66
		RM	0.36	0.66	0.94	1.06	1.06
		SMW	0.27	0.24	0.21	0.20	0.20
		RMW	0.24	0.22	0.20	0.20	0.20

(b)

IV. EXPERIMENTAL RESULTS

The performance of the filters discussed so far is evaluated by applying them to noisy images degraded by additive white noise and then by comparing their respective results. The image under consideration consists of 256×256 pixels with eight bits of resolution. In order to quantitatively compare the performance of filters, the normalized average square errors (NASE) between the original image and the filtered images were evaluated. The NASE is given by

NASE

$$= \frac{\sum_{i=0}^{255} \sum_{j=0}^{255} [Y(i, j) - S(i, j)]^2}{\sum_{i=0}^{255} \sum_{j=0}^{255} [X(i, j) - S(i, j)]^2}, \quad (4)$$

where $S(i, j)$, $X(i, j)$, and $Y(i, j)$ are the original, noisy input, and filtered images, respectively.

The original image is shown in Fig. 3(a). These noisy images were generated by adding zero-mean i.i.d. Gaussian noise of variance 100, 200, and 400 to the original image. (Fig. 3(b) shows the noisy image with noise variance 200.) The noisy images were passed through various filters. In Wiener and MW double smoothing, the original image was assumed to be a covariance stationary process with the covariance function $C_s(m, n) = 0.9^{(|m|+|n|)} \sigma_s^2$, where σ_s^2 was set at 200 because the signal variance estimated from a window of 900 pixels in a flat area of the original image was about 200. In the following, we first compare the NASE's of the filters, and then visually compare some of the filtered images.

Table II shows the NASE's. We can see that MW double smoothers outperformed the others. Between SMW and RMW double smoothers, the latter is preferred to the former. The MW double smoothers employing the 3×3 median filter performed better when $\sigma_n^2 = 100$, but worse when $\sigma_n^2 = 400$ than those employing the 5×5 median filter. This is not surprising since the median filter with

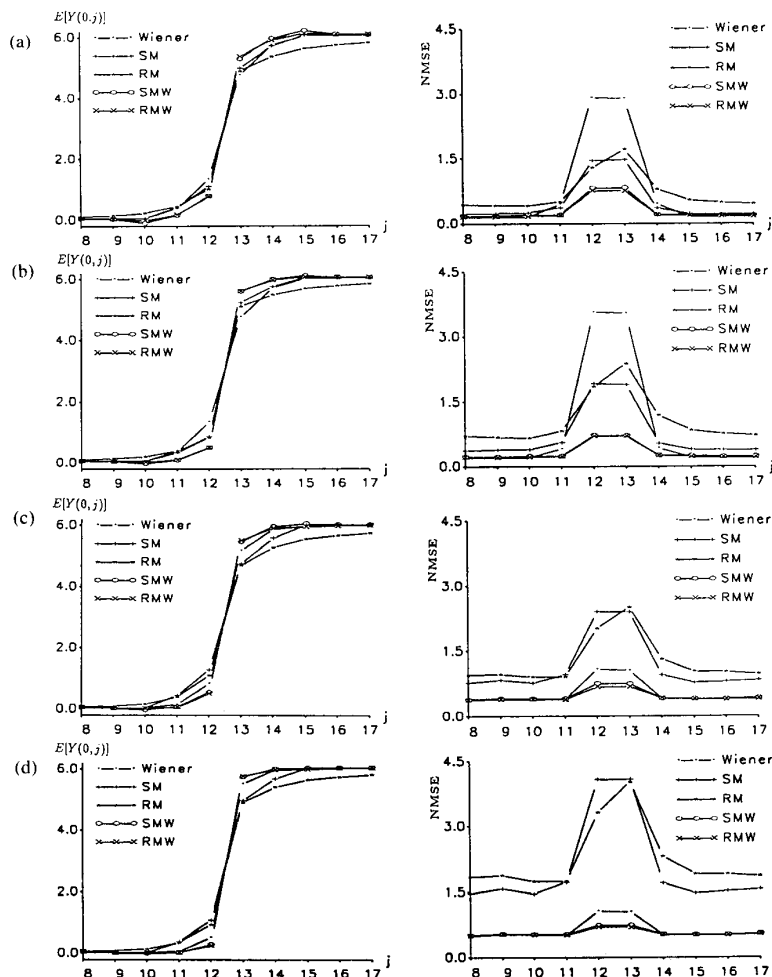


Fig. 2. The output expected values and NMSE's associated with the Wiener, SM, RM filters, the SMW and RMW double smoothers with 5×5 windows. (a) $\rho = 0.9$, SNR = 1, (b) $\rho = 0.9$, SNR = 2, (c) $\rho = 0.5$, SNR = 1, (d) $\rho = 0.5$, SNR = 2.

a larger window size suppresses more noise at the expense of blurring [7].

