Some Statistical Properties of Alpha-Trimmed Mean and Standard Type *M* Filters

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***Abstract*—The statistical properties of two classes of filters generalizing the median filter, the *L* filter, focusing on the alpha-trimmed mean (α-TM) filter, and the standard type *M* (STM) filter are investigated. In particular, results are developed to quantify the white noise suppression and edge preservation characteristics of the filters by considering their output sequence error statistics. It is shown that the α-TM and STM filters can perform better than the running mean and median filters in white noise suppression, while they can be designed to be comparable to the median filter in edge preservation in the presence of noise.**

I. Introduction

THE median filter is a nonlinear filter in which the output signal value at each point is taken as the median of the input signal values in a finite neighborhood (window) centered about that point. The nonlinear median filter yields good edge preservation and impulsive noise suppression characteristics which are quite useful in smoothing applications [1]–[5].

To provide a more powerful set of techniques, median filters have recently been *generalized* to obtain improved rejection of additive Gaussian noise while maintaining good impulsive noise suppression and edge preservation [6]–[9]. In this paper we consider the statistical properties of two useful types of one-dimensional generalized median filters, called the *α-trimmed mean* filter (α-TM filter) and the *standard type M* filter (STM filter). These generalized median filters are defined as follows.

The α-TM filter is a special case of the more general *L* filter (order statistic filter in [6]). The output Y_k of an *L* filter of window size $W = 2N + 1$ at time index k for an input signal $\{X_k\}$ is given by

$$Y_k = \sum_{j=1}^{W} A_j X_{(j)}^k \tag{1}$$

where $X_{(j)}^k$ is the jth smallest sample, the jth-order statistic, among the W samples inside the window centered at k. A particularly simple choice for the A_j coefficients yields an α-TM filter, with output given by

$$Y_k = \sum_{j=T+1}^{W-T} \frac{1}{2(N-T)+1} X_{(j)}^k. \tag{2}$$

Here T is the largest integer which is less than or equal to αW, with α being constrained by $0 \le \alpha \le 0.5$. When $\alpha = 0$, the α-TM filter becomes the running mean filter; when $\alpha = 0.5$ it becomes the median filter.

The output Y_k of an STM filter solves the equation

$$\sum_{i=k-N}^{k+N} \psi(X_i - Y_k) = 0 \tag{3}$$

where

$$\psi(x) = \begin{cases} 1, & x > p \\ x/p, & |x| \le p \\ -1, & x < -p \end{cases} \tag{4}$$

with p some positive constant. The STM filter approaches the running mean filter or the median filter as p approaches infinity or zero, respectively.

The statistical properties of L, α-TM, and STM filters in suppression of additive, symmetric, white noise corrupting two input signal classes, the constant signal and the edge of height H, will be considered. The input sample at time k is taken as

$$X_k = S_k + N_k \tag{5}$$

where

$$S_k = \begin{cases} S, & k \le 0 \\ S + H, & k > 0 \end{cases} \tag{6}$$

with S and H constant. Without loss of generality, S will be taken as zero since the filters under consideration are readily seen to be translation invariant. Note that the constant signal class is the $H = 0$ subclass within the edge signal class.

Four performance measures on filter output at $k = 0$ (that is, at the noisy edge) are considered; with $S \equiv 0$ and $E[\cdot]$ denoting expected value, they are given by

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Mean:

$$\mu = E[Y_0], \tag{7}$$

Mean Absolute Error (MAE):

$$\Delta = E[|Y_0 - S_0|] = E[|Y_0|], \tag{8}$$

Mean Square Error (MSE):

$$m_2 = E[(Y_0 - S_0)^2] = E[Y_0^2], \tag{9}$$

Variance (VAR):

$$\sigma^2 = E[(Y_0 - \mu)^2] = m_2 - \mu^2. \tag{10}$$

The mean, MSE, and VAR have been previously obtained for median filters [10], [11]. In order to evaluate these measures in Section III for the generalized median filters and signal cases under consideration, some statistical results will be derived in Section II. For the STM filter, we derive expressions for the output distribution for (A) the white noise case and (B) the noisy edge case, which permit direct evaluation of the four measures for both input cases. Since for the L filter, μ and m_2 may be computed as

