

Design of Variable Loop Gains of Dual-Loop DPLL

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Abstract—An approach to the derivation of variable loop gain sequences of dual-loop digital phase-locked loop (DPLL) [1] is developed based on some modifications of the Kalman filtering formulation. It is shown that optimal loop gain sequences which are independent of measurement noise statistics can be obtained under a deterministic source model. Computer simulation results demonstrate that the adaptive dual-loop DPLL designed by using the proposed method is more robust to noise variations than the adaptive DPLL in [2].

Index Terms—Digital phase-locked loop, dual-loop DPPL, Kalman filter, variable loop gain.

I. INTRODUCTION

ALTHOUGH Kalman filtering [3] is useful for deriving variable loop gain sequences of an adaptive digital phase-locked loop (DPLL) [2], [4], it often experiences difficulty in practice because of the lack of statistical knowledge of a DPLL input model. In this paper, we shall show that the variable loop gain sequences of a DPLL with dual-loop structure [1] can be efficiently obtained without such statistical knowledge via the Kalman filtering concept. Specifically, the loop gain sequences which are independent of measurement noise statistics are derived under a deterministic source model. Computer simulation results demonstrate the advantage of the adaptive DPLL designed by using the proposed method over the adaptive DPLL in [2].

II. DERIVATION OF VARIABLE LOOP GAINS

Consider the conventional second-order adaptive DPLL model shown in Fig. 1(a). Assuming a zero-crossing DPLL as in [2], the input to this model is the timing offset between the positive going zero-crossings of the incoming signal and those of a locally generated sine wave. To be specific, let $1/T_0$ and $1/T_1$ denote the clock rates of the receiver and the transmitter, respectively. The timing offset $\alpha(k)$ at the k th ($k = 0, 1, 2, \dots$) zero-crossing point can be expressed as

$$\alpha(k) = t_0 + k(T_1 - T_0) \quad (1)$$

where t_0 is the initial timing offset. The input $\alpha_r(k)$ to the DPLL, corrupted by noise, is expressed as

$$\alpha_r(k) = \alpha(k) + v(k) \quad (2)$$

where $v(k)$ is zero-mean white noise with variance σ_v^2 . In [2], the variable gain sequences $K_0(k)$ and $K_1(k)$ are obtained based on the following state-space model:

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{W}(k) \quad (3)$$

$$\alpha_r(k) = \mathbf{C}\mathbf{X}(k) + v(k) \quad (4)$$

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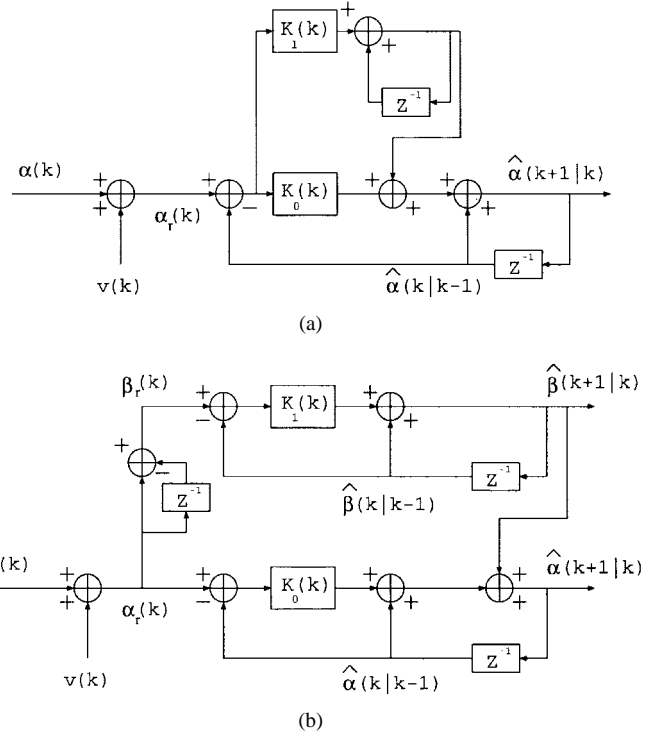


