Modified Leaky LMS Algorithm for Channel Estimation in DS-CDMA Systems

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***Abstract*—A simple adaptive least mean square (LMS) type algorithm for channel estimation is developed based on certain modifications to finite-impulse response (FIR) Wiener filtering. The proposed algorithm is nearly blind since it does not require any training sequence or channel statistics, and it can be implemented using only noise variance knowledge. A condition guaranteeing the convergence of the algorithm and theoretical mean square error (MSE) values are also derived. Computer simulation results demonstrate that the proposed algorithm can yield a smaller MSE than existing techniques, and that its performance is close to that of optimal Wiener filtering.**

***Index Terms*—Channel estimation, DS-CDMA, leaky LMS algorithm.**

I. Introduction

IN DIRECT-SEQUENCE code-division multiple access (DS-CDMA) systems, a pilot channel is usually employed for synchronization and channel estimation. After despreading a pilot channel in a finger of a DS-CDMA RAKE receiver, the received signal $u(k)$ can be expressed as

$$u(k) = h(k) + n(k) \tag{1}$$

where $h(k)$ is a wide-sense stationary channel parameter, $n(k)$ is zero-mean additive white noise, and k is a discrete time index. Estimating $h(k)$ from $\{u(k)\}$ is not a trivial task, because the statistics of $h(k)$ are unknown and vary depending on the velocity of the mobile receiver. The use of an adaptive filter for this estimation is also difficult, since it is impossible to provide a training sequence consisting of channel parameters. A popular approach to the channel estimation is to use a fixed lowpass filter (LPF) in which the cutoff frequency is set to the maximum Doppler frequency [1], [2]. Although such an LPF is simple to implement, it tends to exhibit a poor performance when its cutoff frequency differs from the actual maximum Doppler frequency. The use of optimal filters, such as Wiener and Kalman filters, was previously proposed in [3]–[5], yet these filters require channel statistics, thus their implementation is difficult. In [6], channel parameters are estimated by applying an adaptive linear prediction. The adaptive predictor can perform better than LPFs in slow fading environments, however its performance degrades rather rapidly as the channel fade rate increases.

In this letter, we develop a new adaptive filter for channel estimation based on the FIR Wiener filtering formulation. The proposed adaptation scheme resembles the *leaky* LMS algorithm [7]. It is also shown that the performance of the proposed adaptive estimator is close to that of the optimal Wiener estimator. Consequently, the proposed technique is a useful alternative to existing practical channel estimators, such as LPFs and adaptive predictors.

II. Derivation of Proposed Channel Estimation Algorithm

Suppose that an estimate of $h(k)$ in (1) is given by

$$\hat{h}(k) = \mathbf{w}^H \mathbf{u}(k) \tag{2}$$

where $\mathbf{w}$ is an N-dimensional tap weight vector and $\mathbf{u}(k) = [u(k), u(k-1), \ldots, u(k-N+1)]^T$. The optimal tap weight $\mathbf{w}_o$ minimizing $E[|h(k) - \hat{h}(k)|^2]$ satisfies the Wiener-Hopf equation

$$\mathbf{R}\mathbf{w}_o = \mathbf{p}_{uh} \tag{3}$$

where $\mathbf{R} = E[\mathbf{u}(k)\mathbf{u}^H(k)]$ and $\mathbf{p}_{uh} = E[\mathbf{u}(k)h^*(k)]$. When $\{h(k)\}$ and $\{n(k)\}$ are uncorrelated, $\mathbf{p}_{uh}$ can be rewritten as

$$\mathbf{p}_{uh} = \mathbf{p}_{uu} - [\sigma^2, 0, \ldots, 0]^T \tag{4}$$

where $\mathbf{p}_{uu} = E[\mathbf{u}(k)u^*(k)]$ and σ^2 is the variance of $n(k)$, which is assumed to be known[1]. Using (4) in (3) produces

$$(\mathbf{R} + \mathbf{A})\mathbf{w}_o = \mathbf{p}_{uu} \tag{5}$$

where $\mathbf{A}$ is an $N \times N$ matrix consisting of all zeros with the exception that the (1, 1)th element, say α_o, is nonzero. This parameter α_o is given by

$$\alpha_o = \frac{\sigma^2}{w_{o,0}} \tag{6}$$

where $w_{o,0}$ is the first element of the optimal tap weight vector $\mathbf{w}_o$. Suppose, for the time being, that α_o is known. Then obtaining the optimal weight $\mathbf{w}_o$ from (5) only requires the statistics of the received signal, which are $\mathbf{R}$ and $\mathbf{p}_{uu}$. This fact indicates the possibility that an LMS-type algorithm which does

[1] σ^2 can be easily estimated using a code that is orthogonal to all the codes assigned to users.

