Design of Discrete Coefficient FIR and IIR Digital Filters with Prefilter-Equalizer Structure Using Linear Programming

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***Abstract*—Optimal methods for designing multiplierless finite-impulse response (FIR) and infinite-impulse response (IIR) filters with cascaded prefilter-equalizer structures are proposed. Assuming that an FIR filter consists of a cyclotomic polynomial (CP) prefilter and an interpolated second order polynomial (ISOP) equalizer, in the proposed method, the prefilter and equalizer are simultaneously designed using mixed integer linear programming (MILP). The resulting filter is a cascaded filter with minimal complexity. For IIR filters all-pole IIR equalizers consisting of inverse of interpolated first order polynomials (IIFOP's) are introduced, and a CP-prefilter cascaded with this type of equalizer is designed. Design examples demonstrate that the proposed methods produce a more efficient cascaded prefilter-equalizer than existing methods.**

***Index Terms*—Cyclotomic, equalizer, FIR, interpolated second-order polynomial, IIR, polynomial, prefilter.**

I. Introduction

One approach to the design of efficient finite-impulse response (FIR) filters requiring fewer arithmetic operations than conventional ones is based on a cascade structure composed of a prefilter, which is often multiplerless, followed by an FIR equalizer [1]–[8]. The prefilter provides most of the desired stopband attenuation, then the equalizer adapts the overall filter to meet specifications. Prefilters are based on the use of polynomials such as the recursive running sum (RRS) [1], [2], Chebyshev polynomials [3], and the cyclotomic polynomial (CP) [4] which includes the RRS as a special case. They are designed either in an *ad hoc* manner [4], [5] or by formulating a linear optimization problem [8]. After deriving the prefilter, an equalizer is designed using conventional methods such as the Parks–McClellan algorithm or linear programming.

The two-step process for designing a cascaded prefilter and equalizer generally results in a sub-optimal overall filter, even when each individual filter in the cascade is optimally designed. A more efficient combination of the prefilter and equalizer would seemingly be derived if the two filters were simultaneously designed under the same optimality criterion. Accordingly, this brief outlines a new method for designing a cascaded prefilter and equalizer with minimal complexity. The proposed approach is based on the use of ISOP's [9] and IIFOP's for FIR and infinite-impulse response (IIR) equalizers, respectively. The prefilter and equalizer are designed concurrently by selecting the optimal polynomials from either the set of CP's and ISOP's or the set of CP's and IIFOP's. It will be shown that efficient prefilters and equalizers can be designed by applying the optimal CP selection method in [8] to the proposed problem. Design examples will demonstrate that the proposed method can provide a remarkable reduction in computational complexity.

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II. CP Prefilters, ISOP, and IIFOP Equalizers

A. CP Prefilters

To derive multiplierless prefilters, only those CP's with $\{-1, 0, 1\}$ coefficients are considered. There are 104 such CP's. The roots of each CP lie on the z-plane unit circle. The set of eligible CP's, denoted by $\Omega_{\rm cp}$ consists of CP's that satisfy the given filter specifications, plus various combinations of these CP's that can reduce computational complexity [5]. The system function of the prefilter is represented as $\prod_{q=1}^{Q} F_q(z)^{m_q}$ where $F_q(z) \in \Omega_{\rm cp}, m_q$ is a nonnegative integer and Q is the total number of elements in $\Omega_{\rm cp}$. The roots of $F_q(z)$ lie either within the stopband or within a portion of the transition band.

B. ISOP for FIR Equalizers

An ISOP is an interpolated version of a second order polynomial with symmetric coefficients which guarantee a linear phase characteristic

$$S^I(z) = (a + bz^{-I} + az^{-2I}) \tag{1}$$

where a and b are the CSD coefficients, $b^2 - 4a^2 > 0$ and $a \neq 0$. The magnitude response of $S^I(z)$ is expressed as $|S^I(\omega)| = |b + 2a \cdot \cos I\omega|$ where I is an interpolation factor. Owing to the conditions $b^2 - 4a^2 > 0$ and $a \neq 0$, each $|S^I(\omega)|$ is a sinusoidal wave whose amplitude, frequency and dc gain can be easily adjusted by setting parameters (a, b, I), and the roots of $S^I(z)$ are not located on the unit circle. Consequently, for given filter specifications it is always possible to find a convex waveform $|S^I(\omega)|$ in the passbands[1].