Fig. 1. (a) The conventional second-order DPLL. (b) The dual-loop DPLL. Here, $\hat{\alpha}(k+1|k)$ is a prediction of $\alpha(k+1)$ given $\alpha_r(0), \dots, \alpha_r(k)$.

where state vector $\mathbf{X}(k) = [\alpha(k) \ \beta(k)]^T$, $\beta(k) = T_1 - T_0$, the superscript T denotes transpose, $\mathbf{W}(k)$ is a source noise vector with covariance matrix $\sigma_{\mathbf{W}}^2$, $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, and $\mathbf{C} = [1 \ 0]$. Note that $\beta(k)$ is constant for all k . The optimal gain $K_0(k)$ and $K_1(k)$ minimizing the trace of the prediction error covariance matrix $\mathbf{P}(k+1|k)$ are given by the Kalman gain vector $\mathbf{K}(k) = [K_0(k) \ K_1(k)]^T$ which is obtained by solving the following recurrence equations:

$$\mathbf{K}(k) = \mathbf{P}(k|k-1)\mathbf{C}^T[\mathbf{C}\mathbf{P}(k|k-1)\mathbf{C}^T + \sigma_v^2]^{-1} \quad (5)$$

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{C}\mathbf{P}(k|k-1) \quad (6)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}\mathbf{P}(k|k)\mathbf{A}^T + \sigma_{\mathbf{W}}^2. \quad (7)$$

Evaluation of these equations requires knowledge of $\sigma_{\mathbf{W}}^2$, $\mathbf{P}(0|-1)$, and σ_v^2 .

Now, we consider the dual-loop DPLL shown in Fig. 1(b). This dual-loop DPLL can be thought of as a DPLL with phase and frequency detectors [5]. In [1], the gain sequences $K_0(k)$ and $K_1(k)$ are obtained by solving a minimum mean square error estimation problem in scalar form; as a consequence, the procedure for obtaining them is cumbersome and extension to a higher order DPLL is difficult. In what follows, we shall show that the gain sequences can be efficiently obtained based on the Kalman filtering concept. The state-space model for

the dual-loop DPLL is defined as

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) \quad (8)$$

$$\mathbf{Y}(k) = \mathbf{X}(k) + \mathbf{V}(k) \quad (9)$$

where $\mathbf{X}(k)$ and \mathbf{A} are the same as those in (3) with the exception that the source noise vector term $\mathbf{W}(k)$ is dropped. The state vector $\mathbf{X}(k)$ is now deterministic—this is valid when the local oscillator in the transmitter is sufficiently stable.¹ For the dual-loop DPLL, this source model leads to the derivation of variable gain sequences which are independent of measurement noise variance σ_v^2 . The measurement vector $\mathbf{Y}(k)$ and the noise vector $\mathbf{V}(k)$ in (9) are defined as $\mathbf{Y}(k) = [\alpha_r(k) \ \beta_r(k)]^T$; $\beta_r(k) = \alpha_r(k) - \alpha_r(k-1) = T_1 - T_0 + v(k) - v(k-1)$; and $\mathbf{V}(k) = [v(k) \ v(k) - v(k-1)]^T$. The filtered and predicted state vectors are denoted by $\hat{\mathbf{X}}(k|k)$ and $\hat{\mathbf{X}}(k+1|k)$, respectively. Based on this state model, the dual-loop DPLL operation can be represented by the following Kalman filter update equations:

$$\hat{\mathbf{X}}(k|k) = \hat{\mathbf{X}}(k|k-1) + \mathbf{K}(k)(\mathbf{Y}(k) - \hat{\mathbf{X}}(k|k-1)) \quad (10)$$

$$\hat{\mathbf{X}}(k+1|k) = \mathbf{A}\hat{\mathbf{X}}(k|k) \quad (11)$$

where the loop gain matrix $\mathbf{K}(k) = \begin{bmatrix} K_0(k) & 0 \\ 0 & K_1(k) \end{bmatrix}$. It is rather straightforward to see that the procedure for obtaining $\hat{\mathbf{X}}(k+1|k)$ in (10) and (11) is equivalent to the dual-loop DPLL operation. Now, the optimization of $\mathbf{K}(k)$ in (10) deviates from that of the gain matrix in standard Kalman filtering because $\mathbf{V}(k)$ in (9) is correlated and $\mathbf{K}(k)$ is a diagonal matrix. This fact motivated us to develop a new method for computing $\mathbf{K}(k)$. Since the objective of DPLL is to reduce the phase prediction error, we consider the minimization of the phase prediction error variance $E[(\alpha(k+1) - \hat{\alpha}(k+1|k))^2]$, which is the (1,1)th element of the prediction error covariance matrix $\mathbf{P}(k+1|k)$, with respect to the gain $K_0(k)$ and $K_1(k)$. Here, E denotes expectation. This minimization leads to

