

Data-Aided Frequency Estimation for PSK Signaling in Frequency-Selective Fading

Eui-Rim Jeong, Ginkyu Choi, *Member, IEEE*, and Yong H. Lee, *Senior Member, IEEE*

Abstract—A new data-aided frequency estimator is introduced for phase-shift keying signals transmitted over frequency-selective fading channels. This estimator is developed based on a maximum likelihood criterion. It assumes the use of a special class of pilots, called *near-i.i.d.* (independent identically distributed) sequences, with impulsive fourth-order moments. By the help of such pilots, the proposed method can estimate frequency offsets without the need for channel information. The pilots of GSM and IS-136 mobile communication systems have been observed as are *near-i.i.d.*, and statistical analysis indicates that the proposed estimate is almost unbiased if the pilot is *near-i.i.d.* The advantage of the proposed estimator over conventional methods is demonstrated via computer simulation.

Index Terms—Frequency estimation, frequency-selective fading, *near-i.i.d.* preamble, PSK.

I. INTRODUCTION

IN THE COHERENT demodulation of phase-shift keying (PSK) signals, carrier frequency offset due to either limited oscillator precision or the Doppler effect can cause a significant loss in performance. Correcting the frequency offset in such demodulation will help improve receiver performance and relieve the stringent accuracy requirements on an oscillator. A traditional method for frequency recovery is to employ an automatic frequency control (AFC) loop [1]–[7] in the receiver. Such a loop operates independently of the timing information and can compensate for a large frequency offset, which is of the order of the symbol rate $1/T$; however, the frequency tracking behavior of an AFC loop is rather poor [6], [7]. Therefore, AFC loops are mainly useful only for initial frequency acquisition. When the frequency offset is much smaller than $1/T$, the timing information can be recovered first and then exploited for a frequency estimation.¹ This procedure leads to a class of timing-directed frequency estimators exhibiting excellent tracking behavior. This class encompasses data-aided frequency estimators [6]–[16] which exploit both timing information and a training sequence. A comprehensive review of carrier frequency recovery techniques can be found in [6] and [7].

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E.-R. Jeong and Y. H. Lee are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Yusong-gu, Taejeon, 305-701, Korea (e-mail: yohlee@ee.kaist.ac.kr).

G. Choi is with the Samsung Electronics Co., Ltd., Yongin-City, Kyunggi-Do, Korea (e-mail: gkchoi@bear.telecom.samsung.co.kr).

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¹In certain burst-mode communication systems, the timing is derived during the preliminary bursts and, thereafter, periodically updated [8].

The data-aided estimators in [9]–[15] are based on a maximum likelihood (ML) criterion, which is derived under the assumption of an additive white Gaussian noise (AWGN) channel [9]–[13] or flat fading channel [14], [15]. Consequently, their performances tend to degrade in frequency-selective channels which cause intersymbol interference (ISI). In [16], an ML-based estimator for such dispersive channels is derived by extending the estimator in [12]. It reduces the effect of ISI using autocorrelations of the channel impulse response. However, its performance is degraded where the estimates of the channel autocorrelations become inaccurate.

The objective of this paper is to develop a frequency offset estimator for frequency-selective fading channels. Under the assumption that the channel impulse response is known, an ML-type estimator for frequency-selective fading channels is derived by first extending the results in [12]. Thereafter, the resulting estimate, called the baseline estimate, is modified into the proposed estimator by help of a special class of pilots which are called *near-independent identically distributed* (*i.i.d.*) sequences. It will be shown that the proposed estimate does not require channel information when *near-i.i.d.* pilots are employed. Statistical analysis indicates that the proposed estimator can perform like the baseline estimator.

The organization of this paper is as follows. Section II presents the signal model and the proposed estimator is developed in Section III. The estimator is statistically analyzed in Section IV. Finally, Section V outlines simulation results demonstrating the advantage of the proposed estimator over existing methods.

II. COMMUNICATION SYSTEM MODEL

The baseband system model considered in this paper is shown in Fig. 1. Here, $d(j)$ denotes the transmitted M-ary PSK symbols, $h(t)$ is a baseband pulse shape, $n(t)$ is AWGN, θ is an initial random phase, and Δf represents a carrier frequency offset. Assuming perfect symbol timing recovery, the output of the receive filter sampled at $t = kT$ is

$$r(k) = e^{j(2\pi\Delta f kT + \theta)} \sum_{l=0}^L d(k-l)g_k(l) + \eta(k) \quad (1)$$

where $g_k(l)$ is the impulse response of the equivalent channel at time k due to an impulse that is applied l time units earlier: it describes both $h(t)$ and the Rayleigh fading channel block of Fig. 1 in discrete time domain, and its duration is $L + 1$. $\eta(k)$ is assumed to be AWGN with a variance of $2\sigma_\eta^2$. In the case of

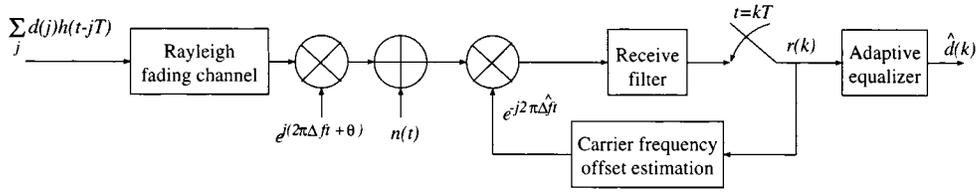


Fig. 1. Baseband system model.

flat fading, $L = 0$ (ISI-free), and for frequency-selective fading channels, $L \geq 1$.

III. DERIVATION OF PROPOSED ESTIMATOR

The derivation of the proposed estimator consists of two steps. First, an ML-type estimator is derived under the assumption that channel parameters are known. This step is a direct extension of the derivation in [12]. Second, the ML-type estimator is modified to an unknown parameter case by the help of a near-i.i.d. preamble.

A. Derivation With Known Channel Parameters

Suppose that the training symbols $\{d(k) | k = 1, \dots, K\}$ are available, and that $\{g_k(l)\}$ is a fixed ISI channel over the training period, i.e., $g_k(l) = g(l)$ for $k = 1, \dots, K$. Assuming that $\{g(l) | 0 \leq l \leq L\}$ are known, the conditional joint probability density function (pdf) of the received samples is given by

$$f(r(1), \dots, r(K) | \theta, \Delta f) = \left(\frac{1}{2\pi\sigma_\eta^2} \right)^K \exp \left(-\frac{1}{2\sigma_\eta^2} \sum_{k=1}^K |r(k) - \bar{r}(k)|^2 \right) \quad (2)$$

where

$$\bar{r}(k) = e^{j(2\pi\Delta f kT + \theta)} \sum_{l=0}^L d(k-l)g(l) \quad (3)$$

and $d(k)$ is assumed to be zero when $k \leq 0$. After dropping the terms irrelevant to the estimation, the log likelihood function corresponding to this pdf becomes

$$\begin{aligned} \Lambda(\tilde{\theta}, \Delta\tilde{f}) &= \sum_{k=1}^K \{r(k)\bar{r}^*(k) + r^*(k)\bar{r}(k) - |\bar{r}(k)|^2\} \\ &= 2\text{Re} \left[\sum_{k=1}^K r(k)e^{-j(2\pi\Delta\tilde{f}kT + \tilde{\theta})} \left(\sum_{l=0}^L d^*(k-l)g^*(l) \right) \right] \\ &\quad - \sum_{k=1}^K \left| \sum_{l=0}^L d(k-l)g(l) \right|^2 \end{aligned} \quad (4)$$

$$\leq 2\text{Re} \left[\sum_{k=1}^K r(k)e^{-j(2\pi\Delta\tilde{f}kT + \tilde{\theta})} \left(\sum_{l=0}^L d^*(k-l)g^*(l) \right) \right] \quad (6)$$

$$\leq 2 \left| \sum_{k=1}^K r(k)e^{-j2\pi\Delta\tilde{f}kT} \left(\sum_{l=0}^L d^*(k-l)g^*(l) \right) \right|. \quad (7)$$

