On the Use of Sigmoid Functions for Multistage Detection in Asynchronous CDMA Systems

Jeong Hoon Ko, Jung Suk Joo, and Yong Hoon Lee

***Abstract*—We consider the use of sigmoid functions for multistage detection in asynchronous code-division multiple-access (CDMA) systems. The sigmoid decision function for each stage of multistage detection is derived under the assumption that the *residual noise* which remains after the multiple-access interference (MAI) cancellation at a stage is Gaussian. It is suggested that the sigmoid function should be adjusted depending on the residual noise power at each stage. In addition, multilevel quantizers that best approximate the sigmoid function are designed. Computer simulation results demonstrate that multistage detectors employing these soft-decision functions perform considerably better than those with hard decision.**

***Index Terms*—Asynchronous CDMA, multiuser detection, sigmoid function.**

I. Introduction

MULTIUSER detection has been recognized as a powerful tool for demodulating signals in direct-sequence code-division multiple-access (CDMA) systems [1]–[10]. By systematically removing multiple-access interference (MAI), multiuser detection can achieve significant performance gains over conventional single-user detection which neglects the presence of MAI. A comprehensive tutorial on multiuser detection can be found in [8]–[10].

Multistage detection [2]–[6] is an effective multiuser detection technique, which is based on the simple idea of successive cancellation. For asynchronous CDMA systems, which is our main concern, its first stage consists of a bank of conventional detectors; in each of the successive stages, the previous decisions are assumed to be correct and the corresponding MAI components are eliminated from the received waveform. Multistage detectors in [2]–[5] are based on hard decisions; in an attempt to improve their performance, some soft-decision functions are considered in [6] and [7]. Use of the asymptotic efficiency, defined in [11], for the design of soft-decision detectors is proposed in [6]. This method is useful for designing some simple soft-decision functions such as dead-zone limiters and linear clippers, but its application to general cases is limited because obtaining the asymptotic efficiency and its maximization for general soft-decision functions is very difficult. In [7], efficient likelihood-based detectors employing a sigmoid function are derived by applying the expectation maximization algorithm to multiuser detection. The sigmoid function in this case is obtained under the assumption that transmitted signal values are known; as a consequence, the receiver tends to be dependent upon reliable initial estimates of the signal values.

In this paper, we develop a new approach to designing soft-decision functions for multistage detection. The sigmoid decision function for each stage of multistage detection is derived under the assumption that the residual noise which remains after the MAI cancellation at a stage is Gaussian. In contrast to the sigmoid function in [7], which is fixed once the background noise level is given, the proposed sigmoid function is adjusted depending on the residual noise power at each stage. Multilevel quantizers that best approximate the sigmoid function are also designed. It is shown that the design of these functions is simple and leads to effective multistage detectors.

In the next section, the multistage detector is briefly reviewed and our notations are introduced. In Section III, we describe the method for designing soft-decision functions. Simulation results are presented in Section IV.

II. Multistage Detection

We consider BPSK transmission through an additive white Gaussian noise (AWGN) channel shared by K asynchronous users employing a direct-sequence spread-spectrum modulation. The kth user's data signal $b_k(t)$ is defined as

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_k^{(i)} p_T(t - iT) \tag{1}$$

where $p_T(t) = 1$ for $0 \le t < T$ and $p_T(t) = 0$, otherwise, $b_k^{(i)} \in \{\pm 1\}$ denotes the ith transmitted bit of the kth user. The kth user's spreading signal is defined as

$$a_k(t) = \sum_{m=0}^{N-1} a_k^{(m)} p_{T_c}(t - mT_c) \tag{2}$$

where $p_{T_c}(t)$ is the spreading chip waveform whose duration $T_c = T/N$ and the spreading sequence $a_k^{(m)} \in \{\pm 1\}$ has a duration of N. The received signal can be written in the form

$$r(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} \sqrt{E_k} b_k^{(i)} s_k(t - iT - \tau_k) + n(t) \tag{3}$$

where E_k is the received energy of the kth user's signal, $n(t)$ is additive white zero-mean Gaussian noise with two-sided power spectral density $N_0/2; s_k(t) = \sqrt{2/T} a_k(t) \cos(\omega_c t)$,

