Optimization of Baud-Rate Timing Recovery for Equalization

In-Seok Hwang, *Member, IEEE,* and Yong Hoon Lee, *Senior Member, IEEE*

***Abstract*—This paper suggests the use of a prefilter before baud-rate timing recovery [1]. It is shown that the timing phase of the baud-rate timing synchronizer can be set to a desirable timing phase, at which point a digital equalizer performs best, by proper prefiltering. The procedure for the optimal design of such a prefilter is developed under the assumption that the channel impulse response and desirable timing phase for equalization are known. Application of the proposed scheme to the gigabit Ethernet system demonstrated that prefiltering can improve receiver performance.**

***Index Terms*—Baud-rate timing recovery, equalization, gigabit Ethernet, prefilter.**

I. Introduction

Although the performance of digital equalization, which compensates for the channel after sampling the received signal, is influenced by a sampling timing phase [2], such equalization has been mostly ignored in developing the baud-rate timing recovery scheme in [1]. Accordingly, to improve receiver performance, it is desirable to employ a baud-rate timing recovery method whose timing phase is set to an optimal phase, at which point the digital equalizer performs best and the timing jitter is minimized. The goal of this paper is therefore to develop such a timing synchronization technique.

The proposed method suggests the use of a prefilter before timing recovery.[1] In particular, it will be shown that the baud-rate timing recovery in [1] has the desirable properties for equalization, if a proper prefilter is employed. A procedure for the optimal design of a prefilter is developed under the assumption that the channel impulse response is known. Application of the proposed scheme to the gigabit Ethernet system (1000BASE-T) [3] demonstrated that prefiltering can improve receiver performance.

II. System Description

A. Description of Clock Synchronizer

The baseband system model considered in this paper is shown in Fig. 1 (for the time being, the prefilter is ignored). The received signal which is sampled at $t = kT + \tau_k$ is expressed as

$$z_k = \sum_{i=-\infty}^{\infty} a_{k-i} g(iT + \tau_k) + n(kT + \tau_k) \tag{1}$$

where $\{a_k\}$ is the transmitted symbol, τ_k is the sampling timing phase, $g(t)$ is the impulse response describing the cascade of the transmission filter, channel, and receiver filter, and $n(t)$ is additive white noise with the variance $\sigma_n^2 = N_o/2$. One of the timing error detectors (TEDs) in [1], which has been recommended for practical applications and which will be considered hereafter, is represented as

$$e_k = \frac{1}{2E\left[a_k^2\right]} \left(\hat{a}_k z_{k-1} - \hat{a}_{k-1} z_k\right) \tag{2}$$

where e_k is the TED output and $\hat{a}_k$ is an estimate of a_k. Let $f(\tau) = E[e_k | \tau_k = \tau]$, then $f(\tau) = (1/2)\left[g(-T+\tau) - g(T+\tau)\right]$. $f(\tau)$ is referred to as the timing function. The sampling phase τ_k is updated according to the following recursive equation:

$$\tau_{k+1} = \tau_k - \gamma e_k \tag{3}$$

where γ is a constant. When $g(t)$ is a raised-cosine pulse, the timing function $f(\tau)$ monotonically increases for $-0.5T \le \tau \le 0.5T$ and it has only one root at $\tau = 0$. In this case, τ_k in (3) tends to converge to zero for proper values of γ. The root of $f(\tau)$ is referred to as the stable equilibrium point and is denoted by τ_o.