The expression in (6) is obtained by discarding the second term in (5) which is irrelevant to the estimation, and the inequality in (7) comes from the fact that $\text{Re}[e^{-j\theta} \cdot x] \leq |x|$ where x is a complex number. Note that the arguments of the log likelihood function are denoted by $\tilde{\theta}$ and $\Delta\tilde{f}$. The ML estimates of θ and Δf are obtained by maximizing $\Lambda(\tilde{\theta}, \Delta\tilde{f})$ with respect to $\tilde{\theta}$ and $\Delta\tilde{f}$. The inequality in (7) becomes equality when

$$\tilde{\theta} = \arg \left[\sum_{k=1}^K r(k)e^{-j2\pi\Delta\tilde{f}kT} \left(\sum_{l=0}^L d^*(k-l)g^*(l) \right) \right]. \quad (8)$$

Therefore, the ML estimate of θ is given by the right-hand side of (8). The ML estimate of Δf can be derived by maximizing (7) with respect to $\Delta\tilde{f}$. Unfortunately, a closed-form solution to the maximization problem cannot be obtained; thus, an alternative suboptimal technique is developed in the sequel.

Differentiating the square of (7) with respect to $\Delta\tilde{f}$ and setting the result equal to zero yields

$$\begin{aligned} \sum_{i=0}^L \sum_{l=0}^L \sum_{m=1}^K \sum_{n=1}^K (m-n)g(i)g^*(l)r(m)r(n)^* \\ \times d(n-i)d^*(m-l)e^{-j2\pi\Delta\tilde{f}(m-n)T} = 0 \end{aligned} \quad (9)$$

or equivalently, as shown in (10) at the bottom of the page, where $R_{i,l}(m)$ denotes the estimated crosscorrelation between sequences $\gamma_i(k)$ and $\gamma_l(k)$, defined as

$$\begin{aligned} R_{i,l}(m) &\triangleq \frac{1}{K-m-\max(i,l)} \\ &\quad \times \sum_{k=m+1+\max(i,l)}^K \gamma_i(k)\gamma_l^*(k-m) \end{aligned} \quad (11)$$

and

$$\gamma_l(k) = r(k)d^*(k-l)g^*(l). \quad (12)$$

When m is close to $K-1-\max(i,l)$, $R_{i,l}(m)$ becomes a poor estimate of the correlation, since the number of terms in

$$\text{Im} \left[\sum_{i=0}^L \sum_{l=0}^L \sum_{m=1}^{K-1-\max(i,l)} m(K-m-\max(i,l))R_{i,l}(m)e^{-j2\pi\Delta\tilde{f}mT} \right] = 0 \quad (10)$$

the sum (11) building up such an estimate is small. In (10), $R_{i,l}(m)$ is weighted by a parabolic window function $w_{i,l}(m) = m(K - m - \max(i, l))$, $m = 1, 2, \dots, K - 1 - \max(i, l)$. In a suboptimum implementation of the frequency estimator, $w_{i,l}(m)$ may be replaced by a rectangular sequence defined as

$$w'_{i,l}(m) = \begin{cases} 1, & 1 \leq m \leq N \\ 0, & N + 1 \leq m \end{cases} \quad (13)$$

for some constant $N \leq K - L - 1$, to discard the unreliable correlation estimates close to $m = K - 1 - \max(i, l)$, while retaining the good correlation samples near $m = 1$ [12]. Equation (10) becomes

$$\text{Im} \left[\sum_{m=1}^N \left(\sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) e^{-j2\pi\Delta\tilde{f}mT} \right) \right] = 0. \quad (14)$$

To solve for $\Delta\tilde{f}$, (14) is rewritten as

$$\begin{aligned} & \text{Im} \left[\sum_{m=1}^N \sum_{m'=1}^N \left(\sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) e^{-j2\pi\Delta\tilde{f}m'T} \right) \right. \\ & \left. - \sum_{m=1}^N \sum_{m'=1, m' \neq m}^N \left(\sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) e^{-j2\pi\Delta\tilde{f}m'T} \right) \right] = 0. \end{aligned} \quad (15)$$

Under the assumption that $\sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) \approx C e^{j2\pi m \Delta f T}$, which is justified in [16], the imaginary parts of the first and second terms inside the bracket in (15) tend to zero if $\Delta\tilde{f} \approx \Delta f$. Using the first term,² an approximate solution for (14) is obtained by solving

$$\text{Im} \left[\left(\sum_{m=1}^N \sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) \right) \left(\sum_{m'=1}^N e^{-j2\pi\Delta\tilde{f}m'T} \right) \right] = 0. \quad (16)$$

Note that for a small $\Delta f T$, i.e., $\Delta f T \ll 1$

$$\arg \left\{ \sum_{m'=1}^N e^{-j2\pi\Delta\tilde{f}m'T} \right\} \approx -\pi\Delta\tilde{f}T(N+1). \quad (17)$$

From (16) and (17), we get a frequency estimator given by

$$\Delta\hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N \sum_{i=0}^L \sum_{l=0}^L R_{i,l}(m) \right\} \quad N \leq K - L - 1. \quad (18)$$

This estimator becomes identical to the one in [16], if $g^*(i)g(l)$ contained in $R_{i,l}(m)$ is replaced by $E[g_k^*(i)g_{k-m}(l)]$. In (18), if $R_{i,l}(m)$, $i \neq l$ are neglected for simplicity, then, the estimate in (18) reduces to

$$\Delta\hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N \sum_{l=0}^L R_{l,l}(m) \right\} \quad N \leq K - L - 1. \quad (19)$$

²The imaginary part of the second term inside the bracket in (15) is close to zero when Δf is given by the solution of (16).

Using (12) in (11), $R_{i,l}(m)$ in (19) can be expressed as

$$\begin{aligned} R_{l,l}(m) &= |g(l)|^2 \cdot \frac{1}{K - m - l} \\ &\times \sum_{k=m+1+l}^K (r(k)d^*(k-l)) \\ &\times (r(k-m)d^*(k-m-l))^* \\ &= |g(l)|^2 \cdot H_l(m) \end{aligned} \quad (20)$$

where

$$\begin{aligned} H_l(m) &= \frac{1}{K - m - l} \\ &\times \sum_{k=m+1+l}^K (r(k)d^*(k-l)) \\ &\times (r(k-m)d^*(k-m-l))^*. \end{aligned} \quad (21)$$

Therefore, $\Delta\hat{f}$ in (19) can be rewritten as

$$\Delta\hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N \sum_{l=0}^L |g(l)|^2 H_l(m) \right\}. \quad (22)$$

This estimate, illustrated in Fig. 2, serves as the baseline system from which the proposed estimator is derived. The estimator in (22) reduces to one in [12] when $L = 0$ and $g(0) = 1$; its acquisition range is $|\Delta f T| \leq 1/(N+1)$. In practice, the use of this estimate in a multipath fading environment is difficult because the magnitude of the channel parameter $|g(l)|$ is unknown and the estimation of such parameters requires frequency offset compensation. Section III-B presents the modification of the estimator in (22) for practical application.