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not need a training sequence can be derived from (5). Accordingly, such an algorithm is derived starting with the following observation.

Observation: The weight minimizing

$$E[|u(k) - \mathbf{w}^H\mathbf{u}(k)|^2 + \mathbf{w}^H\mathbf{A}\mathbf{w}] \tag{7}$$

satisfies (5).

This follows from the fact the gradient of the cost in (7) is

$$\begin{aligned} 2\{-\mathbf{p}_{uu} + (\mathbf{R}+\mathbf{A})\mathbf{w}\} \\ = 2E[-\mathbf{u}(k)u^*(k) + \{\mathbf{u}(k)\mathbf{u}^H(k) + \mathbf{A}\}\mathbf{w}]. \end{aligned} \tag{8}$$

From (8), the steepest descent algorithm for obtaining $\mathbf{w}_o$ in (5) can be written as

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu E[-\mathbf{u}(k)u^*(k) + \{\mathbf{u}(k)\mathbf{u}^H(k) + \mathbf{A}\}\mathbf{w}] \tag{9}$$

where μ is the step-size parameter. By removing the expectation $E[\cdot]$ in (9), the following LMS-type algorithm is derived:

$$\mathbf{w}(k+1) = (\mathbf{I} - \mu\mathbf{A})\mathbf{w}(k) + \mu\mathbf{u}(k)\{u(k) - \mathbf{w}^H(k)\mathbf{u}(k)\}^*. \tag{10}$$

The weight $\mathbf{w}(k)$ in (10) approaches $\mathbf{w}_o$ in (5) (or (3)) as k increases. Although this recursion does not require any training sequence, it does require the knowledge about α_o (or equivalently $\mathbf{A}$). In the proposed algorithm, α_o is also updated at each recursion: it is denoted by $\alpha(k)$ at the kth iteration and updated by

$$\alpha(k) = \frac{\sigma^2}{w_0(k)} \tag{11}$$

where $w_0(k)$ is the first element of $\mathbf{w}(k)$ (compare (11) with (6)). The proposed channel estimation algorithm can be summarized as follows:

1) *Channel estimate:*

$$\hat{h}(k) = \mathbf{w}^H(k)\mathbf{u}(k) \tag{12}$$

2) *Estimation error:*

$$e(k) = u(k) - \hat{h}(k) \tag{13}$$

3) *Tap-weight adaptation:*

$$\mathbf{w}(k+1) = (\mathbf{I} - \mu\hat{\mathbf{A}}(k))\mathbf{w}(k) + \mu\mathbf{u}(k)e^*(k) \tag{14}$$

where $\hat{\mathbf{A}}(k)$ is an $N \times N$ matrix consisting of all zeros with the exception that the $(1,1)$th element is $\alpha(k)$ in (11). The proposed algorithm is referred to as the *modified leaky* LMS algorithm, due to its resemblance to the leaky LMS algorithm.

III. Performance Analysis

In this section, the stability condition and MSE associated with the recursive algorithm in (14) are derived following the approach in [7], then the results are confirmed through computer simulation.

A. Theoretical Derivation

From (11), the recursive equation in (14) can be rewritten as

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\mathbf{u}(k)\{u(k) - \mathbf{w}^H(k)\mathbf{u}(k)\}^* - \mu\mathbf{c} \tag{15}$$

where $\mathbf{c} = [\sigma^2, 0, \ldots, 0]^T$. Subtracting the optimal tap weight $\mathbf{w}_o$ from both sides of (15), we get

$$\begin{aligned} \boldsymbol{\epsilon}(k+1) = \{\mathbf{I} - \mu\mathbf{u}(k)\mathbf{u}^H(k)\}\boldsymbol{\epsilon}(k) \\ + \mu\{\mathbf{u}(k)(e_o(k) + n(k))^* - \mathbf{c}\} \end{aligned} \tag{16}$$

