PAPER

Joint Maximum Likelihood Approach to Frame Synchronization in Presence of Frequency Offset*

Young-Hoi KOO[†] *and* **Yong Hoon LEE**[†a)], ***Nonmembers***

SUMMARY This paper proposes new frame synchronizers that can achieve frame sync in the presence of a frequency offset. In particular, a maximum likelihood (ML) algorithm for joint frame synchronization and frequency estimation is developed for additive white Gaussian noise (AWGN) channels, then the result is extended to frequency selective channels. Computer simulations demonstrate that the proposed schemes can outperform existing methods when a frequency offset exists.
key words: *frame synchronization, maximum likelihood, frequency offset*

1. Introduction

For efficient digital communications using burst transmission, reliable frame synchronization that realizes the start of known symbols in a data frame at the receiver is essential. Popular frame synchronization techniques include the correlation rule, which correlates the received signal with a sync pattern, and its modifications [1]–[4]. These techniques have been derived for AWGN channels under the assumption of a perfect carrier recovery. However, since frame synchronization is usually performed before the carrier recovery is completed, the performances of the synchronizers in [1]–[4] can be degraded in practical situations where frequency and phase errors exist.

To overcome this difficulty, methods of frame synchronization that are tolerant of frequency and phase errors have been developed in [5]–[8], including ML rules, with the exception of an ad hoc rule in [5], for AWGN [6], [7] and frequency selective channels [8]. The ML rules are derived following two different approaches*. The first is the *Bayesian* approach adopted in [6], [7], which involves averaging a probability density function (pdf) over the transmitted data, frequency, and phase offsets, while the second is the *joint* estimation approach adopted in [8], which jointly achieves frame sync, plus a frequency, and channel estimation. In this case, for the derivation, the rule assumes a special sync pattern that is periodically repeated.

In this paper, we first present a new ML rule for AWGN channels based on a joint estimation approach. Then, in relation to the problem discussed in [8], an ML rule is derived that allows an arbitrary sync pattern. Finally, the advantage of the proposed rules over existing schemes is demonstrated through computer simulations. In particular, it is shown that the false acquisition probabilities (FAPs) of the proposed method are smaller than those of existing schemes.

The organization of the remainder of this paper is as follows. Section 2 presents the signal model, the proposed joint ML rules are derived in Sect. 3, and the simulation results are presented in Sect. 4.

2. Signal Model

Consider packet transmission over a dispersive channel where the dispersion spans up to L symbol intervals. It is assumed that a packet starts with a data sequence of length P followed by a training sequence consisting of $L-1$ guard symbols and a sync pattern of duration N_s (Fig. 1). The guard symbols are inserted to avoid the sync pattern being affected by random data. After the sync pattern, the data symbols are continued**. The training sequence is denoted by $\{s_{-L+1}, \ldots, s_{-1}, s_0, \ldots, s_{N_s-1}\}$, in which the first $L-1$ samples represent guard symbols. It is assumed that the symbols are transmitted in an M-ary PSK format. The received baseband signal, which is sampled with a symbol interval T_s, is expressed as

$$r_k = \sum_{l=0}^{L-1} h_l e^{j(\theta_{k-l}+2\pi f_0 T_s k)} + w_k \tag{1}$$

where $e^{j\theta_k}$ is the M-ary phase-modulated symbol at time k; $L-1$ represents the channel memory; $\mathbf{h} = [h_0, h_1, \ldots, h_{L-1}]^T$

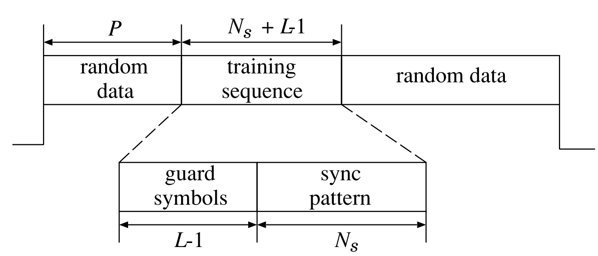


Fig. 1 Frame structure.