B. Extension to Unknown Channel Parameter Case

Suppose that $\eta(k)$ in (1) is negligible. Then $r(k) \approx \bar{r}(k)$ in (3) and $H_l(m)$ in (21) can be approximated as

$$\begin{aligned} H_l(m) &\approx \frac{e^{j2\pi\Delta f m T}}{K - m - l} \\ &\times \sum_{k=m+1+l}^K \left(\sum_{i=0}^L g(i)d(k-i)d^*(k-l) \right) \\ &\times \left(\sum_{n=0}^L g(n)d(k-m-n)d^*(k-m-l) \right)^* \\ &= e^{j2\pi\Delta f m T} \sum_{i=0}^L \sum_{n=0}^L \frac{1}{K - m - l} \\ &\times \sum_{k=m+1+l}^K (g(i)d(k-i)d^*(k-l)) \\ &\cdot (g(n)d(k-m-n)d^*(k-m-l))^*. \end{aligned} \quad (23)$$

To proceed further, the following assumption is made:

$$\begin{aligned} & \frac{1}{K - m - l} \sum_{k=m+1+l}^K (d(k-i)d^*(k-l)) \\ & \cdot (d(k-m-n)d^*(k-m-l))^* \approx \begin{cases} 1, & i = n = l \\ 0, & \text{otherwise} \end{cases} \end{aligned} \quad (24)$$

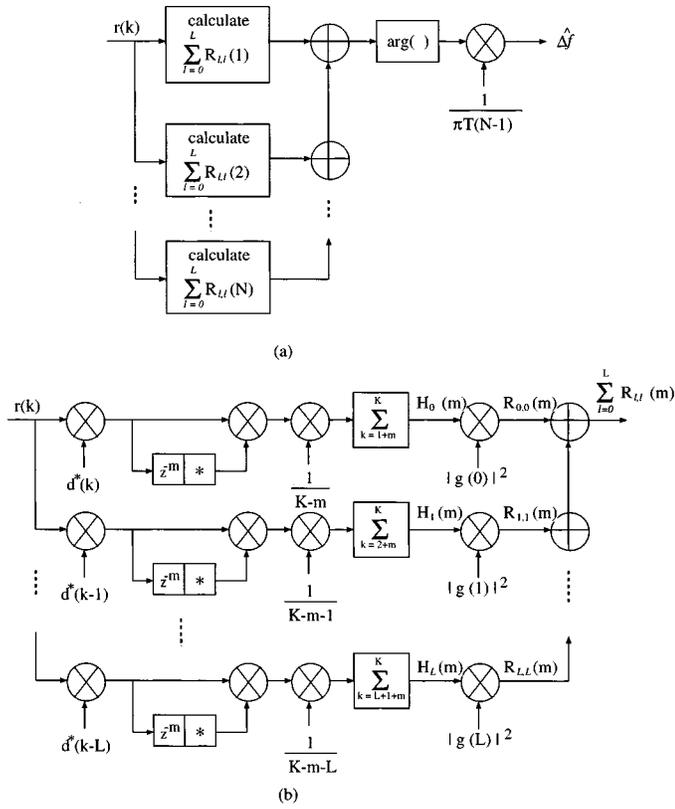


Fig. 2. (a) Frequency estimator in (22). (b) m -th block of (a) evaluating $\sum_{l=0}^L R_{l,l}(m)$.

for $0 \leq i \leq L, 0 \leq l \leq L, 0 \leq n \leq L$ and $1 \leq m \leq N$. This assumption is justified if $d(k)$ can be modeled as an i.i.d. random process with a zero mean and variance of one because for such $d(k)$

$$E[(d(k-i)d^*(k-l))(d(k-m-n)d(k-m-l))^* \times (k-m-l)^*] = \begin{cases} 1, & i=n=l \\ 0, & \text{otherwise.} \end{cases} \quad (25)$$

In general, (24) is satisfied if $d(k)$ is an outcome of such an i.i.d. process. This is called a near-i.i.d. sequence. For a near-i.i.d. training sequence, $H_l(m)$ in (23) reduces to

$$H_l(m) \cong |g(l)|^2 e^{j2\pi \Delta f m T}. \quad (26)$$

Now the channel parameter $|g(l)|^2 \cong |H_l(m)|$ and (22) can be modified as

$$\Delta \hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N \sum_{l=0}^L |H_l(m)| H_l(m) \right\}. \quad (27)$$

This estimate, illustrated in Fig. 3, does not require any information about the channel. Therefore, it should be useful for practical systems if a near-i.i.d. training sequence is available.

A major difficulty encountered in using the estimate in (27) is, of course, caused by the need of near-i.i.d. sequences. In general, such a sequence can be found through an exhaustive search among all possible sequences [18]–[20]. For binary data, pseudonoise (PN) sequences [23] are near-i.i.d. One interesting fact is that the pilot sequences of some mobile communication

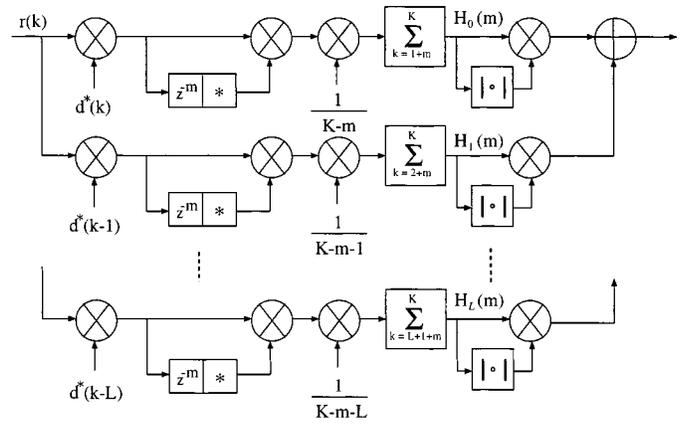


Fig. 3. Proposed estimator in (27).

TABLE I TRAINING SEQUENCES OF GSM AND IS-136

GSM	0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1
IS-136	$-\frac{\pi}{4}, -\frac{\pi}{4}, -\frac{\pi}{4}, 3\frac{\pi}{4}, 3\frac{\pi}{4}, 3\frac{\pi}{4}, -3\frac{\pi}{4}, 3\frac{\pi}{4}, -3\frac{\pi}{4}, -\frac{\pi}{4}, 3\frac{\pi}{4}, \frac{\pi}{4}, -\frac{\pi}{4}, -\frac{\pi}{4}$

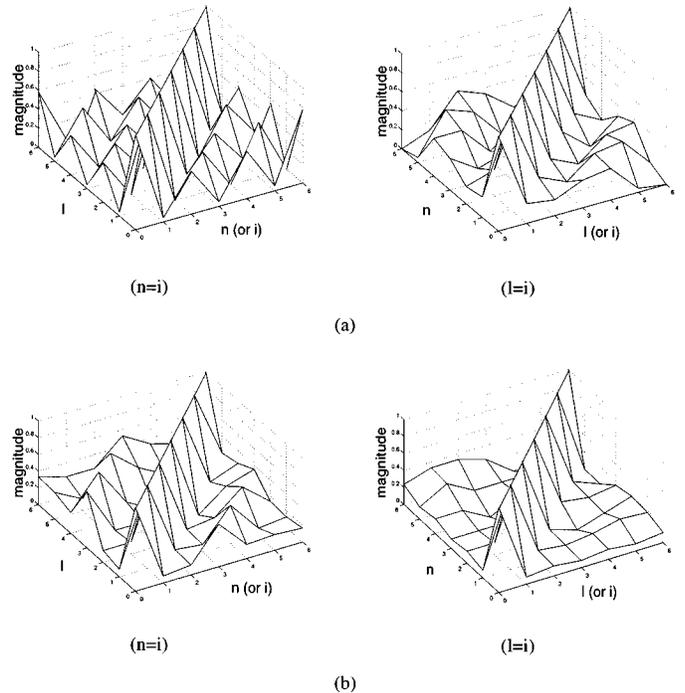


Fig. 4. Values of fourth order moments defined in (27) for $m = 2$. (a) GSM with $L = 6$. (b) IS-136 with $L = 6$.

systems, such as GSM and IS-136, are near-i.i.d. sequence satisfying (24). This is demonstrated in Table I and Fig. 4 which show the pilot sequences and their fourth order moments as defined in (24), respectively. Since the fourth-order moments are nearly impulsive, the sequences indeed satisfy (24); and thus, the proposed estimator in (27) can be applied to GSM³ and IS-136 systems.