B. Prefiltering Received Signal

Suppose that a prefilter with the impulse response $\{w_i;\ -L \le i \le L\}$ is employed (Fig. 1). The prefilter output can be expressed as $\tilde{z}_k = \sum_{i=-L}^{L} w_i \cdot z_{k-i}$ and the TED output e_k can be obtained by replacing $\{z_k\}$ in (2) with $\{\tilde{z}_k\}$. Prefiltering reshapes the received signal. The received pulse shape $g(t)$ is thus modified as $\tilde{g}(t) = \sum_{i=-L}^{L} w_i g_i(t)$ where $g_i(t) = g(t - iT)$. The timing function is rewritten as $\tilde{f}(\tau) = (1/2)\left[\tilde{g}(-T+\tau) - \tilde{g}(T+\tau)\right]$. Using vector notations, $\tilde{f}(\tau) = (1/2)\mathbf{w}^T\left[\mathbf{g}_{-1}(\tau) - \mathbf{g}_1(\tau)\right]$ where $\mathbf{w} = \left[w_{-L}, \ldots, w_L\right]^T$ and $\mathbf{g}_l(\tau) = \left[g_{l+L}(\tau), \ldots, g_{l-L}(\tau)\right]^T$. It is assumed hereafter that $\tilde{f}(\tau)$ is monotonically increasing in $-0.5T \le \tau \le 0.5T$ and has only one root at $\tau = \tau_s$. Then τ_k in (3) approaches τ_s, which is the stable equilibrium point of $\tilde{f}(\tau)$. The purpose of prefiltering is to shape the received signal so as to minimize the variance of the timing jitter at $\tau = \tau_s$. Next, the steady-state jitter variance of the baud-rate clock synchronizer with prefiltering is derived following the procedure in [5]. The loop equation in (3) is rephrased as $\tau'_{k+1} = \tau'_k - \gamma\left[\tilde{f}(\tau_k) + N_k\right]$ where $\tau'_k = \tau_k - \tau_s$, and $N_k = e_k - \tilde{f}(\tau_k)$ is the loop noise. If $\tilde{f}(\tau_k)$ is approximated by $A_s\tau'_k$ in the vicinity of the stable equilibrium point $\tau = \tau_s$ where $A_s = \left[d\tilde{f}(\tau)/d\tau\right]_{\tau=\tau_s}$, then the loop equation can be rewritten as

$$\tau'_{k+1} = (1 - \gamma A_s)\tau'_k - \gamma N_k. \tag{4}$$

The steady-state jitter variance $\sigma_J^2 = \lim_{k\to\infty} E\left[{\tau'_k}^2\right]$ can be found from (4) following standard linear analysis techniques:

Paper approved by L. Vandendorpe, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received April 15, 2000; revised October 15, 2000, and July 15, 2001.

The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, Taejon 305-701, Korea (e-mail: yohlee@ee.kaist.ac.kr).

Publisher Item Identifier S 0090-6778(02)03510-9.

[1]Prefilters that can reduce the timing jitter caused by a data pattern have been introduced in [4] and [5].

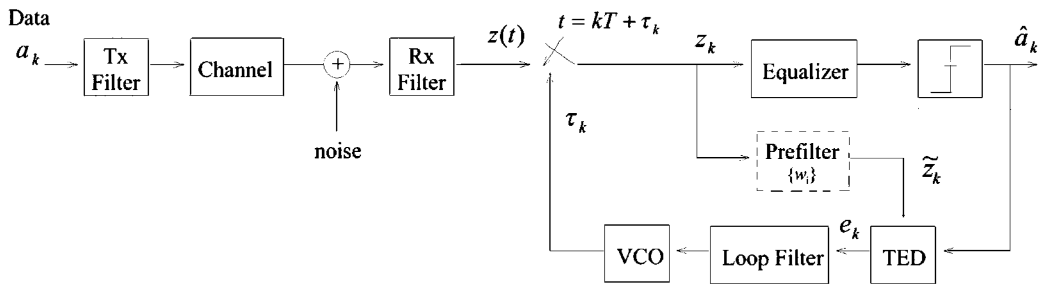

Fig. 1. Baseband system model.