Manuscript received September 18, 2002.
Manuscript revised May 22, 2003.
Final manuscript received December 1, 2003.
[†]The authors are with the Department of Electronical Engineering and Computer Science, Korea Advanced Institute of Science and Technology, 373-1 Kusong-dong, Yusong-gu, Taejon, 305-701, Republic of Korea.
a) E-mail: yohlee@ee.kaist.ac.kr
*This work was supported in part by University IT Research Center project.

*Additional discussion on these approaches may be found in [9].

**The training sequence in this packet is a midamble, thus the proposed synchronizer, derived based on this packet, is most suitable for systems such as GSM [10] that employ a midamble. To consider a preamble some minor modification is necessary.

is a vector containing T_s-spaced samples of the channel response; and f_0 denotes the frequency offset. Complex AWGN is denoted by w_k, and its variance $\sigma_w^2 = N_0/E_s$ where E_s represents the symbol energy and N_0 is the noise power. For the AWGN channels, $L = 1$ and $h_0 = e^{j\phi_0}$ where ϕ_0 is the phase offset. In this case, r_k in (1) is reduced to

$$r_k = e^{j(\theta_k + 2\pi f_0 T_s k + \phi_0)} + w_k. \tag{2}$$

Throughout this paper, frame synchronization is started under the assumption that the position of a packet is roughly known up to an uncertainty of $\mu \in \{0, 1, \ldots, P + L - 1\}$ symbols (such coarse frame sync can be achieved by automatic gain control [8]). A frame is synchronized using N received samples $\{r_0, r_1, \ldots, r_{N-1}\}$. The observation window size $N \geq N_s + P + L - 1$, otherwise some received samples corresponding to the sync pattern may be excluded.

3. Derivation of Proposed Frame Synchronization

3.1 Frame Synchronization in AWGN Channels

Figure 2 illustrates the received sequence $\mathbf{r} = \{r_0, r_1, \ldots, r_{N-1}\}$ in the observation window and corresponding transmitted sequence for an AWGN channel. If the sync pattern starts at the μ-th position, $\mu \in [0, P]$, then the conditional pdf of $\mathbf{r}$ assuming μ, f_0, ϕ_0 and $\mathbf{d}$ is given by

$$p(\mathbf{r} \mid \mu, f_0, \phi_0, \mathbf{d}) = \prod_{k=0}^{N-1} \frac{E_s}{\pi N_0} e^{-|r_k - e^{j(\theta_k + 2\pi f_0 T_s k + \phi_0)}|^2 \frac{E_s}{N_0}}. \tag{3}$$

The joint ML estimates of μ, f_0, and ϕ_0 are obtained by maximizing $p(\mathbf{r} \mid \mu, f_0, \phi_0)$ with respect to μ, f_0, and ϕ_0. Evaluating the average of (3) over all possible data $\{\mathbf{d}\}$ gives

$$p(\mathbf{r} \mid \mu, f_0, \phi_0) = \frac{1}{M^{N-N_s}} \sum_{all\ \mathbf{d}} p(\mathbf{r} \mid \mu, f_0, \phi_0, \mathbf{d}). \tag{4}$$

To simplify this expression, the information symbol $e^{j\theta_k}$ is approximated as $e^{j\alpha}$, where α has a uniform distribution over $(-\pi, \pi)$ [1, p.283]. Then

$$\begin{aligned} p(\mathbf{r} \mid \mu, f_0, \phi_0) &\cong \left(\frac{E_s}{\pi N_0}\right)^N \prod_{k=\mu}^{\mu+N_s-1} e^{-|r_k - s_{k-\mu} e^{j(2\pi f_0 T_s k + \phi_0)}|^2 \frac{E_s}{N_0}} \\ &\cdot \prod_{k \notin \mathbf{\Omega}} \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-|r_k - e^{j(\alpha + 2\pi f_0 T_s k + \phi_0)}|^2 \frac{E_s}{N_0}} d\alpha \\ &= \left(\frac{E_s}{\pi N_0}\right)^N \prod_{k=0}^{N-1} e^{-(|r_k|^2+1)E_s/N_0} \\ &\cdot \prod_{k=\mu}^{\mu+N_s-1} e^{\frac{2E_s}{N_0}\mathrm{Re}\{r_k s_{k-\mu}^* e^{-j(2\pi f_0 T_s k + \phi_0)}\}} \\ &\cdot \prod_{k \notin \mathbf{\Omega}} I_0\left(\frac{2E_s}{N_0} |r_k e^{-j(2\pi f_0 T_s k + \phi_0)}|\right) \end{aligned} \tag{5}$$