In order to maintain the passband convexity of $|S^I(\omega)|, |S^I(\omega)|$ has at most one extremal point in a passband. This property can be satisfied if

$$1 \leq I \leq \left\lfloor \frac{2\pi}{\omega_B} \right\rfloor \tag{2}$$

where ω_B denotes either the bandwidth or twice the bandwidth for bandpass or low-pass filters, respectively.

Since the parameters a and b are CSD coefficients, the number of ISOP's is finite. The set of eligible ISOP's, denoted by Ω_{IS}, is determined by excluding those ISOP's that do not meet the condition in (2) from the set of all possible ISOP's. The number of ISOP's in Ω_{IS} can be further reduced by examining Ω_{IS} elements. When several ISOP's have similar magnitude responses, one can be kept and the others discarded. Moreover, ISOP's with magnitude responses that are too flat can also be discarded.

The system function of an ISOP equalizer is expressed as $\prod_{r=1}^{R} S_r^I(z)^{n_r} = \prod_{r=1}^{R}(a_r + b_r z^{-I} + a_r z^{-2I})^{n_r}$ where $S_r^I(z) \in \Omega_{IS}; n_r$ is a nonnegative integer, a_r and b_r are CSD coefficients, and R is the total number of elements in Ω_{IS}.

C. IIFOP for IIR All-Pole Equalizers

An ISOP cannot be employed as a denominator polynomial of an allpole equalizer because its linear phase characteristic leads to an unstable system. Therefore, instead of ISOP's, a first order polynomial $(a + bz^{-1})$ and its interpolated version $(a + bz^{-I})$, which is called an IFOP, is utilized. An IFOP is a nonlinear phase system whose magnitude response is given by $\sqrt{|b' + 2a' \cos I\omega|}$, where $b' = a^2 + b^2$ and $a' = ab$. This response has the same form as the square root of the ISOP magnitude response. Accordingly, the magnitude square of

[1]This brief only considers the design of single-band filters with one passband. ISOP's can be used for designing multiband filters if eligible ISOP's having convex magnitude responses in passbands are found.

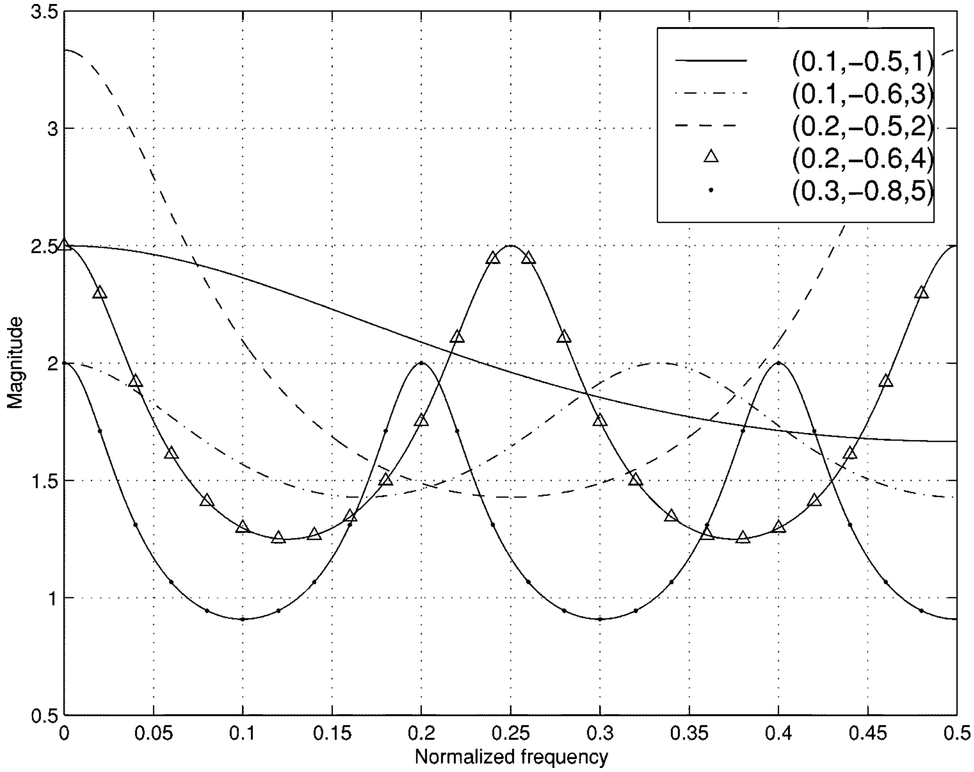

Fig. 1. Waveform of $|A^I(\omega)|$ for some (a, b, I) pairs.