³Gaussian minimum shift keying (GMSK) which is the recommended modulation format in GSM [22], can be regarded as a linear offset modulation, i.e., offset quadrature phase-shift keying (OQPSK) with an appropriate elementary pulse shape [12].

IV. PERFORMANCE ANALYSIS

The first part of this section derives the mean and mean square error (MSE) of the baseline estimate in (22). Thereafter, it is shown through simulation that the mean and MSE values of the proposed estimate in (27) are close to those of the baseline estimate.

A. Derivation of Mean and MSE of Baseline Estimate

The mean and MSE of the estimate in (22) are derived under the assumptions of high carrier-to-noise ratio (CNR) and low-frequency deviation ($\Delta f T \ll 1$). The procedure for deriving the values is outlined in the Appendix. The final results are as shown in (28) and (29) at the bottom of the page, where

$$\begin{aligned}
 \Phi_{\text{IMA}}(k, m, l) &= \text{Im}\{\Phi(k, m, l)\} \\
 \Phi(k, m, l) &= \{g^*(l)\psi(k, l) + g(l)\psi^*(k - m, l) \\
 &\quad + \psi(k, l)\psi^*(k - m, l)\}|g(l)|^2 \\
 N_1(k, m, l) &= \{g^*(l)d^*(k - l) + \psi^*(k - m, l) \\
 &\quad \times d^*(k - l)\}|g(l)|^2 \\
 &\quad \times e^{-j(\pi\Delta f T(2k - 2m + N + 1) + \theta)} \\
 N_2(k, m, l) &= \{g(l)d(k - l - m) + \psi(k, l) \\
 &\quad \times d(k - l - m)\}|g(l)|^2 \\
 &\quad \times e^{j(\pi\Delta f T(2k - N - 1) + \theta)} \\
 \psi(k, l) &= \sum_{i=0, i \neq l}^L d(k - i)d^*(k - l)g(i) \quad (30)
 \end{aligned}$$

and $u(k)$ is the unit step function. The mean estimation error μ_e in (28) represents the bias of the estimate.

Next, it is shown that the baseline estimate is almost unbiased if the preamble $d(k)$ is a near-i.i.d. sequence with impulsive

second- and fourth-order moments. Referring to (28), the first term inside the parenthesis can be rewritten as

$$\begin{aligned}
 &\sum_{m=1}^N \sum_{l=0}^L \frac{1}{K - m - l} \sum_{k=m+1+l}^K \Phi_{\text{IMA}}(k, m, l) \\
 &= \text{Im} \left\{ \sum_{m=1}^N \sum_{l=0}^L \frac{|g(l)|^2}{K - m - l} \right. \\
 &\quad \times \sum_{k=m+1+l}^K [g^*(l)\psi(k, l) + g(l)\psi^*(k - m, l) \\
 &\quad \left. + \psi(k, l)\psi^*(k - m, l)] \right\}. \quad (31)
 \end{aligned}$$

From the definition of $\psi(k, l)$ in (30), it can be seen that the first and second terms inside the bracket in (31) are close to zero if the preamble $d(k)$ has impulsive second-order moments. Similarly, it can be observed that the third term inside the bracket in (31) is close to zero when a preamble with impulsive fourth-order moments is employed. Therefore, the proposed estimate is almost unbiased if the preamble includes impulsive second- and fourth-order moments.

Under the assumption that the bias can be neglected, the Cramér-Rao lower bound (CRLB) for the estimate in (22) can be derived as follows. Using the conditional density function in (2), the Fisher information matrix can be expressed as shown in (32) at the bottom of the next page. The CRLB can be given by the (1, 1)th element of \mathbf{J}^{-1} . Specifically, as shown in (33) at the bottom of the next page, where $\alpha_k = |\sum_{l=0}^L d(k - l)g(l)|^2$.

B. Comparison Between Baseline and Proposed Estimators

Fig. 5 shows the system model used for the simulation. Two kinds of pilots were considered: one is the Gold sequence with $K = 127$ [23] and the other is the IS-136 pilot with $K = 14$. For the Gold sequence, $\{d(k)\}$ was generated via binary PSK. In the

$$\mu_e = E[\Delta \hat{f} - \Delta f] \approx \frac{1}{\pi N(N + 1)T \sum_{l=0}^L |g(l)|^4} \sum_{m=1}^N \sum_{l=0}^L \left(\frac{1}{K - m - l} \sum_{k=m+1+l}^K \Phi_{\text{IMA}}(k, m, l) \right) \quad (28)$$

and

$$\begin{aligned}
 \sigma_e^2 &= E[(\Delta \hat{f} - \Delta f)^2] \approx \frac{1}{\pi^2 N^2 (N + 1)^2 T^2 \left(\sum_{l=0}^L |g(l)|^4 \right)^2} \\
 &\quad \left[\left\{ \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K - m - l} \sum_{k=m+1+l}^K \Phi_{\text{IMA}}(k, m, l) \right\}^2 \right. \\
 &\quad \left. + \sigma_\eta^2 \sum_{k=2}^K \left\{ \left| \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K - m - l} N_1(k, m, l) u(k - m - 1 - l) \right|^2 \right. \right. \\
 &\quad \left. \left. + \left| \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K - m - l} N_2(k, m, l) u(k - m - 1 - l) \right|^2 \right\} \right] \quad (29)
 \end{aligned}$$

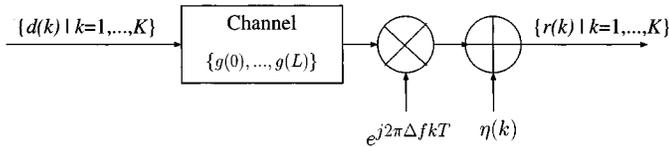


Fig. 5. System model used for simulation.

case of the IS-136 pilot, $\pi/4$ -DQPSK (differentially encoded quadrature PSK) which is the recommended modulation format for IS-136 [21] was used. The channel parameters were assumed to be fixed at $\{g(0), g(1)\} = \{(1/\sqrt{5}), (2/\sqrt{5})\}$ ($L = 1$).⁴ A total of 5000 $\{r(k)\}$ sequences were generated for each Δf . The frequency offset Δf was estimated for various values of the parameter N , which is the maximum time lag of the correlation $R_{i,l}(m)$ in (20).