Fig. 3(c)–(e) shows some results of filtering of the image in Fig. 3(b). The MW double smoother exhibits superior overall performance compared to the Wiener and median filters.

V. CONCLUSION

The MW double smoothers which employ the median and Wiener filters have been introduced in restoring images corrupted by additive white noise. The statistical properties of the MW double smoothers were studied for 2-D covariance stationary inputs. It was shown that the MW double smoother can act like the Wiener filter in reducing additive white Gaussian noise, and preserve edges better than the median filter. It should be noted, however, that MW double smoothing is not effective in suppressing impulsive-type noise, in contrast to median filtering. The MW double smoother is a nonadaptive edge-preserving filter that can outperform some other nonadaptive edge-preserving filters, such as median filters, in suppressing Gaussian-type noise. Finally, more work needs to be done in designing MW double smoothers, particularly on the selection of proper window size of the median filter employed in double smoothing.

TABLE II
THE NASE'S BETWEEN THE ORIGINAL IMAGE AND THE SMOOTHED IMAGES

Filter	Size	σ_n^2		
		100	200	400
SM	3X3	0.47	0.33	0.26
	5X5	0.78	0.45	0.27
RM	3X3	0.62	0.39	0.26
	5X5	1.50	0.88	0.56
SMW	SM: 3X3 Wiener: 5X5	0.39	0.30	0.24
	SM: 5X5 Wiener: 5X5	0.41	0.29	0.20
RMW	RM: 3X3 Wiener: 5X5	0.39	0.27	0.20
	RM: 5X5 Wiener: 5X5	0.43	0.29	0.19
Wiener	5X5	0.48	0.35	0.23

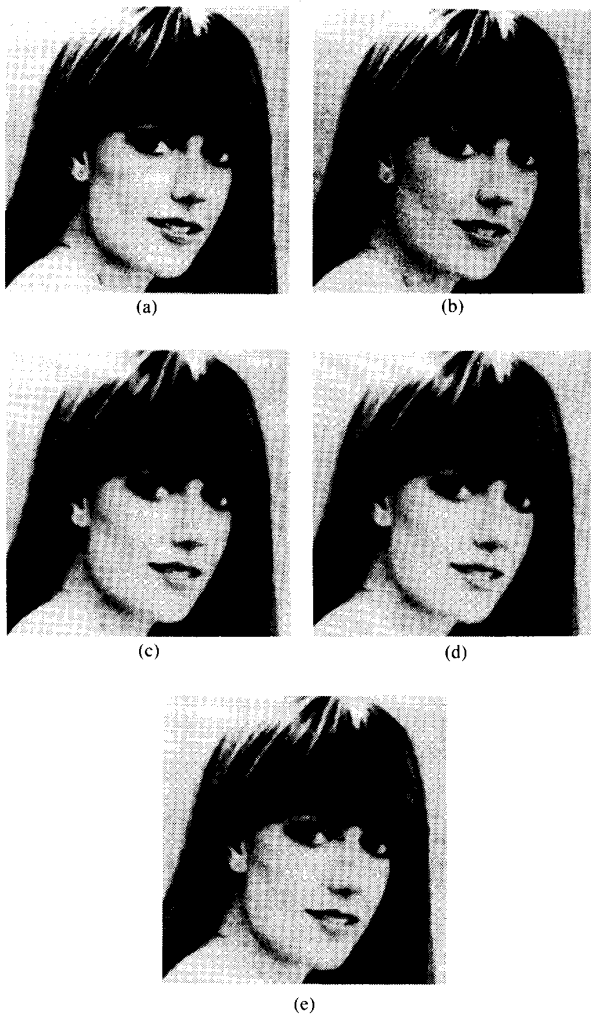


Fig. 3. (a) The original image. (b) The noisy image corrupted by additive white noise, $N(0, 200)$. (c) SM filtered (5×5). (d) Wiener filtered (5×5). (e) SMW double smoothed (SM: 5×5 , Wiener: 5×5).