an IFOP is a sinusoidal wave with period $2\pi/I$ if $b'^2 - 4a'^2 > 0$ and $a' \neq 0$. The inverse of an IFOP (IIFOP), denoted by $A^I(z)$, is

$$A^I(z) = \frac{1}{(a + bz^{-I})} \tag{3}$$

where $|b| \leq |a|$ for stability and its magnitude response is given by $|A^I(\omega)| = 1/(\sqrt{|b' + 2a' \cos I\omega|})$. As shown in Fig. 1, the waveform of $|A^I(\omega)|$ bears a certain resemblance to a sinusoidal wave: it is periodic with period $2\pi/I$ and has two extremal points in each period. Therefore, like ISOP's, a convex waveform $|A^I(\omega)|$ can always be found in passbands by adjusting the parameters (a, b, I). An allpole equalizer is a cascade of IIFOP's represented as $\prod_{r=1}^{R} A_r^I(z)^{n_r} = \prod_{r=1}^{R}(1/(a_r + b_r z^{-I})^{n_r})$ where $A_r^I(z) \in \Omega_{\mathrm{II}}; \Omega_{\mathrm{II}}$ is a set of eligible IIFOPs, n_r is a nonnegative integer, a_r and b_r are CSD coefficients, and R is the total number of elements in Ω_{II}. Eligible IIFOP's are selected as in the case of ISOP's because (2) also holds for IIFOP's.

III. Simultaneous Design of Prefilter and Equalizer

A. CP Prefilter and ISOP Equalizer Design

The system function of a cascaded CP prefilter and ISOP equalizer is given by $H(z) = k \cdot \prod_{q=1}^{Q} F_q(z)^{m_q} \prod_{r=1}^{R} S_r^I(z)^{n_r}$ where k is a constant. To apply the design method in [8], we consider $H_{\mathrm{dB}}(\omega) = 20 \log |H(\omega)|$, which is expressed as

$$H_{\mathrm{dB}}(\omega) = k_{\mathrm{dB}} + \sum_{q=1}^{Q} m_q F_{q,\mathrm{dB}}(\omega) + \sum_{r=1}^{R} n_r S_{r,\mathrm{dB}}^I(\omega) \tag{4}$$

where $k_{\mathrm{dB}} = 20 \log |k|$, $F_{q,\mathrm{dB}}(\omega) = 20 \log |F_q(\omega)|$ and $S_{r,\mathrm{dB}}^I(\omega) = 20 \log |S_r^I(\omega)|$. A cascade of CP prefilter and ISOP equalizer with minimal complexity can be designed by solving the linear programming problem in (5), shown at the bottom of the page, where $e_q^a(e_r^a)$ and $e_q^d(e_r^d)$, respectively, are the number of additions and delays required for subfiltering $F_q(z)(S_r^I(z))$, and c is a constant which is determined depending on the delay and addition complexity. This problem can be solved by MILP, by treating k_{dB}, m_q and n_r as variables.

B. CP Prefilter and IIFOP Equalizer Design

The overall filter is expressed as $H(z) = k \cdot \prod_{q=1}^{Q} F_q(z)^{m_q} \prod_{r=1}^{R} A_r^I(z)^{n_r}$. $H_{\mathrm{dB}}(\omega)$ is given by (4) after replacing $S_{r,\mathrm{dB}}^I(\omega)$ with $A_{r,\mathrm{dB}}^I(\omega)$. Since this filter is an IIR filter, the number of delays should be modified as $\max(O_N, O_D)$, where $O_N = \sum_{q=1}^{Q} m_q e_q^d$ and $O_D = \sum_{r=1}^{R} n_r e_r^d$. ($O_N$ and O_D represent the order of the denominator and numerator of $H(z)$, respectively.) Furthermore, some constraints on group delay

$$\begin{aligned} &\text{minimize} && \sum_{q=1}^{Q} m_q \cdot \left(e_q^a + c \cdot e_q^d\right) + \sum_{r=1}^{R} n_r \cdot \left(e_r^a + c \cdot e_r^d\right) \\ & && \text{(measure of complexity)} \\ &\text{subject to} && 20\log(H_d(\omega) - \delta_p) < H_{\mathrm{dB}}(\omega) < 20\log(H_d(\omega) + \delta_p) \quad \text{(in passbands)} \\ & && H_{\mathrm{dB}}(\omega) < 20 \log \delta_s \quad \text{(in stopbands)} \end{aligned} \tag{5}$$