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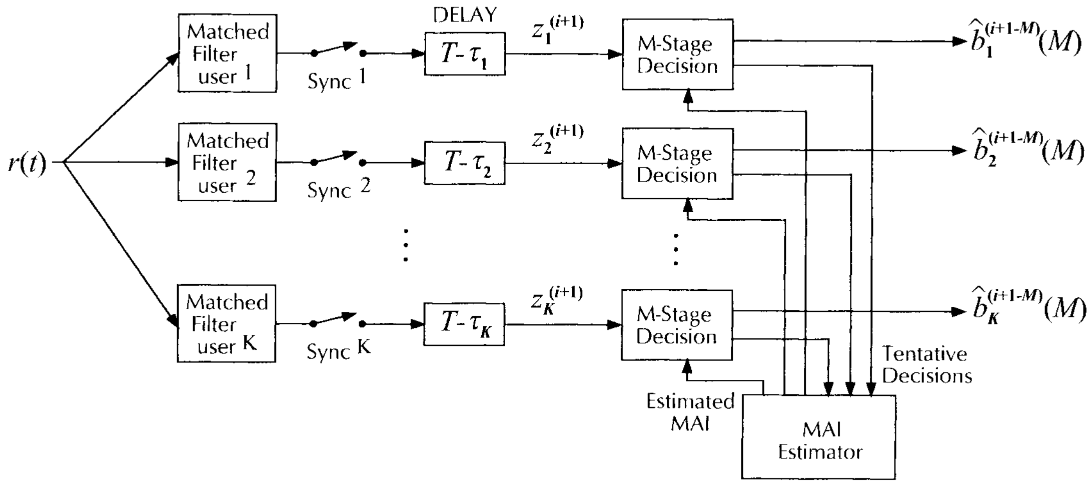

Fig. 1. The multistage detector for the BPSK-CDMA system.

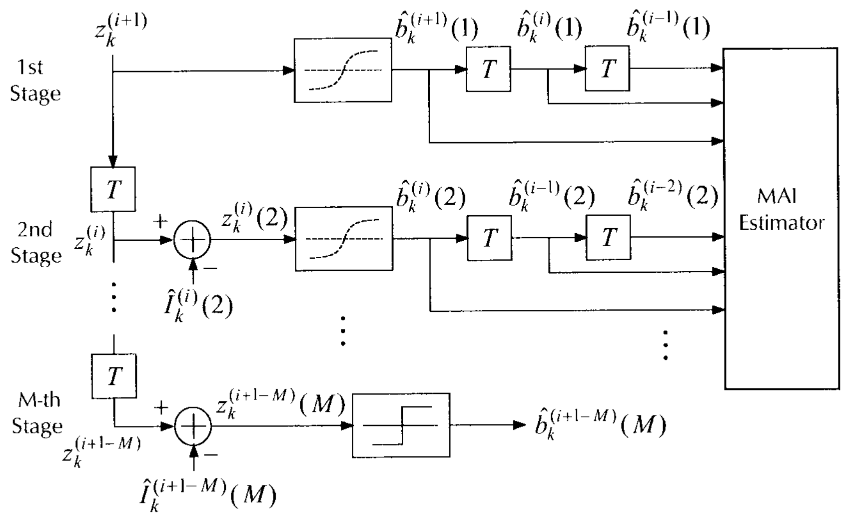

Fig. 2. The M-stage decision device. Here, $\hat{b}_k^{(i)}(m)$ denotes the tentative decision value of $b_k^{(i)}$ at the mth stage.

and τ_k denotes the time delay associated with the kth user. It is assumed that E_k and τ_k are known and, without loss of generality, that $0 \leq \tau_1 \leq \cdots \leq \tau_K \leq T$.