$$\mathbf{M}(k) \begin{bmatrix} K_0(k) \\ K_1(k) \end{bmatrix} = \mathbf{L}(k) \quad (12)$$

where

$$\mathbf{M}(k) = \mathbf{P}(k|k-1) + \mathbf{U}(k|k-1) + \mathbf{U}^T(k|k-1) + \mathbf{R} \quad (13)$$

$$\mathbf{L}(k) = [\mathbf{P}(k|k-1) + \mathbf{U}(k|k-1)]\mathbf{1} \quad (14)$$

with $\mathbf{1} = [1 \ 1]^T$, $\mathbf{R} = E[\mathbf{V}(k)\mathbf{V}^T(k)] = \sigma_v^2\mathbf{D}$, and $\mathbf{D} = \begin{bmatrix} 1 & \\ & 2 \end{bmatrix}$. The matrices $\mathbf{P}(k|k-1)$ and $\mathbf{U}(k|k-1)$ are evaluated from the following recurrence equations:

$$\begin{aligned} \mathbf{P}(k+1|k) &= \mathbf{A}(\mathbf{I} - \mathbf{K}(k))\mathbf{P}(k|k-1)(\mathbf{I} - \mathbf{K}(k))^T\mathbf{A}^T \\ &\quad - \mathbf{A}\mathbf{K}(k)\mathbf{U}(k|k-1)(\mathbf{I} - \mathbf{K}(k))^T\mathbf{A}^T \\ &\quad - \mathbf{A}(\mathbf{I} - \mathbf{K}(k))\mathbf{U}^T(k|k-1)\mathbf{K}^T(k)\mathbf{A}^T \\ &\quad + \mathbf{A}\mathbf{K}(k)\mathbf{R}\mathbf{K}^T(k)\mathbf{A}^T \end{aligned} \quad (15)$$

$$\begin{aligned} \mathbf{U}(k+1|k) &= E[\mathbf{V}(k+1)(\mathbf{X}(k+1) - \hat{\mathbf{X}}(k+1|k))^T] \\ &= \mathbf{B}\mathbf{U}(k|k-1)(\mathbf{I} - \mathbf{K}(k))^T\mathbf{A}^T \\ &\quad - \mathbf{B}\mathbf{R}\mathbf{K}^T(k)\mathbf{A}^T \end{aligned} \quad (16)$$

where \mathbf{I} is the identity matrix and $\mathbf{B} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$.

¹In fact, this deterministic process model is also employed in [2] for the case where the DPLL is in lock.

The gains $K_0(k)$ and $K_1(k)$, which are independent of the measurement noise variance σ_v^2 , can be obtained by setting the initial loop gain matrix $\mathbf{K}(0)$ to the identity matrix. This statement is proved as follows. When $\mathbf{K}(0) = \mathbf{I}$, we get $\mathbf{U}(1|0) = -\mathbf{B}\mathbf{R}\mathbf{A}^T = -\sigma_v^2\mathbf{B}\mathbf{D}\mathbf{A}^T$ and $\mathbf{U}(2|1) = \sigma_v^2[\mathbf{B}(-\mathbf{B}\mathbf{D}\mathbf{A}^T)(\mathbf{I} - \mathbf{K}(1))^T\mathbf{A}^T - \mathbf{B}\mathbf{D}\mathbf{K}^T(1)\mathbf{A}^T]$. In this manner, we can show that σ_v^2 can be factored out from $\mathbf{U}(k+1|k)$ for all k . This argument also holds for $\mathbf{P}(k+1|k)$. As a consequence, σ_v^2 can be factored out from $\mathbf{M}(k)$ and $\mathbf{L}(k)$ as well, and from (12) the gain sequences become independent of σ_v^2 . It is interesting to note that $K_0(k)$ and $K_1(k)$ are also independent of the initial conditions $\mathbf{P}(0|-1)$ and $\mathbf{U}(0|-1)$ when $\mathbf{K}(0) = \mathbf{I}$. This fact eliminates the need of estimating these conditions. In summary, evaluation of the gain sequences using (12)–(16) does not require any statistical knowledge if $\mathbf{K}(0)$ is set at the identity matrix. Once properly designed, the dual-loop DPLL can be applied to various input signals without changing its gain, in contrast to the conventional DPLL which requires rederivation of its gain sequences for each input statistic.

