Frame Synchronization in the Presence of Frequency Offset

Zae Yong Choi and Yong H. Lee

***Abstract*—A new frame synchronization technique, which is robust to carrier frequency and phase errors, is proposed for M-ary PSK systems. This technique is derived through modification of the procedure used for obtaining the maximum-likelihood (ML) rule in the paper by Gansman *et al.* The proposed rule is based on an operation called a *double* correlation which evaluates a correlation after properly multiplying the received signal with a sync pattern. It was shown through computer simulation that the proposed rule generally outperformed the existing rules when a frequency offset existed.**

***Index Terms*—Correlation, frame synchronization, frequency offset.**

I. Introduction

FRAME synchronization is achieved with the aid of a sync pattern which is either injected periodically into the data stream (continuous transmission) or appended at the beginning of each packet (packet transmission). At the receiver, after recovering timing information, sampled input values are typically correlated with a sync pattern and frame synchronization is accomplished by examing the correlation values [1]–[3]. This type of synchronization method, which is generally referred to as the *correlation rule*, has been popular because of its simplicity in implementation and acceptable performance. Frame synchronization can also be achieved using more optimal rules such as the maximum-likelihood (ML) rules in [4]–[7] and their various simplifications. These rules outperform the correlation rules at the expense of additional computation.

Frame synchronization is usually performed before carrier recovery is completed. In particular, popular data-aided methods for estimating carrier frequency and phase [8], [9] require perfect frame sync, and the use of these methods requires frame synchronizers which are tolerant of frequency and phase errors. Although this robustness to a carrier offset is an important characteristic of frame sync rules, only a few existing rules have such a property. The ML rule in [7] is derived under the assumption that frequency and phase errors are uniformly distributed, and it is tolerant of both frequency and phase offsets. The ad hoc rule in [9, p. 487], which evaluates the correlation between a differentially encoded input signal and a differentially encoded sync pattern, also has such tolerance. This rule generally performs worse than the ML rule in [7], but is simpler to implement.

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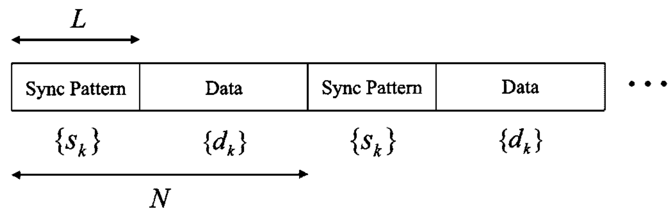

Fig. 1. Frame structure.

This paper attempts to improve the performance of the ML rule in [7], especially for a large frequency offset.[1] Through a certain modification of the procedure for deriving the ML rule, a new rule that can outperform the existing one is proposed. The proposed rule is based on an operation called a *double* correlation which is an extension of the correlation between the differentially encoded input and differentially encoded sync signals in [9].

II. Signal Model

An M-ary PSK signal, which is continuously transmitted over an additive white Gaussian noise (AWGN) channels is considered. The frame structure is shown in Fig. 1. Each frame consists of N M-ary symbols: the first L symbols form a fixed frame synchronization pattern $\{s_k\}$, and the remaining $N-L$ symbols are random data $\{d_k\}$. It was assumed that each data symbol is equally likely to be chosen from the M-ary signal constellation $\{e^{j(2\pi m/M)}\,|\,0 \leq m < M\}$. The received baseband signal is written as

$$r_n = e^{j(\theta_n + n\omega_o T + \phi_o)} + G_n \tag{1}$$

where $e^{j\theta_n}$ is the M-ary phase-modulated symbol, T is the symbol period, $\omega_o = 2\pi f_o$, f_o and ϕ_o are frequency and phase offsets, respectively, G_n is a zero-mean complex white Gaussian noise with the variance $\sigma_G^2 = N_o/E_s$, E_s denotes symbol energy, and n is the time index. In [7], the signal model is given by (1) with the following assumptions: ϕ_o is uniformly distributed over $[-\pi, \pi]$, and the normalized frequency offset f_oT is uniformly distributed over $[-U_m, U_m]$ where $U_m, 0 \leq U_m < 0.5$, is a known constant. This work also starts with (1); however, it assumes that both ϕ_o and $\omega_o T$ $(=2\pi f_o T)$ are uniformly distributed over $[-\pi, \pi]$. This assumption simplifies the derivation and leads to a rule which is reasonably simple to implement.