Fig. 1 illustrates the multistage detection in [2]. The sampled output of the kth matched filter at the end of ith time interval is expressed as

$$z_k^{(i)} = \int_{iT+\tau_k}^{(i+1)T+\tau_k} r(t) s_k(t - iT - \tau_k)\, dt = b_k^{(i)} \sqrt{E_k} + \eta_k^{(i)} + I_k^{(i)} \tag{4}$$

where $I_k^{(i)}$, the MAI, is given by

$$I_k^{(i)} = \sum_{l=k+1}^{K} \sqrt{E_l} h_{kl}(1) b_l^{(i-1)} + \sum_{l \neq k} \sqrt{E_l} h_{kl}(0) b_l^{(i)} + \sum_{l=1}^{k-1} \sqrt{E_l} h_{kl}(-1) b_l^{(i+1)} \tag{5}$$

$$h_{kl}(m) = \int_{-\infty}^{\infty} s_k(t - \tau_k) s_l(t + mT - \tau_l)\, dt, \quad \text{for } m = -1, 0, 1 \tag{6}$$

and $\eta_k^{(i)}$ denotes the filtered noise. The multistage detector removes the MAI by using the M-stage decision devices and the MAI estimator. The former yields tentative and final decision values of $b_k^{(i)}$; the latter estimates the MAI using (5) under the assumption that E_k and $h_{kl}(m)$, called the partial correlation, are known. The details of the M-stage decision device are illustrated in Fig. 2. In the first stage, the tentative decision values are evaluated by directly passing the matched filter outputs through the decision device; they are sent to the MAI estimator. In the successive stages, say the mth stage, the estimated MAI value $\hat{I}_k^{(i+1-m)}(m)$ is subtracted from the matched filter output $z_k^{(i+1-m)}$ and the result, denoted by $z_k^{(i+1-m)}(m)$, is passed through the decision device. Note that the MAI at the mth stage is estimated by using the tentative decision values of the $(m-1)$th stage. There is no MAI estimation for the first stage. The output of the Mth stage is the final decision value.

Now we represent the input and output relation of the decision device as follows:

$$\hat{b}_k^{(j)}(m) = w_{k,m}(z_k^{(j)}(m)) \tag{7}$$

where $w_{k,m}(\cdot)$ is a decision function associated with the kth user's mth stage, $z_k^{(j)}(1) = z_k^{(j)}$, and

$$\begin{aligned} z_k^{(j)}(m) &= z_k^{(j)} - \hat{I}_k^{(j)}(m) \\ &= b_k^{(j)}\sqrt{E_k} + \eta_k^{(j)} + I_k^{(j)} - \hat{I}_k^{(j)}(m) \end{aligned} \tag{8}$$

for $2 \leq m \leq M$. In (8), $z_k^{(j)}(m) - b_k^{(j)}\sqrt{E_k} = \eta_k^{(j)} + I_k^{(j)} - \hat{I}_k^{(j)}(m)$ is the residual noise that remains after the MAI cancellation at the mth stage and $I_k^{(j)} - \hat{I}_k^{(j)}(m)$ is called the residual MAI. In [2]–[5], $w_{k,m}(\cdot)$ is equal to the signum function for all k and m, and a hard decision is made. In [6] and [7] soft decisions are employed in making tentative decisions $(1 \leq m \leq M-1)$; of course, the final decision $(m = M)$ is made by using the signum function. In what follows, we consider the design of some soft-decision functions under the mean square error (MSE) criterion.

III. Design of Soft-Decision Functions

In this section, we shall first derive the sigmoid function minimizing the MSE between the true data $b_k^{(j)}$ and its estimate $\hat{b}_k^{(j)}(m)$, under the assumption that the residual noise in (8) is Gaussian. Then the design of soft-decision functions such as the multilevel quantizer and the dead-zone limiter is considered.