Figs. 6 and 7 show the mean estimation errors that were obtained via the simulation along with the numerical results obtained from (28). For the Gold sequence, all the estimators were almost unbiased in their acquisition range, and a remarkably good agreement between the numerical and simulation results was observed. As expected, the baseline and proposed estimators behaved in a similar manner. For the IS-136 pilot, an observation similar to the above for the Gold sequence was made when $N = 1$. However, some bias appeared in the simulation when $N = 2$ and 3, and it increased as the normalized frequency offset $\Delta f T$ increased. This happened because the baseline estimator is biased unless the preamble is i.i.d. with impulsive second and fourth-order moments. In this case ($N = 2$ and 3), the simulation results for the baseline estimate did not agree with the numerical results from (28). This is because (28) is only valid for $\Delta f T \ll 1$ [when $N = 1$, (28) holds for the larger values of $\Delta f T$ —see the Appendix]. When $N = 3$, even the numerical results exhibited some bias, which was constant irrespective of $\Delta f T$ ($\mu_e = -3.472 \times 10^{-4}$). This bias was caused by the fact that the value on the left side of (24), when the condition $i = n = l$ was not met, increased as N increased. For all

⁴Simulation results for some other channel parameters, which are not reported here, indicate that the baseline and proposed estimators performed in a similar manner irrespective of the channels.

values of N , the baseline and proposed estimators performed in a similar manner.

Figs. 8 and 9 show the normalized MSE, $(\sigma_e T)^2$, plotted as a function of E_b/N_0 for $\Delta f = 0$ (the MSE was not highly dependent upon Δf , as long as $|\Delta f T| \leq 1/(N + 1)$ —see Figs. 10 and 11). As expected, the MSEs decreased as E_b/N_0 increased. The baseline and proposed estimators exhibited similar performances. A remarkably good agreement between the analysis and the simulation was observed. Also shown is the normalized CRLB. The MSEs approached the CRLB as N increased. In Fig. 9, the MSE for $N = 3$ converged to 1.205×10^{-7} as E_b/N_0 increased. This happened because of the bias $\mu_e = -3.472 \times 10^{-4}$ which can be observed in Fig. 7 when $N = 3$ and $\Delta f = 0$. (When $N = 1$ and 2, the bias for $\Delta f = 0$ was negligible and, thus, the corresponding MSE's decreased as E_b/N_0 increased.)

V. APPLICATION TO FREQUENCY-SELECTIVE FADING CHANNEL

In this section, the baseline and proposed estimators are compared with the estimators in [12] and [16] when they are applied to a Rayleigh fading channel, which is slowly time varying. For the simulation, the system model depicted in Fig. 5 was also used. The input sequence was the IS-136 pilot ($K = 14$). The digital channel $\{g_k(l)\}$ was obtained by sampling (T -spaced) the convolution of an analog channel $c(t)$ and the impulse responses of the transmit and receive filters, which were square root raised cosine filters with roll-off factor 0.35. The analog channel was a two-ray Rayleigh fading channel [17], which was modeled as

$$c(t) = \alpha_0(t)\delta(t) + \alpha_1(t)\delta(t - \tau) \quad (34)$$

where $\alpha_0(t)$ and $\alpha_1(t)$ are independent zero-mean complex Gaussian processes and τ is the time delay between the two rays. For simplicity, a normalized uniform delay power profile was assumed, i.e., $E[|\alpha_0(t)|^2] = E[|\alpha_1(t)|^2] = 0.5$. The parameter τ was assumed to be $T/2$ and $3T/4$, which lead to a channel where $L = 2$. The carrier frequency was 900 MHz and

$$\begin{aligned} \mathbf{J} &= -E \begin{bmatrix} \frac{\partial^2 \ln(f(r(1), \dots, r(K)) | \theta, \Delta f)}{\partial \Delta f^2} & \frac{\partial^2 \ln(f(r(1), \dots, r(K)) | \theta, \Delta f)}{\partial \Delta f \partial \theta} \\ \frac{\partial^2 \ln(f(r(1), \dots, r(K)) | \theta, \Delta f)}{\partial \theta \partial \Delta f} & \frac{\partial^2 \ln(f(r(1), \dots, r(K)) | \theta, \Delta f)}{\partial \theta^2} \end{bmatrix} \\ &= \frac{1}{\sigma_\eta^2} \begin{bmatrix} \sum_{k=1}^K 4\pi^2 k^2 T^2 \left| \sum_{l=0}^L d(k-l)g(l) \right|^2 & \sum_{k=1}^K 2\pi k T \left| \sum_{l=0}^L d(k-l)g(l) \right|^2 \\ \sum_{k=1}^K 2\pi k T \left| \sum_{l=0}^L d(k-l)g(l) \right|^2 & \sum_{k=1}^K \left| \sum_{l=0}^L d(k-l)g(l) \right|^2 \end{bmatrix}. \end{aligned} \quad (32)$$

$$\sigma_e^2 = E[(\Delta f - \hat{\Delta f})^2] \geq \frac{\sigma_\eta^2 \sum_{k=1}^K \alpha_k}{4\pi^2 T^2 \left\{ \sum_{k=1}^K k^2 \alpha_k \cdot \sum_{k=1}^K \alpha_k - \left(\sum_{k=1}^K k \alpha_k \right)^2 \right\}} \quad (33)$$

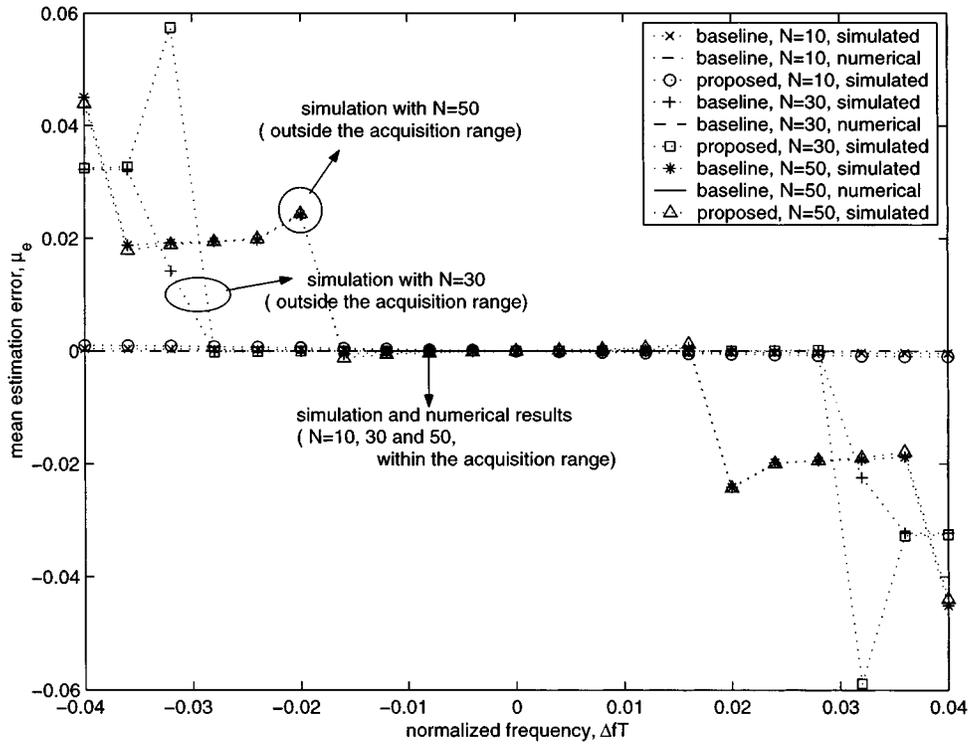


Fig. 6. Mean estimation errors for fixed channel ($L = 1$ and $E_b/N_0 = 25$ dB) with Gold pilot.