$$\mu = \sum_{j=1}^{W} A_j E[X_{(j)}^0] \tag{11}$$

and

$$m_2 = \sum_{r=1}^{W} \sum_{s=1}^{W} A_r A_s E[X_{(r)}^0 X_{(s)}^0], \tag{12}$$

we derive (C) the joint distribution of pairs of order statistics for the noisy edge case, which also provides the univariate distribution of an order statistic. The MAE of the L filter output does not simplify as in (11) and (12), rather it is given by

$$\Delta = E\left[\left|\sum_{j=1}^{W} A_j X_{(j)}^0\right|\right] \tag{13}$$

which may be evaluated using the L filter univariate output distribution which may be derived, in principle, from the multivariate distribution of the order statistics. As this is both lengthy and cumbersome and the MAE results do not differ significantly in character from the MSE results, the MAE will be developed only for STM filter cases.

II. Statistical Derivations

A. STM Filter Output Univariate Distribution for an IID Input

Define $Z_i \equiv X_i - y$ so $F_Z(z) = F_X(z + y)$, where $F_X(x) \equiv P(X \le x)$, the distribution function of X; then the STM filter output univariate distribution for an IID input is given by

$$\begin{aligned} P(Y \le y) &= P\left(\sum_{i=1}^{W} \psi(Z_i) \le 0\right) \\ &= \sum_{r=0}^{W} \sum_{\lambda=0}^{W-r} \binom{W}{r} \binom{W-r}{\lambda} \cdot P\left(\sum_{i=1}^{W} \psi(Z_i) \le 0 \,\middle|\, A\right) \\ &\quad \cdot [F_Z(p) - F_Z(-p)]^r \\ &\quad \cdot [1 - F_Z(p)]^\lambda [F_Z(-p)]^{W-(r+\lambda)} \\ &= \sum_{\lambda=0}^{N} \binom{W}{\lambda} [1 - F_Z(p)]^\lambda [F_Z(-p)]^{W-\lambda} \\ &\quad + \sum_{r=1}^{W} \sum_{\lambda=0}^{W-r} \binom{W}{r} \binom{W-r}{\lambda} P_{r,\lambda} \\ &\quad \cdot [1 - F_Z(p)]^\lambda [F_Z(-p)]^{W-(r+\lambda)} \end{aligned} \tag{14}$$

where

$$\begin{aligned} A = \{(Z_1, Z_2, \cdots, Z_W) \\ \text{with } |Z_i| \le p \quad & \text{for } i = 1, \cdots, r \\ \text{and } Z_i \ge p \quad & \text{for } i = r+1, \cdots, \lambda + r \\ \text{and } Z_i < -p \quad & \text{for } i = \lambda + r + 1, \cdots, W\} \end{aligned}$$

and

$$P_{r,\lambda} = P\left(\sum_{i=1}^{r} Z_i \le C \quad \text{and } |Z_i| \le p \quad \text{for } i = 1, \cdots, r\right)$$

$$\text{with } C = (W - r - 2\lambda)p.$$

The first term in (14) corresponds to the case $r = 0$. The quantity $P_{r,\lambda}$ can be obtained by using the joint distribution of the Z_i's [9].

We can readily obtain the output univariate distribution for the median and running mean filters as special cases of (14) with $p \to 0$ and $p \to \infty$, respectively. For $p \to 0$, $P_{r,\lambda} \to 0$, leaving only the $r = 0$ summation term which may be recognized, by substituting $F_Z(z) = F_X(z + y)$, as the well-known result for the sample median when the input is IID. For $p \to \infty$, only the terms corresponding to $r = W$ and $\lambda = 0$ need be considered, yielding simply $P_{W,0}$ with $C = 0$, which is clearly the running mean filter output distribution.