III. Derivation of the Proposed Frame Synchronization

The frame synchronization problem is the estimation of the frame boundary position in an arbitrarily selected segment of

[1]In [7], a frame synchronizer based on hypothesis testing has been developed as well as the ML rule, and fading channels were also considered. In this work, we attempt to improve only the ML rule operating in an additive white Gaussian noise (AWGN) channel.

AWGN channel output observations corresponding to N transmitted symbols. If the sync pattern starts at the μth position, $\mu \in [0, N-1]$, of the N observations, then the ML estimate $\hat{\mu}$ is the integer that maximizes the conditional probability density $f(\vec{\mathbf{r}} \,|\, \mu)$ of the received signal $\vec{\mathbf{r}} = (r_0, r_1, \ldots r_{N-1})$. To derive $f(\vec{\mathbf{r}} \,|\, \mu)$, the first consideration is

$$f(\vec{\mathbf{r}} \,|\, \mu, \vec{\mathbf{d}}, \omega_o T, \phi_o) = \prod_{k=0}^{N-1} \frac{E_s}{\pi N_o} e^{-|r_k - \exp\{j(\theta_k + k\omega_o T + \phi_o)\}|^2 E_s/N_o} \tag{2}$$

where $\vec{\mathbf{d}}$ denotes an M-ary random data sequence of duration $N-L$. In (2), $\{e^{j\theta_k} \,|\, 0 \le k \le N-1\}$ are PSK symbols consisting of L sync symbols and $N-L$ data symbols. Taking the expectation of (2) with respect to ϕ_o yields

$$f(\vec{\mathbf{r}} \,|\, \mu, \vec{\mathbf{d}}, \omega_o T) = C(\vec{\mathbf{r}}) \cdot I_o\left(\frac{2E_s}{N_o}\left|\sum_{k=0}^{N-1} r_k^* e^{j\theta_k} e^{jk\omega_o T}\right|\right) \tag{3}$$

where $I_o(x) = (1/2\pi)\int_{-\pi}^{\pi} e^{x\cos\theta}\, d\theta$ is the zeroth-order modified Bessel function of the first kind and $C(\vec{\mathbf{r}}) = (E_s/\pi N_o)^N \prod_{i=0}^{N-1} e^{-(|r_i|^2+1)E_s/N_o}$. If the expectation of (3) is taken with respect to $\omega_o T$ and the average over all possible data symbols is evaluated, then the following is produced:

$$f(\vec{\mathbf{r}} \,|\, \mu) = \frac{C(\vec{\mathbf{r}})}{M^{N-L}} \sum_{\text{all}\,\vec{\mathbf{d}}} \int_{-\pi}^{\pi} I_o\left(\frac{2E_s}{N_o}\left|\sum_{k=0}^{N-1} r_k^* e^{j\theta_k} e^{jk\omega_o T}\right|\right) \times \frac{1}{2\pi} d(\omega_o T) \tag{4}$$

where $1/M^{N-L}\sum_{\text{all}\,\vec{\mathbf{d}}}$ represents the averaging over all possible M-ary data sequences of length $N-L$. Maximizing (4) with respect to μ is computationally prohibitive. To obtain a test with much less complexity, $I_0(x)$ is approximated[2] by $(1 + x^2/4 + x^4/64)$ for small x. Then

$$\begin{aligned} f(\vec{\mathbf{r}} \,|\, \mu) \approx \frac{C(\vec{\mathbf{r}})}{M^{N-L}} \sum_{\text{all}\,\vec{\mathbf{d}}} \int_{-\pi}^{\pi} \Bigg\{ 1 &+ \left(\frac{E_s}{N_o}\right)^2 \\ &\times \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} r_p^* e^{j(\theta_p + p\omega_o T)} r_q e^{-j(\theta_q + q\omega_o T)} \\ &+ \left(\frac{E_s}{\sqrt{2}N_o}\right)^4 \sum_{k=0}^{N-1}\sum_{l=0}^{N-1}\sum_{m=0}^{N-1}\sum_{n=0}^{N-1} r_k^* r_l r_m^* r_n \\ &\times e^{j\{\theta_k - \theta_l + \theta_m - \theta_n + (k-l+m-n)\omega_o T\}} \Bigg\} \frac{1}{2\pi} d(\omega_o T). \end{aligned} \tag{5}$$