where $\boldsymbol{\epsilon}(k) = \mathbf{w}(k) - \mathbf{w}_o$ and $e_o(k) = h(k) - \mathbf{w}_o^H\mathbf{u}(k)$. Assuming that $E[\boldsymbol{\epsilon}(k)] = 0$ and that $\boldsymbol{\epsilon}(k)$ and $\mathbf{u}(k)$ are uncorrelated, the correlation matrix for $\boldsymbol{\epsilon}(k)$, defined as $\mathbf{K}(k) = E[\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^H(k)]$, can be computed as follows:

$$\begin{aligned} \mathbf{K}(k+1) = (\mathbf{I} - \mu\mathbf{R})\mathbf{K}(k)(\mathbf{I} - \mu\mathbf{R}) \\ + \mu^2\{(J_{\min} + \sigma^2)\mathbf{R} + \mathbf{c}\mathbf{c}^H\} \end{aligned} \tag{17}$$

where $J_{\min} = E[|e_o(k)|^2]$ denotes the MSE of the Wiener filter. The estimation error can be expressed as

$$\begin{aligned} e(k) &= h(k) - \mathbf{w}^H\mathbf{u}(k) \\ &= e_o(k) - \boldsymbol{\epsilon}^H(k)\mathbf{u}(k). \end{aligned} \tag{18}$$

Under the assumption that $h(k)$ and $\boldsymbol{\epsilon}(k)$ are independent, the MSE at iteration k, say $J(k)$, can be represented as

$$\begin{aligned} J(k) &= E[|e(k)|^2] \\ &= J_{\min} + E[\boldsymbol{\epsilon}^H(k)\mathbf{u}(k)\mathbf{u}^H(k)\boldsymbol{\epsilon}(k)] \\ &= J_{\min} + \mathrm{tr}[\mathbf{R}\mathbf{K}(k)] \end{aligned} \tag{19}$$

where $\mathrm{tr}[\cdot]$ means a trace operation. Following the approach in [7, p. 397], it can be shown that $J(k)$ converges to a constant if and only if

$$0 < \mu < \frac{2}{\lambda_{\max}} \tag{20}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{R}$. When (20) is satisfied, the MSE is given by

$$J(k) = J_{\min} + \boldsymbol{\lambda}^T\{\mathbf{B}^k(\mathbf{x}(0) - \mathbf{x}(\infty)) + \mathbf{x}(\infty)\} \tag{21}$$

where the various terms are defined as follows.

- $\boldsymbol{\lambda}$ is an $N \times 1$ vector whose elements are the eigenvalues of $\mathbf{R}$.
- $\mathbf{B}$ is an $N \times N$ matrix with elements

$$b_{ij} = \begin{cases} (1 - \mu\lambda_i)^2, & i = j \\ \mu^2\lambda_i\lambda_j, & i \neq j. \end{cases}$$

- $\mathbf{x}(0) = \mathrm{diag}\{\mathbf{Q}^H\boldsymbol{\epsilon}(0)\boldsymbol{\epsilon}^H(0)\mathbf{Q}\}$ when $\mathbf{Q}$ is a unitary matrix consisting of the eigenvectors for $\mathbf{R}$.
- $\mathbf{x}(\infty) = \mu^2(\mathbf{I} - \mathbf{B})^{-1}\{(J_{\min} + \sigma^2)\boldsymbol{\lambda} + [\sigma^4, 0, \ldots, 0]^T\}$.

The steady-state MSE is written as

$$J(\infty) = J_{\min} + \boldsymbol{\lambda}^T\mathbf{x}(\infty). \tag{22}$$

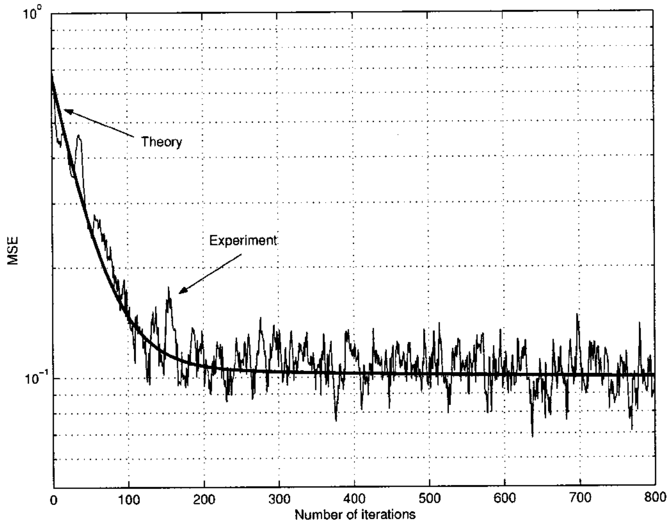

Fig. 1. Comparison of experimental results with theory for the proposed method when $f_dT = 0.03, N = 5, \mu = 0.0025$, and $\mathrm{SNR} = 5$ dB.