B. STM Filter Output Distribution for Independently Distributed Input

Since the noisy edge is not an IID sequence, here we generalize the univariate distribution given by (14) for this non-IID input case. We first consider the distribution of $\psi(Z_i)$ and then of the summation of the $\psi(Z_i)$, where $Z_i \equiv X_i - y$. Denoting the distribution of $\psi(Z_i)$ by $F_\psi(u, i)$, we have

$$F_\psi(u, i) = P(\psi(Z_i) \le u) = \begin{cases} 0, & u < -p \\ F_{Z_i}(u), & |u| \le p \\ 1, & u > p \end{cases} \tag{15}$$

where $F_{Z_i}(u)$ is the univariate distribution function of Z_i. Then the STM filter output distribution is given by

$$P(Y \le y) = P\left(\sum_{i=1}^{W} \psi(Z_i) \le 0\right) = \int_{-\infty}^{0}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} \cdot f_\psi(u_1 - u_2 - \cdots - u_W, i) \cdot f_\psi(u_2, i)\cdots f_\psi(u_W, i) \cdot du_W \cdots du_2 du_1 \tag{16}$$

since the $\psi(Z_i)$'s are independent, where $f_\psi(u, i) = dF_\psi(u, i)/du$ is the probability density function of $\psi(Z_i)$. This convolution of probability density functions provides a general expression for the output univariate distribution of an STM filter when the input samples are independent. For the noisy edge input case, there are exactly two distinct distributions each corresponding to one side of the edge. In our examinations, (16) has been evaluated numerically by transforming to the characteristic functions of the two distributions, multiplying together powers of the characteristic functions, and inverting.

C. Joint and Univariate Distributions of Order Statistics

We now turn to derive the joint distribution of order statistics of a sequence of independent random variables for which each random variable is known to have one of two possible symmetric univariate distributions. Consider that we have m independent random variables $X_1, X_2, \cdots, X_m$ in which ω of them have cumulative distribution function $F_1(x)$ and the rest have cdf $F_2(x)$. Specifically, for our case, $m \equiv W$, $\omega \equiv N + 1$, $F_1(x)$ is the noise cdf, and $F_2(x) \equiv F_1(x - H)$. The joint distribution of order statistics, $P(X_{(r)} \le u, X_{(s)} \le v)$, for $r > s$ can be obtained by symmetry from the results for $r < s$ which is considered now in two cases.

i) For $u < v$,

$$\begin{aligned} &P(X_{(r)} \le u, X_{(s)} \le v) \\ &= P(\text{at least } r\ X_i\text{'s are} \le u \text{ and at least } s\ X_i\text{'s are} \le v) \\ &= \sum_i \sum_j P(\text{exactly } i\ X_i\text{'s} \le u \text{ and exactly } j\ X_i\text{'s} > u \text{ and} \le v) \\ &= \sum_i \sum_j \sum_\lambda \sum_n P(\text{exactly } \lambda\ X_i\text{'s with cdf } F_1(x) \le u, \\ &\qquad \text{exactly } i - \lambda\ X_i\text{'s with cdf } F_2(x) \le u, \\ &\qquad \text{exactly } n\ X_i\text{'s with cdf } F_1(x) > u \text{ and} \le v, \\ &\qquad \text{exactly } j - n\ X_i\text{'s with cdf } F_2(x) > u \text{ and} \le v) \\ &= \sum_i \sum_j \sum_\lambda \sum_n \frac{\omega!}{\lambda!\,n!\,(\omega - \lambda - n)!} F_1^\lambda(u)[F_1(v) - F_1(u)]^n [1 - F_1(v)]^{\omega - \lambda - n} \\ &\quad \cdot \frac{(m - \omega)!}{(i - \lambda)!\,(j - n)!\,[m - \omega - (i - \lambda) - (j - n)]!} \\ &\quad \cdot F_2^{i-\lambda}(u)[F_2(v) - F_2(u)]^{j-n} [1 - F_2(v)]^{m - \omega - (i - \lambda) - (j - n)} \end{aligned} \tag{17}$$

where the summation limits are $r \le i \le m$, $\max(0, s - i) \le j \le (m - i)$, $\max(0, i - m + \omega) \le \lambda \le \min(i, \omega)$, and $\max(0, i - \lambda + j - m + \omega) \le n \le \min(j, \omega - \lambda)$.

ii) For $u \ge v$, the inequality $X_{(s)} \le v$ implies $X_{(r)} \le u$, so that

$$\begin{aligned} &P(X_{(r)} \le u, X_{(s)} \le v) \\ &= P(X_{(s)} \le v) \\ &= \sum_i \sum_\lambda \frac{\omega!}{\lambda!(\omega - \lambda)!} F_1^\lambda(v)[1 - F_1(v)]^{\omega - \lambda} \\ &\quad \cdot \frac{(m - \omega)!}{(i - \lambda)!(m - \omega - i + \lambda)!} \\ &\quad \cdot F_2^{i-\lambda}(v)[1 - F_2(v)]^{m - \omega - i + \lambda} \end{aligned} \tag{18}$$

for limits of summation $s \le i \le m$ and $\max(0, i - m + \omega) \le \lambda \le \min(i, \omega)$. This also is the univariate distribution of the order statistic $X_{(s)}$ which provides the needed result for the $r = s$ case.