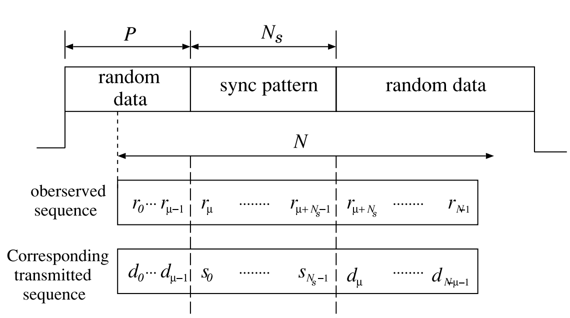


Fig. 2 Observed sequence and corresponding transmitted data in AWGN channel, where $0 \leq \mu \leq P$.

where $\mathbf{\Omega} = \{\mu, \mu+1, \ldots, \mu+N_s-1\}$ and $I_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x cos\theta} d\theta$ is a zeroth-order modified Bessel function of the first kind. The conditional pdf in (5) is further simplified by approximating $I_0(x)$ as $e^{|x|}/\sqrt{2\pi}$ [4]:

$$\begin{aligned} p(\mathbf{r} \mid \mu, f_0, \phi_0) &\cong C(\mathbf{r}) \cdot \prod_{k \notin \mathbf{\Omega}} e^{\left(\frac{2E_s}{N_0}|r_k|\right)} \\ &\cdot \prod_{k=\mu}^{\mu+N_s-1} e^{\frac{2E_s}{N_0} Re\{r_k s_{k-\mu}^* e^{-j(2\pi f_0 T_s k + \phi_0)}\}} \end{aligned} \tag{6}$$

where $C(\mathbf{r}) = \left(\frac{E_s}{\pi N_0}\right)^N \prod_{k=0}^{N-1} e^{-\frac{(|r_k|^2+1)E_s}{N_0}} \frac{1}{(\sqrt{2\pi})^{N-L}}$. After taking the logarithm, dropping the terms independent of μ,f_0, and ϕ_0 and subtracting a constant $\sum_{k=0}^{k=N-1}|r_k|$, the following test function is obtained:

$$\begin{aligned} &\Lambda(\mu, f_0, \phi_0) \\ &\equiv \sum_{k=\mu}^{\mu+N_s-1} \mathrm{Re}\{r_k s_{k-\mu}^* e^{-j(2\pi f_0 T_s k + \phi_0)}\} - \sum_{k=\mu}^{\mu+N_s-1} |r_k| \\ &= \sum_{l=0}^{N_s-1} \mathrm{Re}\{r_{l+\mu} s_l^* e^{-j(2\pi f_0 T_s l + \phi_0')}\} - \sum_{l=0}^{N_s-1} |r_{l+\mu}| \\ &\equiv \Lambda(\mu, f_0, \phi_0') \end{aligned} \tag{7}$$

where $l = k - \mu$ and $\phi_0' = \phi_0 + 2\pi f_0 T_s \mu$. When the frequency and phase offsets are zero, this expression reduces to the ML rule for a high SNR derived in [2]. The maximization of $\Lambda(\mu, f_0, \phi_0')$ with respect to μ,f_0, and ϕ_0' can be achieved through the following three-step procedure [1, p.248]†: First, maximize $\Lambda(\mu, f_0, \phi_0')$ with respect to ϕ_0' for each possible μ and f_0. Specifically, an estimate of ϕ_0' is obtained as a function of μ and f_0: $\hat{\phi}_0'(\mu, f_0) = \arg\max_{\phi_0'} \Lambda(\mu, f_0, \phi_0')$. Second, derive an estimate of f_0 as a function of μ, which is expressed as $\hat{f}_0(\mu) = \arg\max_{f_0} \Lambda(\mu, f_0, \phi_0' = \hat{\phi}_0'(\mu, f_0))$. Third, select μ with the largest likelihood $\hat{\mu} = \arg\max_{\mu} \Lambda(\mu, f_0 = \hat{f}_0(\mu), \phi_0' = \hat{\phi}_0'(\mu, \hat{f}_0(\mu)))$. To derive $\hat{\phi}_0'(\mu, f_0)$, $\Lambda(\mu, f_0, \phi_0')$ is differentiated with respect to ϕ_0' and the result is set to zero. This yields