Since $\int_{-\pi}^{\pi} e^{jk\theta}\, d\theta = 2\pi\delta(k)$, only the terms corresponding to $p = q$ and $k - n = l - m$ remain after the above integration. Let $i = k - n = l - m$. Then $i \in [-N+1, N-1]$, and

$$\begin{aligned} f(\vec{\mathbf{r}} \,|\, \mu) \approx \frac{C(\vec{\mathbf{r}})}{M^{N-L}} \sum_{\text{all}\,\vec{\mathbf{d}}} \Bigg\{ 1 &+ \left(\frac{E_s}{N_o}\right)^2 \sum_{p=0}^{N-1} |r_p|^2 \\ &+ \left(\frac{E_s}{\sqrt{2}N_o}\right)^4 \Bigg(\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} |r_k|^2 |r_l|^2 \\ &+ 2\sum_{i=1}^{N-1}\sum_{k=i}^{N-1}\sum_{l=i}^{N-1} r_k^* r_l r_{k-i} r_{l-i}^* \\ &\times e^{j(\theta_k - \theta_l - \theta_{k-i} + \theta_{l-i})} \Bigg) \Bigg\}. \end{aligned} \tag{6}$$

After dropping the terms independent of μ, a test function is obtained as

$$\begin{aligned} L(\mu) = \frac{1}{M^{N-L}} \sum_{\text{all}\,\vec{\mathbf{d}}} \Bigg(&\sum_{i=1}^{L-1}\sum_{k=i}^{N-1}\sum_{l=i}^{N-1} r_k^* r_l r_{l-i}^* r_{k-i} \\ &\times e^{j(\theta_k - \theta_l + \theta_{l-i} - \theta_{k-i})} \\ &+ \sum_{i=L}^{N-1}\sum_{k=i}^{N-1}\sum_{l=i}^{N-1} r_k^* r_l r_{l-i}^* r_{k-i} e^{j(\theta_k - \theta_l + \theta_{l-i} - \theta_{k-i})} \Bigg). \end{aligned} \tag{7}$$

In (7), $\sum_{\text{all}\,\vec{\mathbf{d}}} \sum_{i=L}^{N-1} \sum_{k=i}^{N-1} \sum_{l=i, l\neq k}^{N-1} r_k^* r_l r_{l-i}^* r_{k-i} e^{j(\theta_k - \theta_l + \theta_{l-i} - \theta_{k-i})} = 0$ because at least one of the $(e^{j\theta_k}, e^{j\theta_{k-i}})$'s is not a sync pattern symbol but a random data symbol, and the sum of such a symbol for all possible M-ary symbols is equal to zero ($\sum_{m=0}^{M-1} e^{j(2\pi m/M)} = 0$). Therefore, $L(\mu)$ can be rewritten as

$$\begin{aligned} L(\mu) = \frac{1}{M^{N-L}} \sum_{\text{all}\,\vec{\mathbf{d}}} \Bigg\{ &\sum_{i=1}^{L-1}\sum_{k=i}^{N-1}\sum_{l=i}^{N-1} r_k^* r_l r_{l-i}^* r_{k-i} \\ &\times e^{j(\theta_k - \theta_l + \theta_{l-i} - \theta_{k-i})} \Bigg\} + \sum_{i=L}^{N-1}\sum_{m=i}^{N-1} |r_m|^2 |r_{m-i}|^2. \end{aligned} \tag{8}$$

After some simplification, again by using the fact that $\sum_{m=0}^{M-1} e^{j(2\pi m/M)} = 0$, we get the following test:

$$L_0(\mu) = \sum_{i=1}^{L-1} \left\{ \left| \sum_{k=i}^{L-1} r_{\mu+k}^* s_k r_{\mu+k-i} s_{k-i}^* \right|^2 - \sum_{k=\mu+i}^{\mu+L-1} |r_k|^2 |r_{k-i}|^2 \right\} \tag{9}$$

where s_k is the frame sync pattern and $L(\mu)$ in (8) is expressed as $L(\mu) = L_0(\mu) + \sum_{i=1}^{N-1}\sum_{m=i}^{N-1} |r_m|^2 |r_{m-i}|^2$. The first term inside the bracket in (9) is the magnitude square of the correlation between $r_{\mu+k} s_k^*$ and $r_{\mu+k-i} s_{k-i}^*$. This correlation $\sum_{k=i}^{L-1} r_{\mu+k} s_k^* (r_{\mu+k-i} s_{k-i}^*)^*$ will be referred to as the *double* correlation with lag i. The second term inside the bracket in (9) can be thought of as the random data correction term [4]–[7].