A. The Sigmoid Function

Our objective is to find a decision function $w_{k,m}^*(\cdot)$, $1 \leq m \leq M-1$, minimizing the conditional MSE $E\{(b_k^{(j)} - \hat{b}_k^{(j)}(m))^2 | z_k^{(j)}(m) = z\}$, where $\hat{b}_k^{(j)}(m) = w_{k,m}(z_k^{(j)}(m))$ at each stage. By differentiating with respect to $w_{k,m}(z)$, we get the function

$$\begin{aligned} w_{k,m}^*(z) &= E\{b_k^{(j)} | z_k^{(j)}(m) = z\} \\ &= P\{b_k^{(j)} = 1 | z_k^{(j)}(m) = z\} \\ &\quad - P\{b_k^{(j)} = -1 | z_k^{(j)}(m) = z\}. \end{aligned} \tag{9}$$

Note that $w_{k,m}^*(z)$ is the minimum MSE estimate of $b_k^{(j)}$ at the mth stage given $z_k^{(j)}(m) = z$ [12]. From the Bayes theorem, (9) can be rewritten as

$$w_{k,m}^*(z) = \frac{0.5\{f_k^{(m)}(z|b_k^{(j)} = 1) - f_k^{(m)}(z|b_k^{(j)} = -1)\}}{f_k^{(m)}(z)} \tag{10}$$

assuming that $P\{b_k^{(j)} = 1\} = P\{b_k^{(j)} = -1\}$, where $f_k^{(m)}(z)$ is the probability density function (pdf) of $z_k^{(j)}(m)$. Now we assume that the residual noise has identical pdf for all j, and furthermore that it has Gaussian density with zero-mean and variance $\sigma_k^2(m) = E\{(\eta_k^{(j)} + I_k^{(j)} - \hat{I}_k^{(j)}(m))^2\}$—this will be justified through computer simulation in the following section. Under these assumptions, we obtain

$$w_{k,m}^*(z) = \tanh(z\sqrt{E_k}/\sigma_k^2(m)), \qquad 1 \leq m \leq M-1 \tag{11}$$

which is the sigmoid function. This function is adjusted depending on the variance of the residual noise $\sigma_k^2(m)$. When the bit energy to noise power ratio E_b/N_o is high, $\sigma_k^2(m)$ tends to decrease as m increases, and the sigmoid function approaches the signum function. In practice, the variance $\sigma_k^2(m)$ should be estimated for each m. The procedure for the estimation will be discussed in Section IV. Finally, in this section it should be pointed out that the sigmoid function considerably increases the computational load for the MAI estimation as compared with the hard-decision case. The main reason for this is because estimating $I_k^{(i)}$ using (5) requires additional $2(K-1)$ multiplications when the sigmoid function is employed. A soft-decision function that does not increase multiplications in estimating the MAI is discussed next.

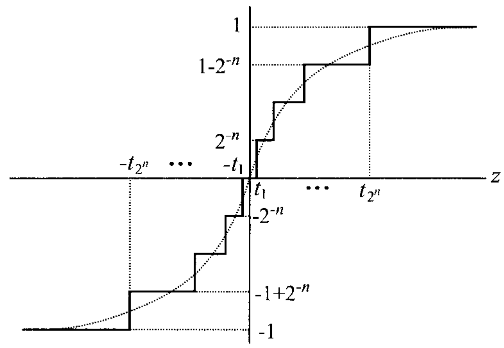

Fig. 3. A multilevel quantizer (the dotted curve shows a sigmoid function).

B. Multilevel Quantizer

Fig. 3 shows a multilevel quantizer that approximates the sigmoid function. We define the set of quantizer output levels as integer multiples of powers-of-two terms that can be expressed as the sums and differences of powers-of-two terms [13], which leads to multiplier-free MAI estimators. An important special case is the quantizer with $n = 0$ having output levels with $\{0, \pm 1\}$, which is called the *dead-zone* limiter. Following the procedure for designing optimal quantizers in [14], we define the average distortion

$$D = 2\sum_{i=0}^{2^n} \int_{t_i}^{t_{i+1}} (w_{k,m}^*(z) - i \cdot 2^{-n})^2 f_k^{(m)}(z)\, dz \tag{12}$$

where $t_0 = 0, t_{2^n+1} = \infty$. By differentiating D with respect to t_i, we get t_i values minimizing D. The results are

$$t_i = w_{k,m}^{*\,-1}(\{2^{-n-1}(2i-1)\}), \qquad i = 1, \cdots, 2^n. \tag{13}$$

For the *dead-zone* limiter, $t_1 = (\ln\ 3/2) \cdot \{\sigma_k^2(m)/\sqrt{E_k}\}$.