$$\hat{\phi}_0'(\mu, f_0) = \arg\left\{\sum_{l=0}^{N_s-1} r_{l+\mu} s_l^* e^{-j2\pi f_0 T_s l}\right\}. \tag{8}$$

†Since μ, f_0, and ϕ_0' are finite, the maximum of $\Lambda(\mu, f_0, \phi_0')$ can also be found through an exhaustive search [9].

Using (8) in (7) gives

$$\Lambda(\mu, f_0, \phi_0' = \hat{\phi}_0'(\mu, f_0)) \equiv \Lambda'(\mu, f_0) - \sum_{l=0}^{N_s-1} |r_{l+\mu}| \tag{9}$$

where $\Lambda'(\mu, f_0) = \left|\sum_{l=0}^{N_s-1} r_{l+\mu} s_l^* e^{-j2\pi f_0 T_s l}\right|$. Now $\hat{f}_0(\mu)$ can be obtained by maximizing $\Lambda'(\mu, f_0)$. Differentiating the *square* of $\Lambda'(\mu, f_0)$ with respect to f_0 and setting the result equal to zero yields

$$\sum_{l=1}^{N_s-1} \sum_{m=1}^{N_s-1} (l-m) r_{l+\mu} s_l^* r_{m+\mu}^* s_m e^{-j2\pi f_0 T_s (l-m)} = 0. \tag{10}$$

Then, following the procedure for deriving the frequency estimator in [11],

$$\hat{f}_0(\mu) = \frac{1}{\pi T_s (R+1)} \arg\left\{\sum_{l=1}^{R} \rho_\mu(l)\right\} \tag{11}$$

where

$$\rho_\mu(l) = \frac{1}{N_s - l} \sum_{m=l}^{N_S-1} r_{m+\mu} s_m^* r_{m-l+\mu}^* s_{m-l}. \tag{12}$$

When μ is known, this estimator is identical to the one in [11]. The design parameter R enables the adjustment of the acquisition range of the frequency estimate, which is given by $|\hat{f}_0(\mu)| < \frac{1}{(R+1)T_s}$. The proposed frame synchronization rule is then obtained from (9) and (11):

$$\hat{\mu} = \arg\max_{\mu} \left\{ \left| \sum_{l=0}^{N_s-1} r_{l+\mu} s_l^* e^{-j2\pi \hat{f}_0(\mu) T_s l} \right| - \sum_{l=0}^{N_s-1} |r_{l+\mu}| \right\}. \tag{13}$$

where $\hat{f}_0(\mu)$ is given by (11). When $\hat{f}_0(\mu) = 0$ and the phase offset $\phi_0 = 0$, this rule reduces to

$$\hat{\mu} = \arg\max_{\mu} \left\{ \sum_{l=0}^{N_s-1} r_{l+\mu} s_l^* - \sum_{l=0}^{N_s-1} |r_{l+\mu}| \right\}. \tag{14}$$

The ML rule in (14) is identical to the ML rule in [2], developed for AWGN channels without any frequency and phase offset. The proposed rule in (13) compensates for the frequency offset of $\{r_k\}$ and then evaluates the correlation with the sync pattern $\{s_k\}$.