III. α-TM and STM Filter Performance

The results developed in Section II are now applied to the characterization of α-TM and STM filter performance by numerically evaluating the output performance measures for both white noise and noisy edge input cases as set forth in Section I. In general, the noise is taken as zero mean, unit variance noise. Implicitly, edge height H and STM filter p parameter are considered normalized relative to the noise standard deviation, as are also the output measures, except the output MSE and VAR, which are accordingly normalized by the noise variance. First, the white noise input is considered for the Gaussian and more impulsive double exponential noise distributions. Second, the output measures are evaluated at an edge of height H in Gaussian noise. Finally, an overall consideration is given to filter and filter parameter selection.

For white noise input with symmetric distribution, the α-TM and STM filter outputs are unbiased, that is, the mean output error is zero. Fig. 1 plots output MSE for STM filters of different p parameters for $W = 3$ and $W = 5$ window sizes. For *Gaussian noise* suppression, the optimal MSE of $1/W$ is achieved by the running mean filter, while the median filter exhibits significantly larger MSE of about $(1/(W + (\pi/2) - 1))\ \pi/2$, about 57 percent more than the running mean for large W [10]. This may be expected since the sample mean is the maximum likelihood estimator of the mean for Gaussian noise. Hence, for a variety of error criteria, the mean filter will be optimal for large W. For MSE criteria, it is optimal for all W. The STM filter shows substantially the same performance as the running mean filter for $p > 1.5$.

For more impulsive *double exponential noise* suppression, the maximum likelihood estimator is the sample median. For the finite windows of sizes $W = 3$ and $W = 5$, the STM filter with $p \simeq 0.8$ yields the least output MSE, while the running mean filter again achieves an MSE of $1/W$ and the median filter achieves an MSE of about $1/(2W - 1)$, about 50 percent less than the running mean filter for large W [10]. The $W = 3$ and $W = 5$ curves show that as the window size increases, the optimal p parameter selection moves toward that of the median filter ($p \rightarrow 0$).

For comparison, Fig. 2 plots the same cases for the MAE performance criteria. The character of the results is the same, with the notable change that for double exponential noise, the median filter is optimal for all W. These results suggest that a selection of $p = 1.5$ for an STM filter yields noise suppression that is fairly robust over variations in noise distribution, from Gaussian to the more impulsive double exponential, and over both MAE and MSE performance measures at a slight penalty to optimal performance. When the noise is more impulsive and MAE performance more emphasized, the optimal selection of the STM filter p parameter moves toward that of the median filter, $p \rightarrow 0$. When the noise is more Gaussian and the MSE criteria more emphasized, the optimal STM p selection moves toward that of the running mean filter, $p \rightarrow \infty$.

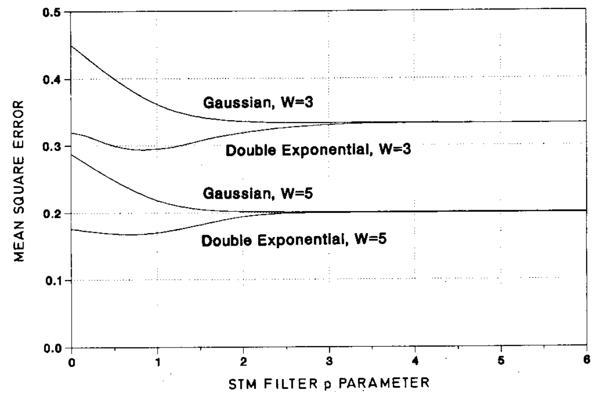

Fig. 1. STM filter output MSE with $W = 3$ and $W = 5$ for zero mean, unit variance Gaussian, and double exponential white noise inputs.