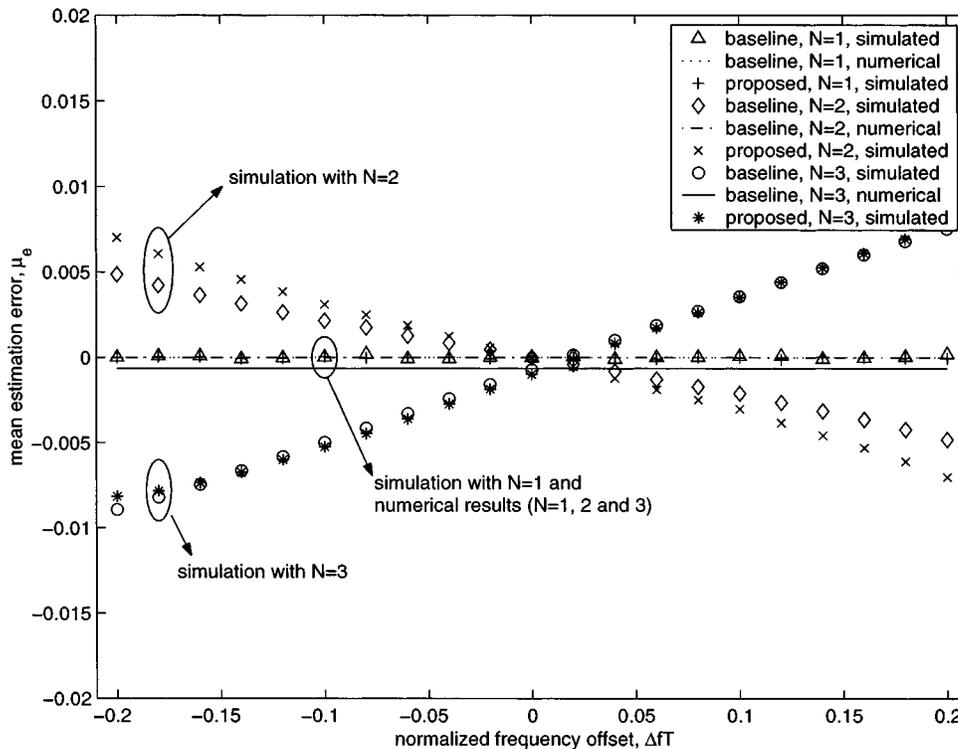


Fig. 7. Mean estimation errors for fixed channel ($L = 1$ and $E_b/N_0 = 25$ dB) with IS-136 pilot.

the vehicle speed was 100 km/h. The normalized frequency offset, ΔfT , varied from 0 to 0.5.

After obtaining the time-varying channel $\{g_k(l)\}$, the estimators were implemented as follows:

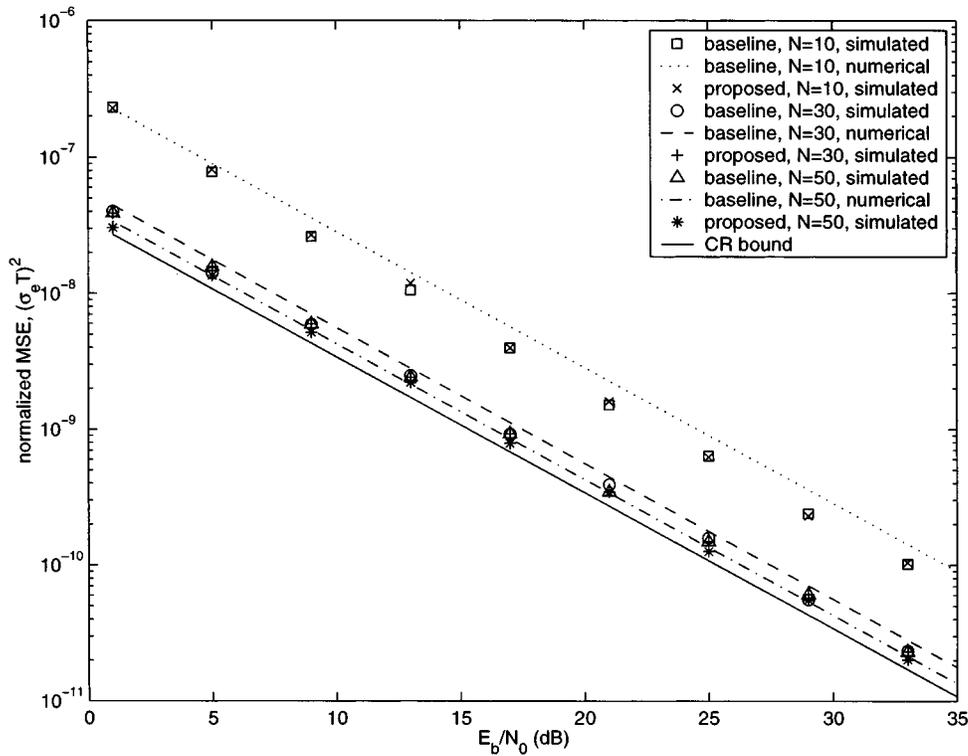


Fig. 8. Normalized MSE for fixed ISI channel ($L = 1, \Delta f = 0$) with Gold pilot.

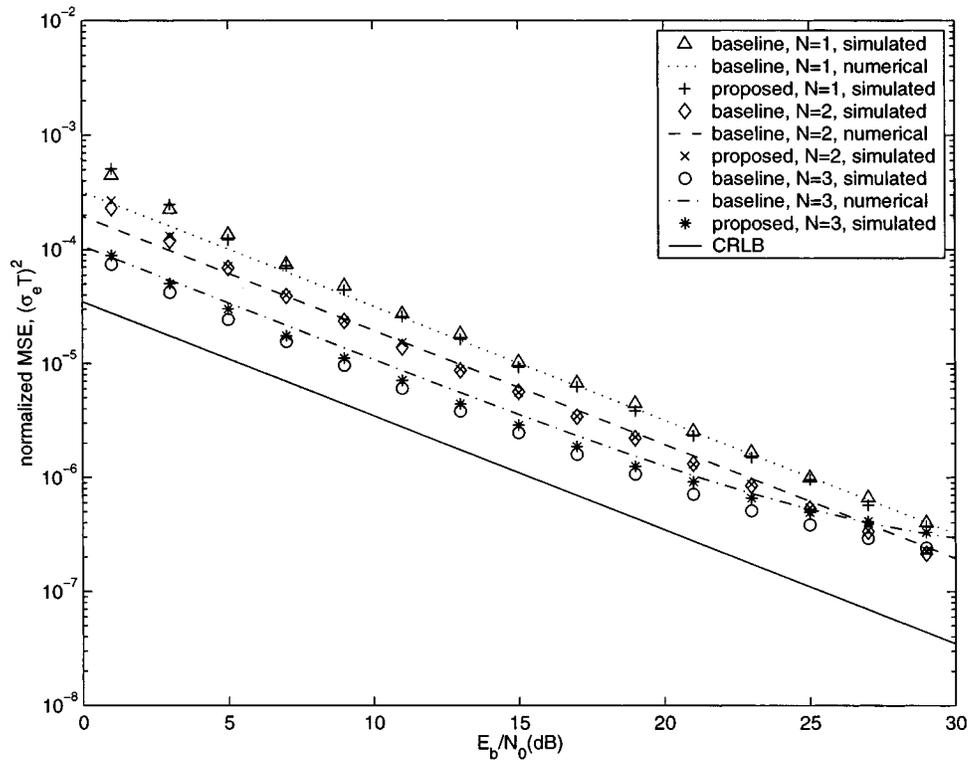


Fig. 9. Normalized MSE for fixed ISI channel ($L = 1, \Delta f = 0$) with IS-136 pilot.

1) *Baseline Estimator*: Equation (22) was evaluated. The time-invariant channel impulse response $\{|g(l)|^2\}$, which is required by this estimator, was evaluated by averaging $\{|g_k(l)|^2\}$ over the preamble period, i.e., $|g(l)|^2 = (1/K) \sum_{k=1}^K |g_k(l)|^2$

for $0 \leq l \leq L$. This estimate is impractical but it can provide an upper bound on the performance of the proposed estimator.