IV. Simulation Results

The performances of three types of multistage detectors employing hard decision, the sigmoid function, and the multilevel quantizer are compared by simulating a 31-user $(K = 31)$

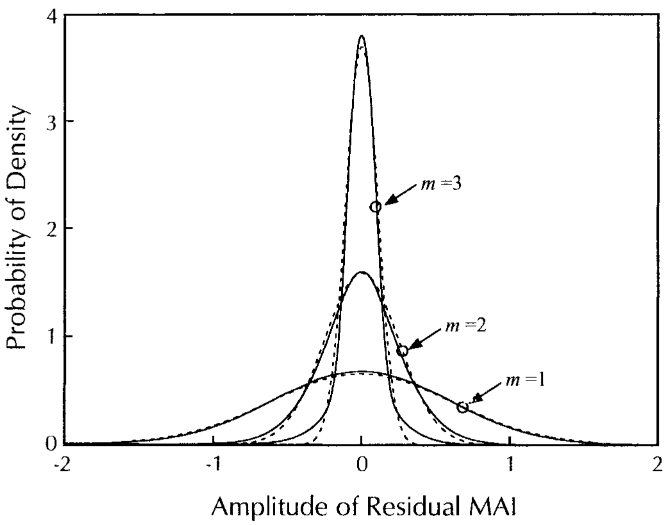

Fig. 4. Comparison between empirically estimated pdf's of the residual MAI in each stage (solid curves) and corresponding Gaussian pdf's (dotted curves). Here, the sigmoid function in (11) is employed, $E_b/N_o = 8$ dB, and $M = 3$.

CDMA system. Each user was assigned a unique spreading sequence from a set of Gold codes of length 31. The bit error rate (BER) values were estimated through 200 simulation runs. For each run, we generated random binary input sequences $b_k^{(i)}$ of length 20 000, $k = 1, \cdots, 31$, and the time delay τ_k has uniform density in $[0, T]$. For these input values, outputs of the multistage detectors were evaluated. Empirical BER values at each run were obtained by counting the number of errors at the output (the errors occurred in the first 200-symbol period were ignored, because we consider this as a preamble period). After 200 simulation runs, the resulting BER values for each case were averaged.

In this simulation, $\sqrt{E_k}$ as well as $\sigma_k^2(m)$ were estimated as follows: to obtain the output $\hat{b}_k^{(i+1-M)}(M)$ (see Fig. 2), we evaluated

$$\sqrt{\hat{E}_k} = \frac{1}{J} \sum_{l=i+1-M-J}^{i-M} |z_k^{(l)}(M)| \tag{14}$$

$$\hat{\sigma}_k^2(m) = \frac{1}{J} \sum_{l=i+1-M-J}^{i-M} \{z_k^{(l)}(m) - \sqrt{\hat{E}_k}\hat{b}_k^{(l)}(M)\}^2 \tag{15}$$

where $1 \le m \le M-1$, $z_k^{(l)}(1) = z_k^{(l)}$, and J is a positive integer which was set to 30. These are essentially the sample mean and the sample variance under the MAI-free condition.

To show the validity of the assumption that the residual noise in (8) is Gaussian, we empirically estimated the pdf's of the residual MAI $I_k^{(i)} - \hat{I}_k^{(j)}(m)$ and found Gaussian pdf's which are close to them. The results are shown in Fig. 4. The empirical pdf's look very close to the corresponding Gaussian pdf's; this justifies our assumption.