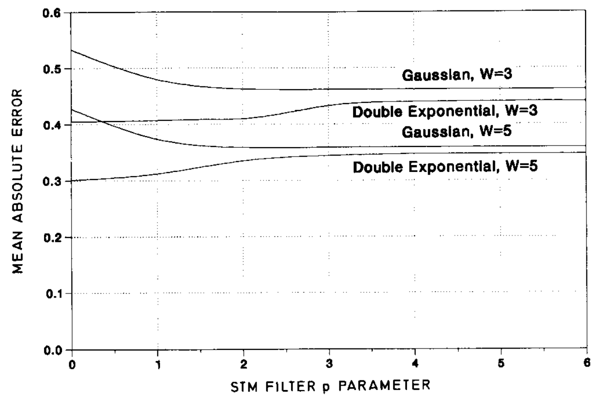

Fig. 2. STM filter output MAE with $W = 3$ and $W = 5$ for zero mean, unit variance Gaussian, and double exponential white noise inputs.

Similar quantitative comparisons for α-TM filters using an MSE criterion have been previously presented [6]–[8], [12], [13], which indicate behavior similar to that of the STM filters and hence are omitted here for brevity.

Figs. 3–8 examine α-TM and STM filters operating upon an edge of height H corrupted by additive white Gaussian noise. The mean, standard deviation, rmse (square root of the MSE), and MAE of the output at the edge are considered as performance measures. While both the output mean and the standard deviation contribute to the rmse in this case, they can be thought of as measuring mainly the filter edge preservation and white noise suppression characteristics, respectively. The reason is that the means of the α-TM and STM filtered sequences are the same as that of the input sequence when the input is IID (i.e., $H = 0$, no edge case), while the output standard deviations are zero if the input is a noise-free step edge. Thus, the rmse measure provides a composite measure of edge preservation and noise suppression.

Figs. 3 and 4 give the output mean, standard deviation, and rmse performance measures for the window size $W = 5$ α-TM filters versus input edge height H. Specifically, the median filter, α-TM filter with $\alpha = 0.2$ or $T = 1$, and

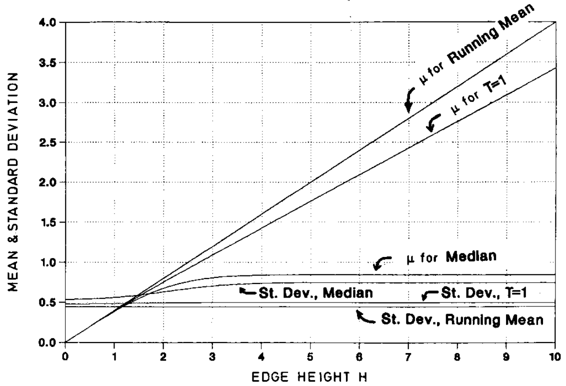

Fig. 3. α-TM filter output mean and standard deviation with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

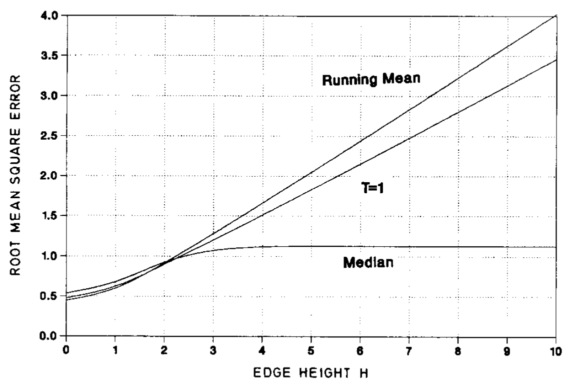

Fig. 4. α-TM filter output rmse with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

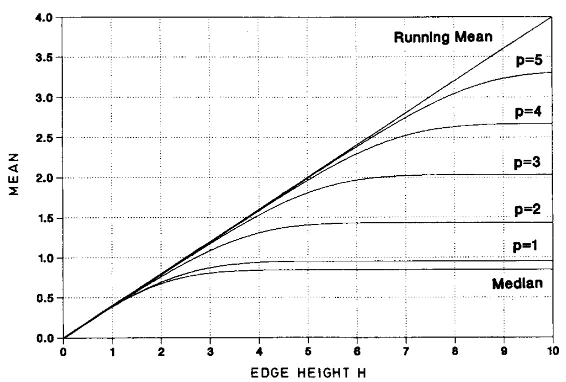

Fig. 5. STM filter output mean with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