σ_J^2 can be obtained by manipulating the correlation of N_k. Let $r_l(\tau_s) = \lim_{k\to\infty} E[N_k \cdot N_{k-l}]$. Then, it can be shown that

$$r_l(\tau_s) = \mathbf{w}^T \mathbf{R}_l(\tau_s)\mathbf{w} \tag{5}$$

where

$$\begin{aligned}\mathbf{R}_0(\tau_s) =& \frac{1}{2}\sum_{i\neq 0} \mathbf{g}_i(\tau_s)\mathbf{g}_i(\tau_s)^T - \left\{\frac{1}{2} - \frac{1}{4}\frac{E\left[a_k^4\right]}{E^2\left[a_k^2\right]}\right\}\\ &\cdot\left\{\mathbf{g}_{-1}(\tau_s)\mathbf{g}_{-1}(\tau_s)^T + \mathbf{g}_1(\tau_s)\mathbf{g}_1(\tau_s)^T\right\}\\ &+ \frac{1}{2}\frac{\sigma_n^2}{E\left[a_k^2\right]}\mathbf{J}_0\\ \mathbf{R}_1(\tau_s) =& -\frac{1}{4}\sum_{|i|>1} \mathbf{g}_{i-1}(\tau_s)\mathbf{g}_{i+1}(\tau_s)^T - \frac{1}{4}\frac{E\left[a_k^4\right]}{E^2\left[a_k^2\right]}\\ &\cdot \mathbf{g}_{-1}(\tau_s)\mathbf{g}_1(\tau_s)^T - \frac{1}{4}\frac{\sigma_n^2}{E\left[a_k^2\right]}\mathbf{J}_2\end{aligned}$$

and

$$\begin{aligned}\mathbf{R}_l(\tau_s) =& \frac{1}{4}\left\{\mathbf{g}_{1-i}(\tau_s)\mathbf{g}_{1+i}(\tau_s)^T + \mathbf{g}_{-1-i}(\tau_s)\mathbf{g}_{-1+i}(\tau_s)^T\right.\\ &\left.-2\cdot\mathbf{g}_{-i}(\tau_s)\mathbf{g}_i(\tau_s)^T\right\}, \quad \text{for } l>1. \end{aligned}\tag{6}$$

The matrices $\mathbf{J}_i$, $i = 0, 2$ in the above equations are $(2L+1)\times(2L+1)$ matrices of the form

$$\mathbf{J}_i = \begin{bmatrix} \mathbf{0}_{2L+1-i,i} & \mathbf{I}_{2L+1-i,2L+1-i}\\ \mathbf{0}_{i,i} & \mathbf{0}_{i,2L+1-i}\end{bmatrix} \tag{7}$$

where $\mathbf{0}_{i,j}$ is the i by j zero matrix and $\mathbf{I}_{i,i}$ is the i by i identity matrix. The steady-state jitter variance σ_J^2 is given by $\sigma_J^2 = \sum_{l=-\infty}^{\infty} r_l(\tau_s)\cdot[h_l \otimes h_{-l}]$ where $\otimes$ denotes convolution, h_l is the impulse response of the recursive equation (4) and is expressed as $h_l = \gamma(1-\gamma A_s)^{l-1}$, $l \geq 1$. After some calculation, σ_J^2 is represented by

$$\sigma_J^2 = \frac{2B_LT}{A_s^2}\mathbf{w}^T\left[\sum_{l=-\infty}^{\infty}\mathbf{R}_l(\tau_s)\cdot(1-\gamma A_s)^{|l|}\right]\mathbf{w} \tag{8}$$

where $B_LT = \gamma A_s/2(2-\gamma A_s)$ is the normalized equivalent noise bandwidth of the loop.