2) *Proposed Estimator*: Equation (27) was evaluated, ignoring the time-varying nature of the channel.

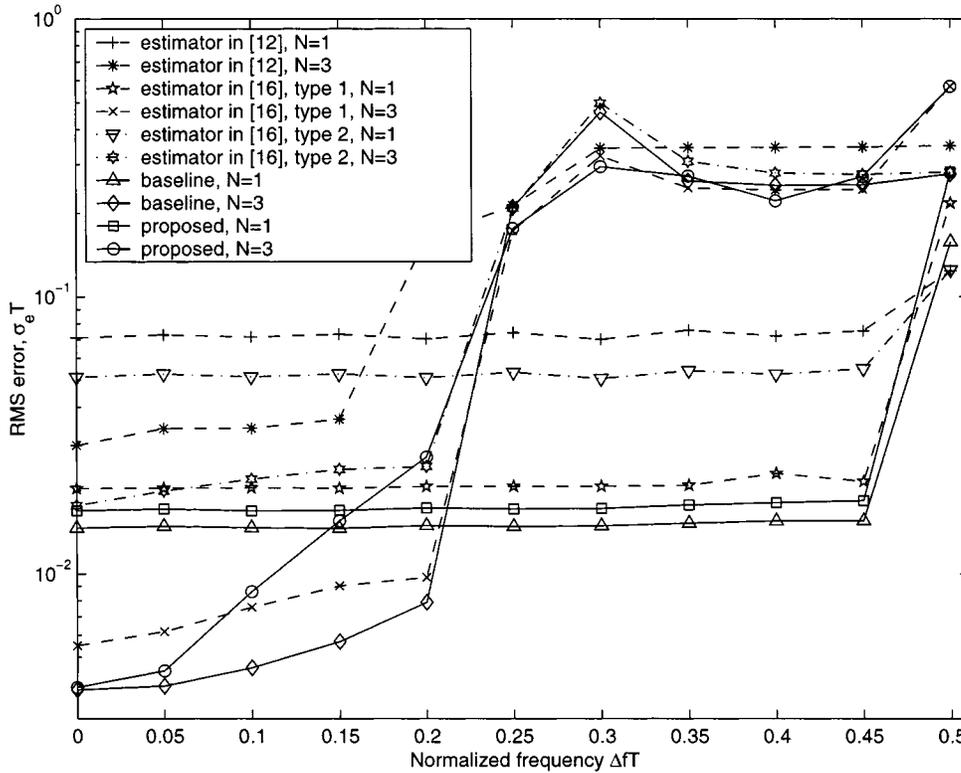


Fig. 10. Frequency offset estimation performance in frequency-selective fading channel, $\tau = T/2$ ($L = 2$ and $E_b/N_0 = 25$ dB).

3) *Estimator in [12]*: This is given by

$$\Delta \hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N R'(m) \right\} \quad (35)$$

where $R'(m) = (1/K - m) \sum_{k=m+1}^K r(k)r^*(k-m)$.

4) *Estimator in [16]*: This estimator is expressed as

$$\Delta \hat{f} = \frac{1}{\pi T(N+1)} \arg \left\{ \sum_{m=1}^N \sum_{i=0}^L \sum_{l=0}^L R''_{i,l}(m) \right\} \quad (36)$$

where

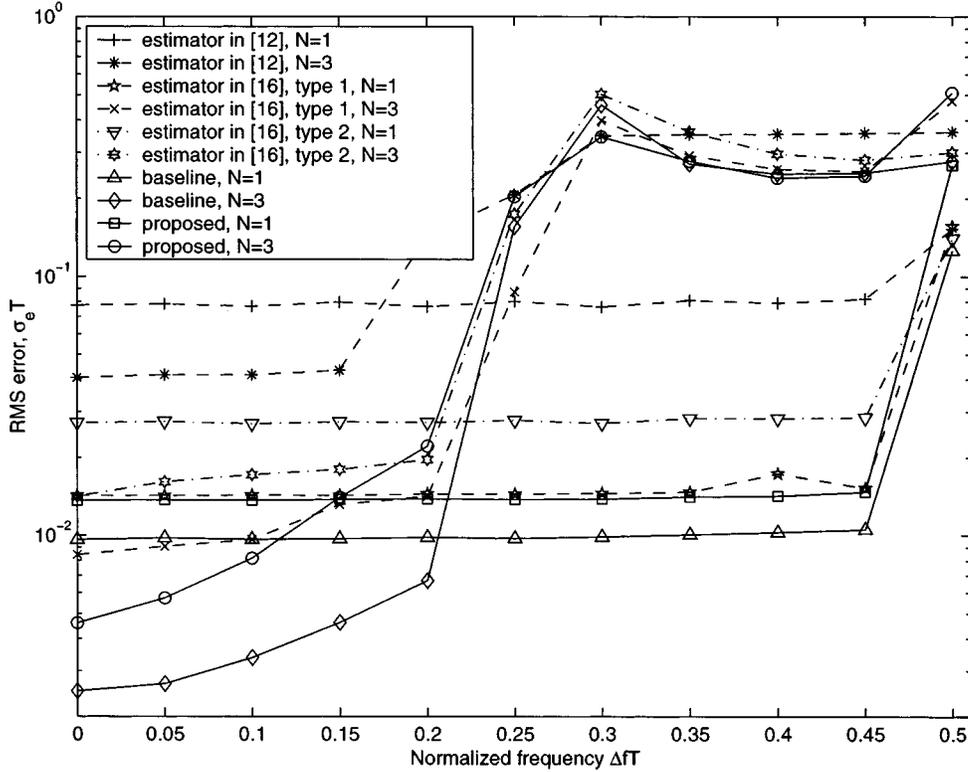
$$\begin{aligned} R''_{i,l}(m) &= \frac{1}{K - m - \max(i,l)} \\ &\times \sum_{k=m+1+\max(i,l)}^K r(k)r^*(k-m) \\ &\times d^*(k-i)d(k-m-l)\hat{E}[g_k^*(i)g_{k-m}(l)] \end{aligned} \quad (37)$$

and $\hat{E}[g_k^*(i)g_{k-m}(l)]$ denotes an estimate of channel autocorrelation. Two types of channel autocorrelation estimates were considered. One was an empirical estimate which was obtained by averaging $g_k^*(i)g_{k-m}(l)$ over one million samples, i.e., $\hat{E}[g_k^*(i)g_{k-m}(l)] = (1/10^6) \sum_{k=1}^{10^6} g_k^*(i)g_{k-m}(l)$. The other was based on an exponential model given by $\hat{E}[g_k^*(i)g_{k-m}(l)] = e^{(-iT/\tau)}\delta(i-l)$, where τ is the time delay in (34) [16]. The empirical and model-based estimates will be referred to as the types 1 and 2 estimates, respectively. The type 1 estimate is impractical but it should be more accurate than the type 2. It is expected that the estimator in [16] associated

with the type 1 estimate can provide an upper bound on the performance of this estimator.

Figs. 10 and 11 show the simulation results. An increased N resulted in more accurate estimates for all the estimators at the expense of a narrower acquisition range. When $N = 1$, the rms errors were independent of $\Delta f T$, as long as $\Delta f T$ was within the acquisition range ($|\Delta f T| \leq 1/(N+1)$). When $N = 3$, however, this was not true: some increase in the rms errors was observed even within the acquisition range, as $\Delta f T$ increased. This increase was caused by the bias of the estimators (Fig. 7). As expected, the baseline estimator performed the best. The estimator in [12] performed the worst, because it was designed for AWGN channels. Among the estimators from [16], those with the type 1 channel autocorrelation estimate outperformed those with the type 2. This happened because the empirical estimate (type 1) was more accurate than the model-based estimate (type 2). The performance of the estimator in [16] became comparable with that of the proposed estimator, when the type 1 estimate was employed. However, use of the type 1 estimate is impracticable, and these results indicate that the proposed estimator can outperform the estimator in [16] in practical situations. Although the estimator in [16] would be a useful alternative to the proposed estimator when a near-i.i.d. preamble is unavailable, its performance can be severely degraded if channel statistics are inaccurate. Among the frequency estimators that can be implemented in practice, the proposed one equipped with a near-i.i.d. preamble performed the best.