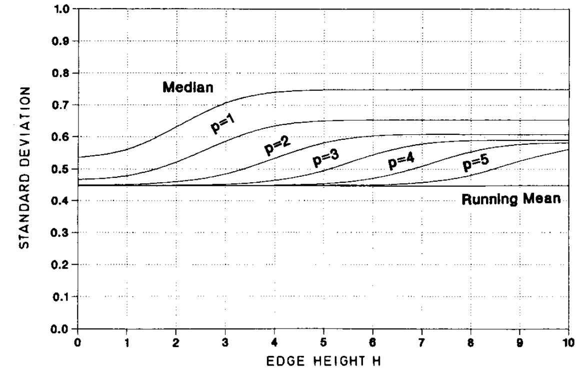

Fig. 6. STM filter output standard deviation with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

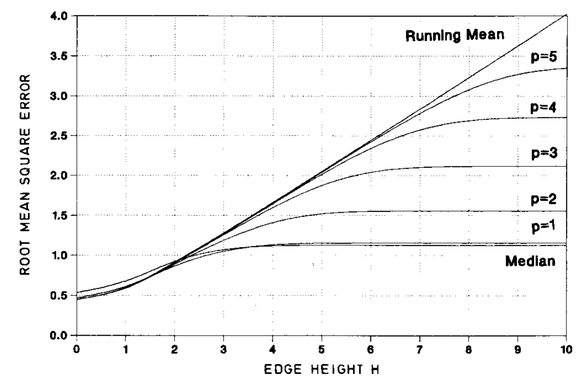

Fig. 7. STM filter output rmse with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

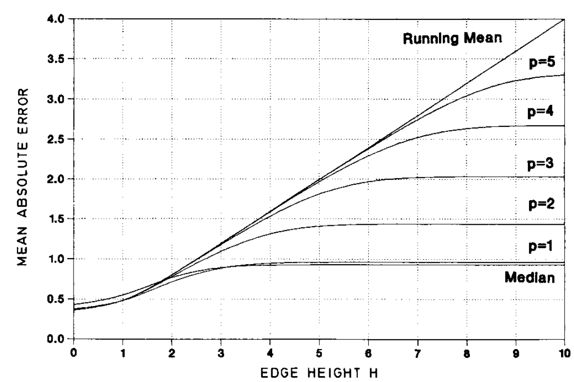

Fig. 8. STM filter output MAE with $W = 5$ for edge of height H in additive, white, zero mean, unit variance Gaussian noise.

running mean filter are shown. It may be seen that the output mean of only the median filter is bounded as $H \to \infty$ and is less than or equal to those of the other two filters, while its standard deviation is always greater than those of the other two filters for all edge heights. For large H relative to the noise standard deviation, the α-TM filter output mean is close to $(N - T)H/(W - 2T)$. As an α-TM filter with maximum trimming of $T = N$ samples from above and below the sample median, the median filter maximizes its resistance to outlying sample values. In the case of large edges with respect to the noise standard deviation, the median filter can reject influence of the samples from the opposing portion of the edge as outliers, while any less trimmed α-TM filter can resist only some outliers (except the running mean filter which resists none) and must average the remaining outliers, yielding unbounded output mean as $H \to \infty$. This outlier resistance allows the median filter to limit, but not totally eliminate, the influence of the samples from across the edge, as the median filter output for large H will be taken as the extreme of the $N + 1$ noisy samples to one side of the edge. The standard deviation results correspondingly indicate the enhanced noise suppression due to the increased averaging as the trimming T is reduced toward the running mean filter case of $T = 0$. The Fig. 4 results confirm that minimizing rmse emphasizes noise suppression for small edges, favoring the running mean filter, and edge retention for large edges, favoring the median filter. The α-TM filters for $0 < T < N$ strictly compromise mean and median filter performance characteristics.