III. Prefilter Optimization

Suppose that an adaptive equalizer such as a decision feedback equalizer (DFE) is employed (Fig. 1) and that the optimal timing phase τ_{opt}, at which the MSE of the equalizer is minimized, is known. The prefilter is therefore, designed to minimize the steady-state jitter variance σ_J^2 in (8) under the constraint that $\tilde{f}(\tau_{\text{opt}}) = 0$ (this leads to $\tau_s = \tau_{\text{opt}}$). In addition, to avoid the trivial solution $\mathbf{w} = \mathbf{0}$, it is necessary to impose another constraint. A useful constraint for this purpose is: $A_s \triangleq (d\tilde{f}(\tau)/d\tau)|_{\tau=\tau_{\text{opt}}} = A_o$ where $A_o \triangleq [df(\tau)/d\tau]_{\tau=\tau_o}$, $f(\tau)$ is the timing function without prefiltering, and τ_o is the root of $f(\tau)$ [5].[2] The optimization problem can be summarized as follows:

$$\begin{aligned}&\underset{w}{\text{minimize}}\ \sigma_J^2 \text{ in (8) (after replacing } \tau_s \text{ and } A_s\\ &\qquad \text{in (8) with } \tau_{\text{opt}} \text{ and } A_o, \text{ respectively)}\\ &\text{subject to } \tilde{f}(\tau_{\text{opt}}) = \frac{1}{2}\mathbf{w}^T\left[\mathbf{g}_{-1}(\tau_{\text{opt}}) - \mathbf{g}_1(\tau_{\text{opt}})\right] = 0\\ &\qquad \left.\frac{d\tilde{f}(\tau)}{d\tau}\right|_{\tau=\tau_{\text{opt}}} = \frac{1}{2}\mathbf{w}^T\left[\acute{\mathbf{g}}_{-1}(\tau_{\text{opt}}) - \acute{\mathbf{g}}_1(\tau_{\text{opt}})\right]\\ &\qquad = A_o\end{aligned} \tag{9}$$

where $\acute{\mathbf{g}}_l(\tau)$ denotes the gradient of $\mathbf{g}_l(\tau)$. To solve this constrained optimization problem, the method of Lagrange multipliers was used. The cost function $L(\mathbf{w}, \boldsymbol{\lambda})$ that combines the two parts of this problem is written as $L(\mathbf{w},\boldsymbol{\lambda}) = \mathbf{w}^T\mathbf{R}\mathbf{w} + \lambda_1\cdot(1/2)\mathbf{w}^T\mathbf{g} + \lambda_2\cdot((1/2)\mathbf{w}^T\acute{\mathbf{g}} - A_o)$ where $\mathbf{R} = (2B_LT/A_o^2)\left[\sum_{l=-\infty}^{\infty}\mathbf{R}_l(\tau_{\text{opt}})\cdot(1-\gamma A_o)^{|l|}\right]$, $\mathbf{g} = \mathbf{g}_{-1}(\tau_{\text{opt}}) - \mathbf{g}_1(\tau_{\text{opt}})$, $\acute{\mathbf{g}} = \acute{\mathbf{g}}_{-1}(\tau_{\text{opt}}) - \acute{\mathbf{g}}_1(\tau_{\text{opt}})$, and λ_1 and λ_2 are the Lagrange multipliers. The optimal solution $\mathbf{w}_{\text{opt}}$ is obtained through the differentiation of $L(\mathbf{w},\boldsymbol{\lambda})$ with respect to $\mathbf{w}$ and $\boldsymbol{\lambda}$. The solution is expressed as

$$\begin{aligned}\mathbf{w}_{\text{opt}} =& \frac{A_o}{\mathbf{g}^T\mathbf{R}^{-1}\mathbf{g}\cdot\acute{\mathbf{g}}^T\mathbf{R}^{-1}\acute{\mathbf{g}} - \left(\mathbf{g}^T\mathbf{R}^{-1}\acute{\mathbf{g}}\right)^2}\\ &\times\left(\mathbf{g}^T\mathbf{R}^{-1}\mathbf{g}\cdot\mathbf{R}^{-1}\acute{\mathbf{g}} - \mathbf{g}^T\mathbf{R}^{-1}\acute{\mathbf{g}}\cdot\mathbf{R}^{-1}\mathbf{g}\right).\end{aligned} \tag{10}$$