REFERENCES

- [1] J. W. Tukey, *Exploratory Data Analysis*. Reading, MA: Addison-Wesley, 1977 [preliminary edition, 1971].
- [2] L. R. Rabiner, M. R. Sambur, and C. E. Schmidt, "Applications of a nonlinear smoothing algorithm to speech processing," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-23, pp. 552-557, Dec. 1975.
- [3] T. A. Nides and N. C. Gallagher Jr., "Median filters: Some modifications and their properties," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-30, pp. 739-746, Oct. 1982.
- [4] C. F. N. Cowan, "Optimum estimation techniques," in *Adaptive Filters*, C. F. N. Cowan and P. M. Grant, Eds. Englewood Cliffs, NJ: Prentice-Hall, 1985.
- [5] B. R. Hunt, "Nonstationary statistical image models (and their application to image data compression)," *Comput., Graphics, Image Processing*, vol. 12, pp. 173-186, 1980.
- [6] A. Rosenfeld and A. C. Kak, *Digital Picture Processing*, vol. 1, 2nd ed. New York: Academic, 1982.
- [7] G. R. Arce and R. E. Foster, "Multilevel median filters," in *Proc. 25th Annu. Allerton Conf. Commun., Contr. Comput.*, Monticello, IL, Sept. 30-Oct. 2, 1987, pp. 753-762.

Variable Parameter Window Families for Digital Spectral Analysis

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Abstract—In this correspondence, two different window function families, namely, the first-order Bessel (I_1 -cosh) family and raised-cosine family, which have variable parameters and hence make them flexible in digital spectrum analysis applications, have been considered. Closed-form expressions have been derived/computed, which facilitate the tradeoffs between record length, spectral resolution, leakage suppression, bandwidth, etc. Simple expressions relating to mainlobe width and maximum sidelobe level have been given for the two families considered. The results obtained have been compared to that obtained by Kaiser and Schafer in the case of zeroth-order Bessel (I_0 -sinh) family.

I. INTRODUCTION

Spectral analysis has received considerable attention in the past [1]–[4], especially with the advent of the fast Fourier transform [5]. It is well known that the application of fast Fourier transform (FFT) to a finite length data gives rise to leakage [6]. Thus, while estimating power spectra, via FFT, the input frequency components that are not integer multiples of the reciprocal of the sample length tend to leak out of their expected positions and thus yield nonzero coefficients over the entire spectrum of interest. This spreading of energy is commonly referred to as "frequency leakage." The energy spread throughout the spectrum due to leakage can mask the output of desired low amplitude spectral lines, thus giving rise to an incorrect discrete spectra estimate. It has been established that by the application of appropriate windows, the problem of leakage can significantly be reduced. Therefore, the selection of good data windows in spectral estimation of stationary random processes is an important consideration and has received much attention [1], [2], [6]–[8]. Rigorous attempts have been made in establishing good window functions having better sidelobe behavior [9], [10]. However, most of the window functions considered have no variable parameter in their design equations, i.e., the windows are fixed and cannot be altered, as, for example, in the Hamming and Hanning windows.

A class of windows that maximizes the energy into some selected frequency interval to the total energy is derived by Slepian, Pollak, and Landau [11], [12] and is called prolate-spheroidal wave functions. They are ideally suited for use as spectral windows. However, they are very difficult to compute. Kaiser has discovered two families of window functions that are relatively simple approximations to these complicated functions [13]. They are: (a) modified zeroth-order (I_0 -sinh) family and (b) modified first-order Bessel (I_1 -cosh) family. Prabhu *et al.* have introduced raised-cosine family, which also has a variable parameter [14].

The Kaiser-Bessel window families have been used extensively in digital signal processing applications, since their introduction. This is because of the fact that by a careful choice of one parameter, one can conveniently trade its desirable mainlobe (resolution) and sidelobe rejection properties (leakage). In addition, these window functions can be generated easily and also provide a close approximation to the prolate-spheroidal wave functions. The raised-cosine family of windows are still simpler, similar to that of a Hamming window.

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