[2]In [7], $I_0(x)$ was approximated by x^2.

The test $L_0(\mu)$ is "unbalanced" in the sense that the difference between the double correlation term and the correction term is nonzero even when the perfect sync is achieved in a noise-free environment $(r_{k+\mu} = s_k)$. Such unbalance, which was caused by the approximation of $I_0(x)$, may degrade the test performance. A "balanced" test can be obtained by dropping the squares in (9). The resulting test function, denoted by $L_1(\mu)$, is

$$L_1(\mu) = \sum_{i=1}^{L-1} \left\{ \left| \sum_{k=i}^{L-1} r^*_{\mu+k} s_k r_{\mu+k-i} s^*_{k-i} \right| - \sum_{k=\mu+i}^{\mu+L-1} |r_k||r_{k-i}| \right\}. \tag{10}$$

If only the case where $i = 1$ in (10) is considered, then the test becomes

$$L_2(\mu) = \left| \sum_{k=1}^{L-1} r^*_{\mu+k} s_k r_{\mu+k-1} s^*_{k-1} \right| - \sum_{k=\mu+1}^{\mu+L-1} |r_k||r_{k-1}|. \tag{11}$$

Finally, dropping the data correction term (second term) in (11), the following is obtained:

$$L_3(\mu) = \left| \sum_{k=1}^{L-1} r_{\mu+k} s^*_k r^*_{\mu+k-1} s_{k-1} \right|. \tag{12}$$

This rule evaluates the correlation between differentially encoded inputs and differentially encoded sync symbols and is identical to the ad hoc rule in [9, p. 487].

Before concluding this section, it is worth describing the ML rule in [7]. For PSK signals this rule, denoted by $L_4(\mu)$, is given by

$$L_4(\mu) = \sum_{k=0}^{L-1} \sum_{l=0}^{L-1} r_{\mu+k} s^*_k r^*_{\mu+l} s_l \cdot \mathrm{sinc}\{2\pi U_m (k-l)\} - \sum_{k=0}^{L-1} |r_{k+\mu}|^2 \tag{13}$$

where $\mathrm{sinc}\{\cdot\}$ represents the sinc function and $0 \le U_m < 0.5$.

IV. Performance Evaluation

In the simulation, the received signal was assumed to be QPSK symbols $(M = 4)$ distorted by AWGN noise, phase, and frequency offsets. The frame length $N = 162$ and the sync pattern length $L = 15$. Ten million independent frames were generated and the false acquisition probability was empirically estimated by counting the number of frame sync failures.[3] The frame synchronizers considered in the simulation were $\{L_i(\mu), i = 0, 1, \ldots, 4\}$ and the conventional correlation rule, which is given by $|\sum_{k=0}^{L-1} r_{\mu+k} s^*_k|$. For each rule, frame synchronization is declared at a position where its test function is maximized. The robustness of the frame synchronizers was examined by estimating the false acquisition probabilities for various normalized frequency offsets $(f_o T)$ in between 0 and 0.2, while fixing E_b/N_o at 6 dB. The results are shown in Fig. 2. For the rule $L_4(\mu), U_m$ was assumed to be 0.02, 0.08, 0.15, and 0.3. The performance of $L_4(\mu)$ indicated some tradeoff between the false acquisition probability and robustness to a frequency offset: an increased U_m enhanced the robustness yet degraded the false acquisition probability. The performances of the conventional correlation and $L_4(\mu)$ when $U_m = 0.02$ degraded rapidly as $f_o T$ increased. As expected, $L_0(\mu), L_1(\mu), L_2(\mu)$ and $L_3(\mu)$ were tolerant of a frequency offset. It was interesting to note that $L_1(\mu)$ and $L_2(\mu)$ performed better than did $L_0(\mu)$. This is attributed to the fact that $L_0(\mu)$ is "unbalanced." The rules associated with $L_1(\mu)$ and $L_2(\mu)$ outperformed the others. When comparing $L_1(\mu)$ and $L_2(\mu)$, the former produced a better performance than the latter at the expense of more computation.