Finally, it should be pointed out that the estimation accuracy can be trivially improved by averaging the number of consecutive single-burst estimates [12]. Improvements can also be gained by using space diversity [16].


 Fig. 11. Frequency offset estimation performance in frequency-selective fading channel, $\tau = 3T/4$ ($L = 2$ and $E_b/N_0 = 25$ dB).

VI. CONCLUSION

A new baseband signal processing method for estimating frequency offset in a frequency-selective fading channel was proposed. By the help of near-i.i.d. sequences, the proposed method estimates frequency offsets without the need for channel information. Therefore, it should be suitable for fast acquisition in the initial stage of communications. Statistical analysis indicates that the proposed estimate is almost unbiased and its MSE approaches the CRLB if the pilot is near-i.i.d. Simulation results indicate that the proposed frequency estimator can perform better than existing techniques in frequency-selective fading channel environments.

APPENDIX

Using (11), the frequency estimator in (19) [or equivalently (22)] can be rewritten as

$$\Delta \hat{f} = \frac{1}{\pi(N+1)T} \arg \left\{ \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \sum_{k=m+1+l}^K \gamma_u(k) \gamma_u^*(k-m) \right\}. \quad (\text{A1})$$

To simplify the notations, let $\hat{\xi} \triangleq e^{j\pi\Delta f(N+1)T}$, $\xi \triangleq e^{j\pi\Delta f(N+1)T}$, and $\epsilon \triangleq \arg\{\hat{\xi}\xi^*\}$, then (A2)–(A4).

$$\epsilon = \arg \left\{ \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \sum_{k=m+1+l}^K \gamma_u(k) \gamma_u^*(k-m) \cdot e^{-j\pi\Delta f(N+1)T} \right\} \quad (\text{A2})$$

$$E[\Delta \hat{f} - \Delta f] = \frac{1}{\pi(N+1)T} E[\epsilon] \quad (\text{A3})$$

and

$$E[(\Delta \hat{f} - \Delta f)^2] = \frac{1}{\pi^2(N+1)^2 T^2} E[\epsilon^2]. \quad (\text{A4})$$

Using (12) and $r(k) = e^{j(2\pi\Delta f k T + \theta)} \sum_{l=0}^L d(k-l)g(l) + \eta(k)$ in (A2), ϵ can be approximated by

$$\begin{aligned} \epsilon \approx \arg \left\{ e^{j\pi\Delta f(1-N)T} \frac{1 - e^{j2\pi\Delta f N T}}{1 - e^{j2\pi\Delta f T}} \sum_{l=0}^L |g(l)|^4 \right. \\ \left. + \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \cdot \sum_{k=m+1+l}^K \{ \Phi(k, m, l) + N_1(k, m, l)\eta(k) \right. \\ \left. + N_2(k, m, l)\eta^*(k-m) \} \right\} \quad (\text{A5}) \end{aligned}$$

$$\begin{aligned} \approx \arg \left\{ N \sum_{l=0}^L |g(l)|^4 + \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \right. \\ \left. \cdot \sum_{k=m+1+l}^K \{ \Phi(k, m, l) + N_1(k, m, l)\eta(k) \right. \\ \left. + N_2(k, m, l)\eta^*(k-m) \} \right\} \quad (\text{A6}) \end{aligned}$$

$$\begin{aligned}
E[\epsilon^2] \approx & \frac{1}{N^2 \left(\sum_{l=0}^L |g(l)|^4 \right)^2} \left[\left\{ \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \sum_{k=m+1+l}^K \Phi_{\text{IMA}}(k, m, l) \right\}^2 \right. \\
& + \sigma_\eta^2 \sum_{k=2}^K \left\{ \left| \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} N_1(k, m, l) u(k-m-1-l) \right|^2 \right. \\
& \left. \left. + \left| \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} N_2(k, m, l) u(k-m-1-l) \right|^2 \right\} \right] \quad (\text{A9})
\end{aligned}$$

$$\begin{aligned}
\approx & \text{Im} \left\{ \frac{1}{N \sum_{l=0}^L |g(l)|^4} \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \right. \\
& \cdot \sum_{k=m+1+l}^K \{ \Phi(k, m, l) + N_1(k, m, l) \eta(k) \\
& \left. + N_2(k, m, l) \eta^*(k-m) \} \right\} \quad (\text{A7})
\end{aligned}$$

where $\Phi(k, m, l)$, $N_1(k, m, l)$, $N_2(k, m, l)$ and $\psi(k, l)$ are defined in (30). Equation (A5) is obtained from ϵ by neglecting $\eta(k)\eta^*(k-m)$, and (A6) approximates (A5) when $\Delta fT \ll 1$. (When $N = 1$, (A6) becomes identical to (A5). As a result, (28) holds for large values of ΔfT .) (A7) is obtained from (A6) under the assumption that the first term inside the brace is much larger than the magnitude of the second term. (This assumption holds when $\{d(k)\}$ is near-i.i.d., i.e., $\Phi(k, m, l) \approx 0$.) From (A7)

$$\begin{aligned}
E[\epsilon] \approx & \frac{1}{N \sum_{l=0}^L |g(l)|^4} \sum_{m=1}^N \sum_{l=0}^L \frac{1}{K-m-l} \\
& \times \sum_{k=m+1+l}^K \Phi_{\text{IMA}}(k, m, l). \quad (\text{A8})
\end{aligned}$$

After squaring (A7) and taking the expectation, (A9), shown at the top of the page, is left, where $u(k)$ is the unit step function.

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Eui-Rim Jeong received the B.S. and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Taejeon, Korea, in 1995 and 1997, respectively. He is currently working toward the Ph.D. degree in electrical engineering at KAIST.

His research interests include synchronization, modem structure of wireless digital communication, and equalization.



Ginkyu Choi (S'95–M'96) received the B.S. degree in electronic engineering from Sung Kyun Kwan University, Seoul, Korea, in 1983, the M.S. degrees in electrical engineering and computer science, and the Ph.D. degree in electrical engineering in 1990, 1994, and 1996, respectively, all from the Polytechnic University, Brooklyn, NY.

From 1983 to 1985, he was with Gold Star Company, Korea, as a Technical Engineer. In 1985, he was also with Motorola as a Process Engineer. Since 1996, he has been with the Telecommunication R&D

Center, Samsung Electronics Company, Ltd., Bundang-Gu, Korea, where he is now working on the IMT2000 system and wireless ATM Modem development. His current research interests include wireless digital communication, real-time digital signal and multirate processing, wavelets and applications, and image processing.



Yong H. Lee (S'81–M'84–SM'98) was born in Seoul, Korea, on July 12, 1955. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in electrical engineering from the University of Pennsylvania, Philadelphia, in 1984.

From 1984 to 1988, he was an Assistant Professor with the Department of Electrical and Computer Engineering, State University of New York, Buffalo.

Since 1989, he has been with the Department of Electrical Engineering, KAIST, where he is currently a Professor. His research activities are in the area of communication signal processing, which includes synchronization, interference cancellation, and efficient filter design.