We now solve (12)–(16) starting with $\mathbf{K}(0) = \mathbf{I}$. When $k = 1$, (12) becomes

$$\begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} K_0(1) \\ K_1(1) \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix} \quad (17)$$

which is an underdetermined problem. For this case, we choose the minimum norm solution² $K_0(1) = K_1(1) = 2/3$. In subsequent steps ($k = 2, 3, 4, \dots$) a unique solution exists. Through numerical computation, we observed an interesting fact that $K_0(k) = K_1(k) = 2/(k+2)$ for all k . These gain values are applied to the dual-loop DPLL in our simulation.

III. SIMULATION RESULTS

In this section, we compare the dual-loop DPLL designed through (12)–(16) with the conventional DPLL designed using (5)–(7). The input to the DPLL is generated under the assumption that the initial phase $\alpha(0)$ and the initial phase change $\beta(0)$ have uniform density in $[-T_0/2, T_0/2]$ and $[-T_0/10, T_0/10]$, respectively. For these initial statistics³ $\mathbf{P}(0|-1)$ required for computing (5)–(7) is $\begin{bmatrix} T_0^2/12 & 0 \\ 0 & T_0^2/300 \end{bmatrix}$. The noise vector $\mathbf{W}(k)$ in (3) is dropped as in (8). As a performance measure, the phase prediction error variances are evaluated both theoretically and experimentally for several values of signal-to-noise ratio (SNR), which is defined as $10 \log_{10}(T_0^2/\sigma_v^2)$ dB. The variable loop gain sequences obtained through (5)–(7) and (12)–(16) are depicted in Fig. 2 and some of the corresponding phase prediction error variances evaluated theoretically are shown in Fig. 3. Since the gain sequences of the dual-loop DPLL are independent of σ_v^2 , $K_0(k)$ and $K_1(k)$ obtained for 15 dB are the same as those for 30 dB ($K_0(k) = K_1(k) = 2/(k+2)$). The phase prediction error variances demonstrate that the performance of the dual-loop DPLL is comparable to that of the conventional DPLL.

²Our simulation results indicate that this choice leads to the dual-loop DPLL exhibiting the best performance.

³Another initial statistic corresponding to $\beta(0)$ which is uniform in $[-T_0/5, T_0/5]$ was also considered. Simulation results were very similar to those presented in this section.

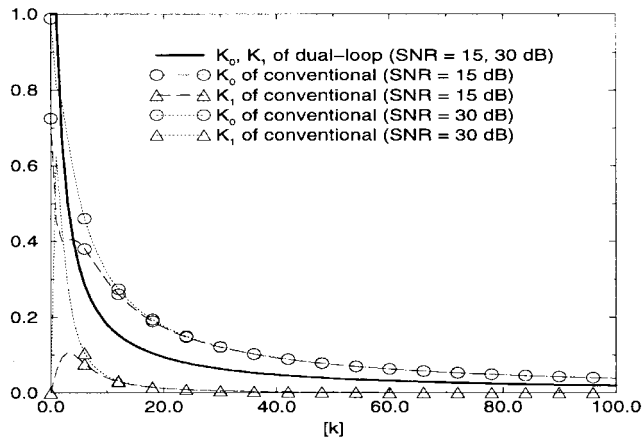


Fig. 2. The variable loop gain sequences.