TABLE I
COMPARISON WITH OTHER METHODS IN EXAMPLE 1

	Modified K-H and Remez Equalizer [1]	Tapped Cascaded Identical Subfilters [12]	Interpolated FIR Filter [11]	CP Prefilter/ Remez Equalizer [5]	CP Prefilter/ Subset Selection Equalizer [5]	CP Prefilter and ISOP Equalizer	CP Prefilter and IIFOP Equalizer
multiplications	17	4	5	10	4	(1)	(1)
additions	37	17	15	28	16	16	10
delays	73	100	78	71	85	82	50

TABLE II
COMPARISON WITH OTHER METHODS IN EXAMPLE 2

	Modified K-H and Remez Equalizer [1]	Cabezas-Diniz Design 2 [2]	CP Prefilter/ Remez Equalizer [4]	CP Prefilter/ Remez Equalizer [5]	CP Prefilter/ Subset Selection Equalizer [5]	CP Prefilter and ISOP Equalizer	CP Prefilter and IIFOP Equalizer
multiplications	43	7	31	17	13	(1)	(1)
additions	163	48	82	58	49	24	25
delays	162	217	163	161	180	172	104

of $H(\omega)$ are necessary. The proposed method which minimizes the complexity under the group delay and ripple constraints is described in (6), shown at the bottom of the page, where ct represents the complexity of delays, G_d represents a constant group delay, ϵ is a given allowable group delay deviation, $G_q^F(\omega)$ and $G_r^A(\omega)$ are the group delay responses of $F_q(z)$ and $A_r^I(z)$, respectively, and the last four inequalities regarding O_N and O_D are added for realizing $\max(O_N, O_D)$ [10]. This optimization can be achieved using MILP, treating $k_{\rm dB}, m_q, n_r, t$, and y as variables.

IV. DESIGN EXAMPLES

In design exmaples, the parameters (a, b) for the ISOP's and IIFOP's were assumed to be 8-bit CSD coefficients consisting of a *single* powers-of-two term. The constant c in (5) was set at 0.5.

Example 1 (Low-pass Filter): The specifications in normalized frequency were: passband $f \in [0, 0.021]$, stopband $f \in [0.07, 0.5], \delta_{p,\rm dB} \leq 0.1$ dB in passband, $\delta_{s,\rm dB} \leq -60$ dB in stopband, and $\epsilon = 1.125$. Given these specifications, 33 eligible CP's, 72 eligible ISOP's, and 31 eligible IIFOP's were selected. The computation time was less than 10 minutes in an Ultra Sparc II. The resulting CP-prefilter and ISOP-equalizer were[2]

$$P(z) = (1+z^{-3})(1+z^{-4})^2(1+z^{-6})\left(\frac{1-z^{-10}}{1-z^{-1}}\right) \times \left(\frac{1-z^{-13}}{1-z^{-1}}\right)\left(\frac{1-z^{-14}}{1-z^{-1}}\right),$$

[2]CP prefilters in (7) and (8) which have RRS subfilters are stable, because exact pole-zero cancellation is guaranteed in the RRS [5].

$$\begin{aligned}
&\text{minimize} && \sum_{q=1}^{Q} m_q e_q^a + \sum_{r=1}^{R} n_r e_r^a + ct \qquad \text{(measure of complexity)}\\
&\text{subject to} && \left|\sum_{q=1}^{Q} m_q G_q^F(\omega) + \sum_{r=1}^{R} n_r G_r^A(\omega) - G_d\right| \leq \epsilon\\
& && \text{(group delay constraint in passbands)}\\
& && \text{the ripple constraints in (5)}\\
& && -(t - O_D) \leq My, \quad O_D - O_N \leq M(1-y)\\
& && -(t - O_N) \leq M(1-y), \quad O_N - O_D \leq My
\end{aligned} \tag{6}$$

$$E(z) = (2^{-3} - 2^{-1}z^{-4} + 2^{-3}z^{-8})^2 \times (2^{-3} - 2^{-1}z^{-6} + 2^{-3}z^{-12}). \tag{7}$$