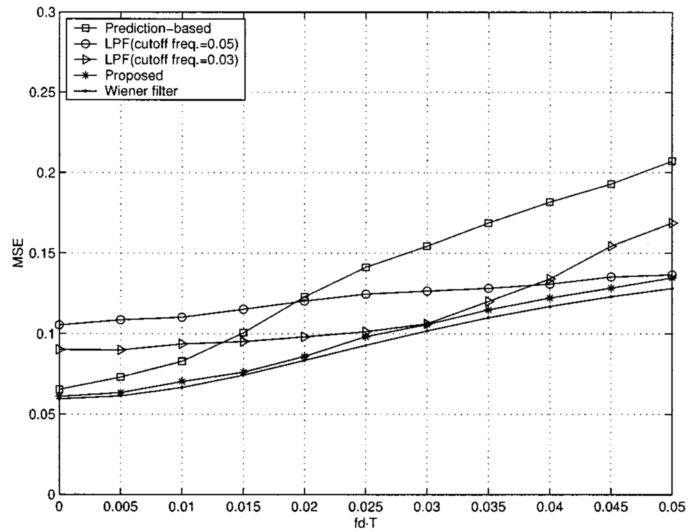

Fig. 2. MSE comparison when $N = 5, \mu = 0.0025$, and $\mathrm{SNR} = 5$ dB.

B. Comparison of Experimental Results With Theory

To confirm the theoretical result in (21), the modified leaky LMS algorithm was applied to channel estimation and empirical MSEs obtained. The signal $u(k)$ in (1) was generated under the following assumptions: $h(k)$ was a Rayleigh fading gain with $f_dT = 0.03$ and $n(k)$ was complex additive white Gaussian noise, where f_dT is the normalized maximum Doppler frequency[2] . The parameters of the modified leaky LMS algorithm were as follows: the number of taps $N = 5$; the initial tap weight vector $\mathbf{w}(0) = [1/N, 0, \ldots, 0]^T$; and the step-size $\mu = 0.0025$. Fig. 1 shows the learning curves when $\mathrm{SNR} = 5$ dB. The experimental curve was obtained by ensemble-averaging the squared error over 100 independent trials. The result demonstrated a good agreement between the theory and the experiment.

IV. Application to Channel Estimation

The signal $u(k)$ in (1) was generated as in Section III-B, with the exception that various f_dT values between 0 and 0.05 were considered. The parameters of the proposed algorithm remained the same. For comparison, two LPFs designed for $f_dT = 0.03$ and 0.05, an adaptive linear predictor in [6], and an optimal Wiener filter were considered along with the proposed method. The parameters of these filters were the same as the corresponding parameters of the modified leaky LMS algorithm. The LPFs were FIR filters designed using the Parks–McClellan algorithm. The Wiener filter was designed for each f_dT, thereby providing a minimum bound. The empirical MSEs of the proposed and the adaptive linear predictor were obtained by averaging the steady-state squared error values.

Fig. 2 compares the empirical MSE values. The performance of the proposed algorithm was found to be close to that of the Wiener filter. The proposed method always outperformed the adaptive linear predictor and the LPF. The adaptive linear predictor worked better than the LPFs for slow fading channels, yet its MSE values rapidly increased as the f_dT increased. As expected, the LPFs only performed well in the neighborhood of the f_dT values for which they were designed.

[2]In the case of W-CDMA [8], f_dT was 0.037 when the chip rate was 3.84 Mcps, the symbol rate was 15.0 ksps, the carrier frequency was 2.0 GHz, and the maximum mobile velocity was 300 km/h.

V. Conclusion

An adaptive algorithm, called the modified leaky LMS algorithm, for channel estimation was developed. It was shown that the algorithm requires neither a training sequence nor channel statistics, and it can be implemented once the noise variance is known. The convergence characteristic of the proposed algorithm was analyzed, and its advantage over existing techniques was demonstrated through computer simulation. The proposed scheme may be applied to signal enhancement as it can restore a signal corrupted by additive white noise. In this area, extending the proposed algorithm to the case of colored noise would be an interesting topic for further research.

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