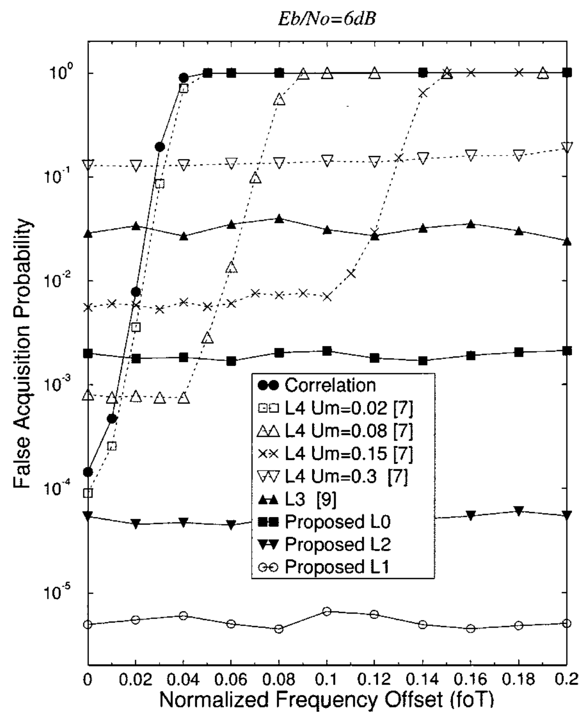

Fig. 2. False acquisition probability versus frequency offset when E_b/N_o is 6 dB.

In the simulation shown in Fig. 3, the behaviors of the sync rules with respect to E_b/N_o were investigated under the assumption that the normalized frequency offset was uniformly distributed over $[-U_m, U_m]$, where $U_m \in \{0.01, 0.04, 0.1\}$. It was assumed that the value of U_m was known to $L_4(\mu)$ (for the other rules this knowledge is unnecessary). As in the case of Fig. 2, the performances of the correlation and $L_4(\mu)$ degraded as $f_o T$ increased. The proposed rules are robust to a frequency offset[4] and $L_1(\mu)$ outperforms the others.

V. Conclusion

New ML-type frame synchronizers which are robust to frequency offset were proposed for PSK signaling and their

[3]Since the sync pattern starts at the μth position, $\mu \in [0, N-1]$, of the N observations, the synchronizer has only two states: it either acquires or false locks. Therefore, the probability of false acquisition (or false lock) completely characterizes the synchronizer performance.

[4]This robustness may be attributed to the fourth power term, $x^4/64$, which is included in approximating $I_0(x)$.

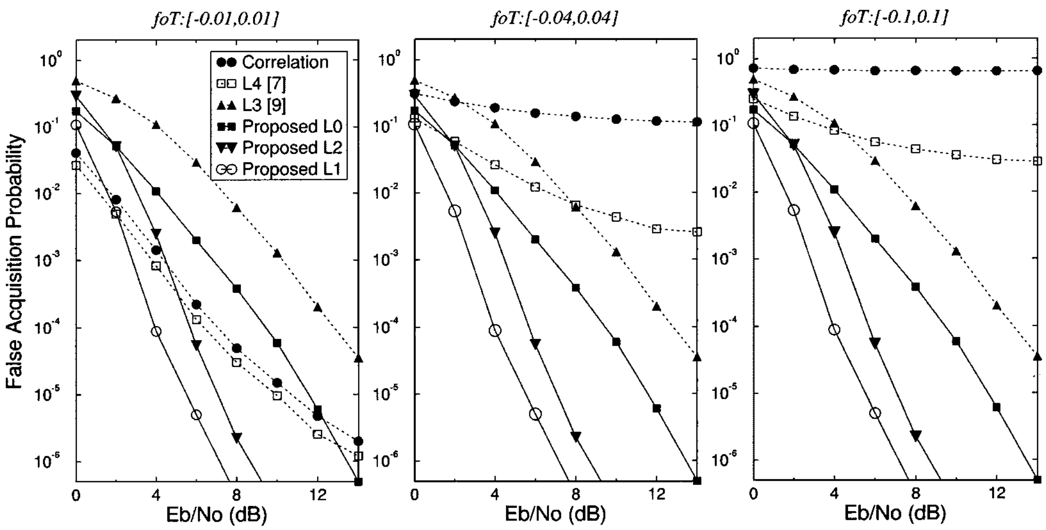

Fig. 3. False acquisition probability versus SNR.

performances were examined through computer simulation. These synchronizers are based on the double correlation which is an extension of the correlation between the differentially encoded input and differentially encoded sync symbols. The proposed synchronizers generally performed better than conventional techniques when a frequency offset existed. An extension of the proposed frame synchronizers to a case with QAM signaling and discussions regarding the use of multiple frames for synchronization can be found in [10].

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