The CP/IIFOP cascade was

$$P(z)E(z) = (1 + z^{-2})\left(\frac{1 + z^{-3} + z^{-6}}{1 + 2^{-1}z^{-13}}\right)\left(\frac{1 - z^{-9}}{1 - z^{-1}}\right) \times \left(\frac{1 - z^{-12}}{1 - z^{-1}}\right)\left(\frac{1 - z^{-14}}{1 - z^{-1}}\right). \tag{8}$$

In Table I, the computational complexity of the overall filter $H(z) = k \cdot P(z)E(z)$ is compared with those of the low-pass filters designed in [1], [5], [11], and [12]. The number of multiplications in parentheses represents scaling with a constant k. The proposed CP prefilter/ISOP(IIFOP) equalizer implementation required only 16(10) additions and 82(50) delays, ignoring scaling. The IIR filter required less computation than the FIR filters in Table I. The filters designed using the proposed method were simpler to implement than the other filters.

Example 2 (Bandpass Filter): The specifications in a normalized frequency were: passband $f \in [0.189, 0.211]$, stopband $f \in [0, 0.168] \bigcup [0.232, 0.5]$, ripple $\delta_{p,\,\mathrm{dB}} \leq 0.25$ dB in passband, $\delta_{s,\,\mathrm{dB}} \leq -60$ dB in stopband, and $\epsilon = 3.000$. These specifications were previously considerded in [1], [2], [4], [5]. 40 eligible CP's, 38 eligible ISOP's, and 69 eligible IIFOP's were selected. It took a few hours to solve the MILP problem. The designed CP prefilter and ISOP equalizer were

$$\begin{aligned} P(z) &= (1 - z^{-2})(1 - z^{-3})(1 - z^{-2} + z^{-4})^2 \\ &\quad \times (1 + z^{-5})(1 - z^{-3} + z^{-6})^2(1 - z^{-7}) \\ &\quad \times (1 + z^{-10})(1 + z^{-5} + z^{-10})(1 + z^{-15}) \\ &\quad \times (1 + z^{-10} + z^{-15} + z^{-20} + z^{-30}), \\ E(z) &= (2^{-3} + 2^{-1}z^{-17} + 2^{-3}z^{-34}) \\ &\quad (2^{-3} + 2^{-1}z^{-18} + 2^{-3}z^{-36}). \end{aligned} \tag{9}$$

The CP/IIFOP cascade was

$$\begin{aligned} P(z)E(z) &= \left(\frac{1}{1 + 2^{-1}z^{-2}}\right)^2 \left(\frac{1 + z^{-1}}{1 + 2^{-1}z^{-2}}\right) \\ &\quad \times \left(\frac{1 + z^{-5}}{1 + 2^{-1}z^{-2}}\right)\left(\frac{1 - z^{-3} + z^{-6}}{1 + 2^{-1}z^{-2}}\right) \\ &\quad \times \left(\frac{1 - z^{-7}}{1 + 2^{-2}z^{-3}}\right)\left(\frac{1 - z^{-8}}{1 + 2^{-1}z^{-3}}\right) \\ &\quad \times \left(\frac{1 - z^{-12}}{1 + 2^{-1}z^{-3}}\right)\left(\frac{1 - z^{-13}}{1 + 2^{-1}z^{-20}}\right) \\ &\quad \times \left(\frac{1 + z^{-10} + z^{-20}}{1 + 2^{-1}z^{-20}}\right) \\ &\quad \times \left(\frac{1 + z^{-5} + z^{-10} + z^{-15} + z^{-20}}{1 + 2^{-1}z^{-20}}\right). \end{aligned} \tag{10}$$

Table II compares the relative complexities of the various methods. The proposed CP prefilter/ISOP(IIFOP) equalizer cascade required only 24(25) additions and 172(104) delays. The IIR filter required one more addition than the FIR filter employing the ISOP equalizer; however, it needed considerably fewer delays than the FIR filter. The filters designed using the proposed method required remarkably less computation than the others.

V. Conclusion

An optimal procedure for designing multiplierless cascaded form FIR/IIR digital filters with minimal complexity was presented. Specifically, CP prefilters and an ISOP (or allpole IIFOP) equalizer with CSD coefficients were considered and methods for the simultaneous design of such prefilters and equalizers were developed.

The proposed methods can be extended to designing general cascade filters as long as an eligible set of filter polynomials can be defined. The search for finding other classes of polynomials which can be useful for cascade filter design remains as an area for further development.

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