Fig. 5 shows the BER values when the received energy E_k are identical $(E_1 = E_2 = \cdots = E_{31})$. As expected, the multistage detectors perform better as the number of stages increases. The soft-decision multistage detectors outperform those with hard decision. The difference between them becomes more conspicuous as E_b/N_0 increases. This is because

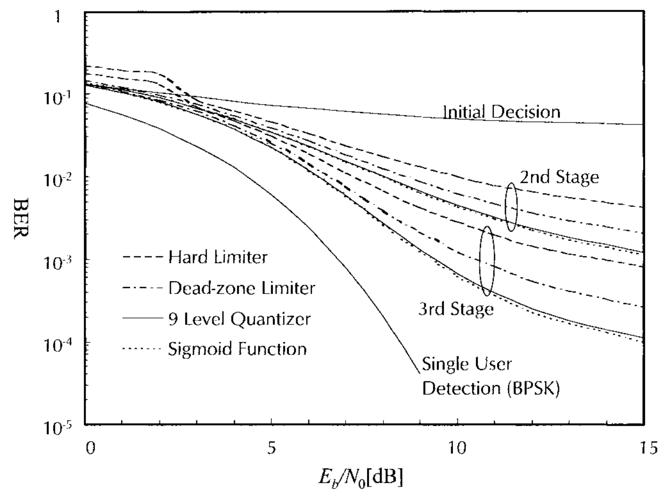

Fig. 5. BER versus E_b/N_o for 31-user systems when $E_1 = E_2 = \cdots = E_{31}$.

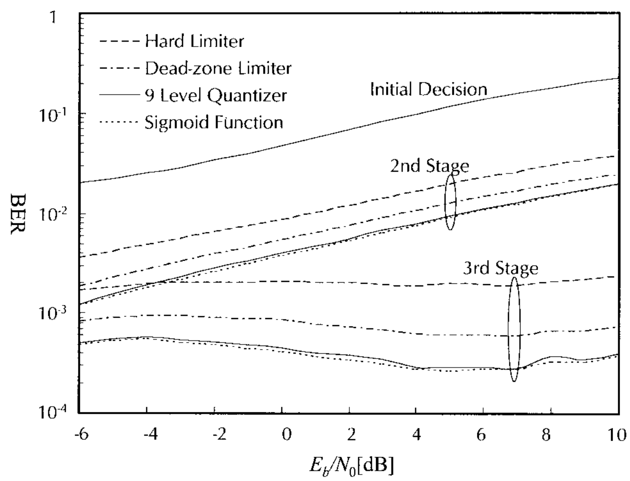

Fig. 6. BER of the users in group 1 when $E_2 = E_4 = \cdots = E_{30} = E_{g1}$ (group 1) and $E_1 = E_3 = \cdots = E_{31} = E_{g2}$ (group 2). The E_b/N_o for group 1 is set at 10 dB.

the MAI dominates the background noise for high SNR. The power gain obtained through soft decision can be significant. For example, in the case of three-stage detection more than 5-dB gain can be achieved at BER $= 10^{-3}$. Among the soft-decision devices, the nine-level quantizer acts like the sigmoid function, and the performance of the dead-zone limiter is in between those of the sigmoid function and the hard limiter.

Fig. 6 shows the BER values when received energy values are not identical. Specifically, we partition the users into two groups and assume that $E_2 = E_4 = \cdots = E_{30} = E_{g1}$ (group 1) and $E_1 = E_3 = \cdots = E_{31} = E_{g2}$ (group 2). The E_b/N_0 for group 1 was set at 10 dB and the E_b/N_o for group 2 was varied. It is seen that the soft-decision multistage detectors consistently perform better than hard-decision ones in the wide range of SNR differences between the two groups.

V. CONCLUSIONS

The sigmoid function for multistage detection was derived under the assumption that the residual noise at each stage is Gaussian. In addition, multilevel quantizers that best approximate the sigmoid function were designed. Computer simulation results showed that multistage detectors employing the proposed soft-decision functions can greatly outperform those with hard decision. Further work in this direction will concentrate on applications of the proposed design scheme to multipath environments described in [5] and to multistage detection methods such as those in [3] and [4].

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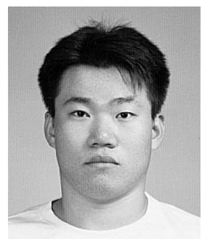

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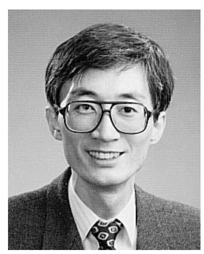

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