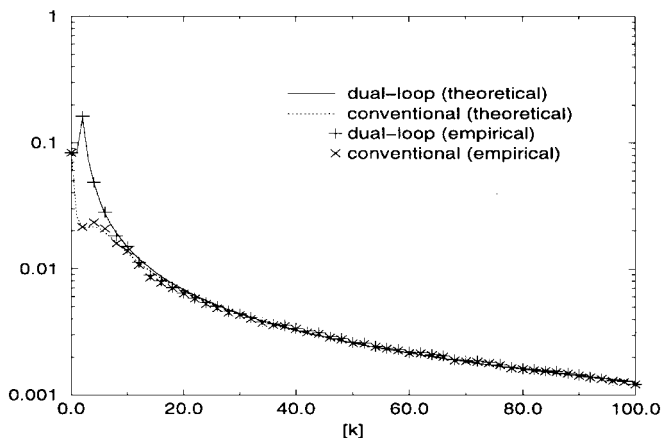


Fig. 3. Theoretical and empirical phase prediction error variances (normalized to T_0^2) when SNR = 15 dB.

The dual-loop DPLL would be more convenient to use because it can be applied to the 15- and 30-dB cases without changing its gain sequences.

The DPLL's with variable gains shown in Fig. 2 are applied to computer-generated input signals. We generated 1000 sequences of length 100 assuming that $v(k)$ was Gaussian. Each sequence was applied to the DPLL's and the phase prediction error variances were estimated. The results are also shown in Fig. 3. Note that the empirical variances are very close to the corresponding theoretical ones.

To further demonstrate the advantages of the dual-loop DPLL over the conventional one, we designed the DPLL's under the assumption that SNR = 15 dB and applied them to the case where SNR = 30 dB. The resulting error variances are illustrated in Fig. 4. Clearly, the dual-loop DPLL outperformed the conventional DPLL. Now, the DPLL's designed for SNR = 30 dB were applied to the case with SNR = 15 dB. The simulation results, which are not reproduced due to space limitation, indicate that, in this case, performance of the two DPLL's is comparable.

In summary, the conventional and the dual-loop DPLL generally perform in a similar manner, but the dual-loop DPLL

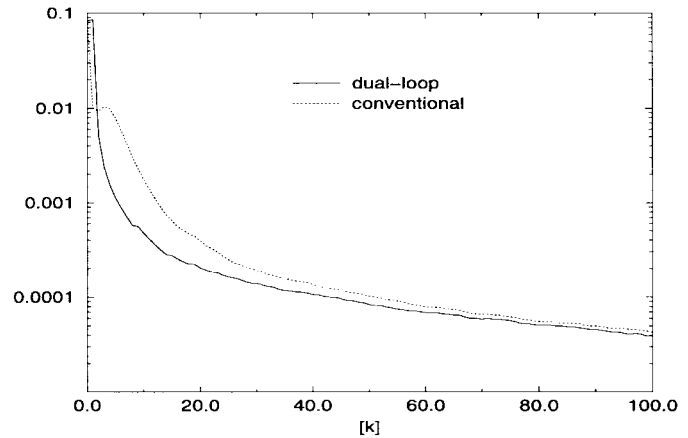


Fig. 4. Empirical phase prediction error variances (normalized to T_0^2) when the DPLL's designed for SNR = 15 dB are applied to the case where SNR = 30 dB.

is more robust to erroneous estimates of measurement noise variance.

IV. CONCLUSION

An algorithm to derive time-varying loop gains of dual-loop DPLL has been developed based on the Kalman filtering concept. This algorithm produces loop gains insensitive to measurement noise variance and leads to an adaptive DPLL, which is more robust to noise variations than the conventional one.

The variable gain dual-loop DPLL should be useful for timing recovery of burst mode data transmission, which requires fast acquisition. By simply resetting the gain matrix $\mathbf{K}(k)$ to the identity matrix and restarting with $\{\mathbf{K}(0), \mathbf{K}(1), \mathbf{K}(2), \dots\}$ whenever a data burst is received, the dual-loop DPLL can maintain the rapid acquisition characteristic. The time instants at which a data burst arrives may be detected by using a simple lock detection algorithm as in [2].

Finally, it should be pointed out that extension of the proposed algorithm to a higher order DPLL is straightforward. In essence, (12)–(16) can be used after extending their matrices and vectors to a proper dimension.

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