Figs. 5–8 plot the MAE in addition to the above measures for window size $W = 5$ STM filters with several p parameter values versus input edge height H. It may be seen that the output means and standard deviations of the STM filters lie between those of the running mean filter ($p \to \infty$) and the median filter ($p \to 0$). Thus, as with α-TM filters, the STM filters can correspondingly compromise between the edge retention and noise suppression characteristics of the mean and median. However, since the STM filter can tolerate as many extreme samples above and below the sample median as can the median filter, it can achieve bounded output mean as $H \to \infty$, as does the median filter, and as cannot the α-TM filters (excluding the median filter). For a noise-free edge, the STM filter output is independent of the height of the edge for edges larger than $(1 + (N/(N + 1)))\, p$ [8]. In this case, the values across the edge are always treated as outliers. For noisy edges, the output performance measures for STM filters saturate to virtually constant values for

$$H \geq \left(1 + \left(N/(N + 1)\right)\right) p + 3\sigma_{\text{noise}}. \quad (19)$$

The additional $3\sigma_{\text{noise}}$ term reflects a requirement for larger edge height in the presence of noise than for the noise-free case. This condition generalizes the condition $H > 3\sigma_{\text{noise}}$ which was noted by Justusson for median filtering ($p \to 0$) [10]. Note that the STM filter output mean shows improved edge retention (lower mean error) performance in comparison to the running mean filter when $H > p$. However, Gaussian noise suppression is poorer in comparison to the running mean filter when $H > p$, as exhibited by the STM filter output standard deviations deviating noticeably from those of the running mean filter for $H > p$. Hence, we observe that the tradeoff between noise suppression and edge retention in selection of the p parameter occurs in the vicinity of $H = 1$ for Gaussian noise with unity variance. In Fig. 7, the rmse results for STM filters do not strictly compromise between those of the median and running mean filters. Observe that the STM filter with $p \simeq 1$ exhibits smaller rmse than other filters when $H \simeq 2$. The overall performance of the STM filter with $p \simeq 1$ may be said to be superior to those of the other filters in the sense that its rmse is very close to that of the running mean filter when $H \simeq 0$, and to that of the median filter for $H > 3$, while the rmse is smaller around $H \simeq 2$. That is, for $p = 1$, the STM filter exhibits fairly robust output rmse performance over small to large edges. STM filter output performance exhibits similar performance under an MAE criterion, as shown in Fig. 8.

Now let us consider the results collectively to compare the α-TM and STM filters along with the special case running mean and median filters. For small edges relative to the noise standard deviation, little can be done to extract the two distinct levels on each side of the edge; the filters must effectively ignore the edge and attempt noise suppression. The best Gaussian noise suppression is achieved not surprisingly by the running mean filter, while the median filter exhibits the worst suppression, and while the α-TM and STM filters compromise performance. For more impulsive noise, the α-TM, STM, and median filters, with their resistance to extremal or outlying input samples, can perform better. The median filter performs best for double exponential noise under the mean absolute error criteria and maximizes resistance to outliers.

For larger edges, the running mean filter severely corrupts the edge since the mean output error increases with edge height, while the median and STM filters exhibit bounded mean output error. As the α-TM filters strictly compromise between the median and running mean filter performance, they also exhibit unbounded error. The STM filters can sometimes exhibit better performance than either the running mean or median filters, indicating them to be more than a strict compromise between the performance characteristics of the latter filters. In general, to effectively filter in the presence of large edges thus requires the nonlinear behavior of the α-TM filter or, better, that of the median or STM filters.

Both the α-TM and STM filters provide a free parameter which may be utilized to adapt the filter performance more toward the Gaussian noise suppression of the running mean filter or more toward the edge retention of the median filter. The STM filter with $p \simeq 1$ exhibits particularly good, and in some cases best, behavior for several different output performance measures over the continuum of possible edge heights.

These results provide the nonlinear filter designer with an effective guide for selecting from these generalized median filters, and tuning their output performance for desired noise suppression and edge retention under various criteria, by considering the expected edge heights and noise characteristics of the input.

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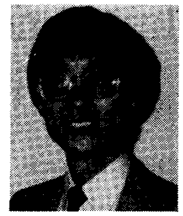

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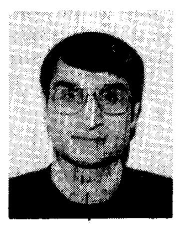

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