IV. Application to 1000BASE-T Systems

The 1000BASE-T Ethernet system bidirectionally transmits 5-PAM signals at the rate of 125 Msymbols/s through unshielded twisted pair (UTP) category-5 cables. The maximum distance between the transmitter and receiver is 100 m. The received signal is sampled at the baud rate (125 Msamples/s). In the receiver, a DFE, echo canceller, and near–end cross-talk (NEXT) canceller are usually employed. The timing functions $\{f(\tau)\}$ for different channel length are shown in Fig. 2. These functions were monotonic in $-0.5T \leq \tau \leq 0.5T$ and each $f(\tau)$ had only one root at about $0.25T$ ($\tau_o \approx 0.25T$).

The receiver employed a DFE with 16 feedforward taps and 12 feedback taps. The DFE coefficients were optimized to minimize the mean square error (MSE) [6]. The MSEs of this DFE were numerically evaluated for various values of τ when echo and cross-talk signals were absent and $E_s/N_o = 30$ dB. The

[2]When $A_s = A_o$, the normalized equivalent noise bandwidth [B_LT in (8)] of the loop with prefiltering is identical to that of the loop without prefiltering.

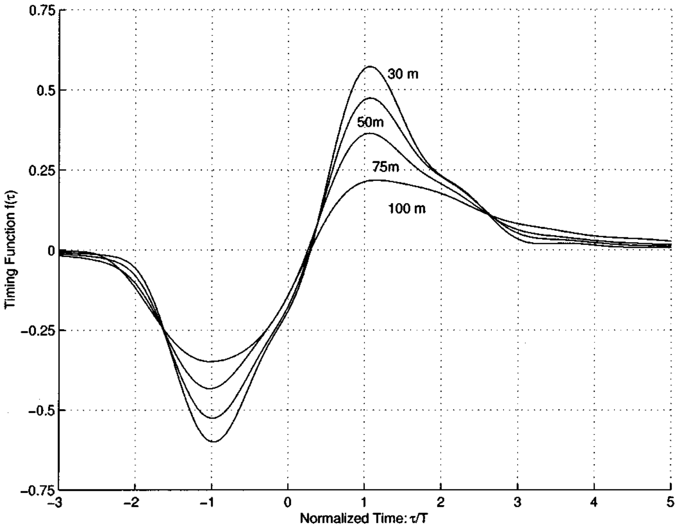

Fig. 2. Timing functions $\{f(\tau)\}$ for different channel length.

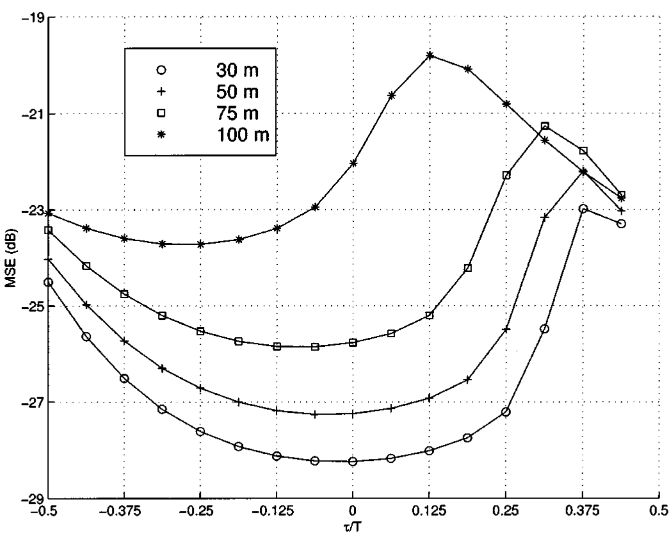

Fig. 3. Numerically calculated MSEs of the DFE.

results are shown in Fig. 3. It can be seen that the influence of the sampling phase on the MSE is significant. For example, for the 30-m channel the difference between the maximum and minimum MSE values was larger than 5 dB. As expected, the MSEs increased as the channel distance increased. The optimal timing phase $\tau_{\rm opt}$ varied depending on the channel distance. For example, $\tau_{\rm opt} = 0$ for the 30-m channel and $\tau_{\rm opt} = -0.125T$ for the 75-m channel. Consequently, it is impossible to design an optimal prefilter that can improve the receiver performance for all channels. Accordingly, in what follows, a prefilter was designed that optimized the timing phase of the 100-m channel, which was the worst. For the other channels, this optimization will produce a nonoptimal timing phase; however, in most practical cases, after prefiltering the MSEs should still be smaller than those associated with the 100-m channel without prefiltering. This is demonstrated through the following simulation.

Suppose that $E_s/N_o = 30$ dB, $B_LT = 0.01$, and $L = 2$ (5-tap prefilter). For the 100-m channel, $\tau_{\rm opt} = -5/16T$ and $A_o = 0.5$. From (10), the optimal weight was evaluated as $\mathbf{w}_o = [-0.1368\ 0.0864\ 1.1827\ -0.9947\ 0.1752]^T$. The steady-state jitter variance and MSE of the DFE were empirically estimated through computer simulation, assuming that echo and cross-talk signals were absent. The results are listed in Table I. It can be seen that a considerable reduction in σ_J^2 and MSE were achieved for the 100-m channel with the optimal prefilter: σ_J^2 after prefiltering was about 1/20 of the original and the MSE gain was about 6 dB. Some performance improvement was also achieved for the 75-m channel.[3] However, the prefiltering somewhat increased the σ_J^2 and MSE in the case of the 30-m channels. The largest MSE after prefiltering was −20.1 dB (50 m channel), which was 5.6 dB smaller than the largest MSE before prefiltering (−14.5 dB, 100 m channel); therefore, the prefilter decreased the largest MSE by 5.6 dB.[4]

TABLE I
STABLE EQUILIBRIUM POINTS, JITTER VARIANCE σ_J^2 AND MSE OF DFE WHEN $E_s/N_o = 30$ dB, $B_LT = 0.01$ AND $L = 2$

channel	No Prefilter			5-tap Prefilter		
	τ_o	σ_J^2	MSE	τ_s	σ_J^2	MSE
30 m	$0.262T$	2.14×10^{-4}	-22.1 dB	$-0.383T$	2.44×10^{-4}	-20.3 dB
50 m	$0.251T$	3.41×10^{-4}	-20.1 dB	$-0.370T$	2.75×10^{-4}	-20.1 dB
75 m	$0.226T$	6.12×10^{-4}	-18.0 dB	$-0.369T$	2.82×10^{-4}	-20.3 dB
100 m	$0.259T$	2.94×10^{-3}	-14.5 dB	$-0.312T$	1.27×10^{-4}	-20.8 dB

V. CONCLUSION

The design of a prefilter for baud-rate timing recovery was investigated. An optimization problem that minimizes the steady-state jitter variance under a constraint regarding the location of the steady-state timing phase was formulated, and a closed-form expression of the optimal filter coefficients was derived using Lagrange multipliers. The prefilter was applied to the 1000BASE-T Ethernet system. Accordingly, it was shown through simulation that prefiltering before the timing recovery can reduce the MSE of the equalizer.

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[3]This fact indicates the robustness of prefiltering to channel variations: the prefilter performance is not sensitive to small variations of the channel and the optimal timing phase for which the prefilter is optimized. When channel variations are large, one may design multiple prefilters and select one of them.

[4]In some applications, use of a single prefilter may not significantly reduce the MSE. For those cases, employing multiple prefilters and selecting one of them, which causes the smallest MSE, may be considered.