3.2 Extension to Frequency Selective Channels

For frequency selective channels, following the approach in [8], a subwindow of length N_s observing $\mathbf{r}_\mu = [r_\mu, r_{\mu+1}, \ldots, r_{\mu+N_s-1}]^T$ for each $\mu \in \{0, 1, \ldots, P + L - 1\}$ is employed and the conditional pdf of $\mathbf{r}_\mu$ is derived. Under the assumption that μ is the correct sync pattern start position, $\mathbf{r}_\mu$ only corresponds to the training sequence (Fig. 3), thus its conditional pdf does *not* dependent on the random data within the packet. This fact greatly simplifies the derivation, because averaging the conditional pdf over all possible data sequences becomes unnecessary. The synchronizer derived

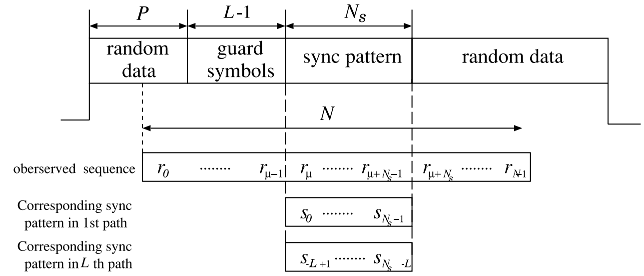


Fig. 3 Observed sequence and corresponding transmitted data in frequency selective channel, where $0 \le \mu \le P + L - 1$.

below detects the sync pattern start position of the 1st path of a multipath fading channel.

The received vector $\mathbf{r}_\mu$ can be written in matrix form [9] as

$$\mathbf{r}_\mu = \mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h} + \mathbf{w}_\mu \tag{15}$$

where $\mathbf{\Gamma}(f_0)$ is a diagonal matrix given by

$$\mathbf{\Gamma}(f_0) = \mathrm{diag}\{1, e^{j2\pi f_0}, e^{j4\pi f_0}, \ldots, e^{j2\pi(N_s-1)f_0}\}, \tag{16}$$

$\mathbf{A}$ is an $N_s \times L$ matrix with entries

$$[\mathbf{A}]_{i,j} = s_{i-j} \quad 0 \le i \le N_s - 1,\ 0 \le j \le L - 1, \tag{17}$$

and $\mathbf{w}_\mu = [w_\mu, w_{\mu+1}, \ldots, w_{\mu+N_s-1}]^T$ is a zero-mean Gaussian vector with the covariance matrix $\frac{N_0}{E_s}\mathbf{I}_{N_s}$. Here $\mathbf{I}_{N_s}$ is the $N_s \times N_s$ identity matrix. For a given pair $(\mu, f_0, \mathbf{h})$, the vector $\mathbf{r}_\mu$ is Gaussian with the mean $\mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h}$ and covariance matrix $\frac{N_0}{E_s}\mathbf{I}_{N_s}$. Thus the conditional pdf of $\mathbf{r}_\mu$ assuming μ, f_0, and $\mathbf{h}$ takes the form

$$\begin{aligned} p(\mathbf{r}_\mu \mid \mu, f_0, \mathbf{h}) &= \left(\frac{E_s}{\pi N_0}\right)^{N_s} \\ &\cdot \exp\left\{-(\mathbf{r}_\mu - \mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h})^H(\mathbf{r}_\mu - \mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h})\frac{E_s}{N_0}\right\}. \end{aligned} \tag{18}$$

After taking the logarithm and dropping the terms independent of μ, f_0, and $\mathbf{h}$, this yields

$$\Lambda(\mu, f_0, \mathbf{h}) = -(\mathbf{r}_\mu - \mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h})^H(\mathbf{r}_\mu - \mathbf{\Gamma}(f_0)\mathbf{A}\mathbf{h}). \tag{19}$$

To maximize $\Lambda(\mu, f_0, \mathbf{h})$ with respect to μ, f_0, and $\mathbf{h}$, the three-step approach introduced in the previous section is applied. First a channel estimate is found by setting the derivative of (19) equal to zero. The result is:

$$\hat{\mathbf{h}}(\mu, f_0) = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{\Gamma}(f_0)^H\mathbf{r}_\mu. \tag{20}$$

When μ and f_0 are known, (20) reduces to the ML channel estimate in [12]. Substituting $\hat{\mathbf{h}}(\mu, f_0)$ in (20) for $\mathbf{h}$ in (19),

$$\begin{aligned} \Lambda(\mu, f_0, \mathbf{h} = \hat{\mathbf{h}}(\mu, f_0)) &= \mathbf{r}_\mu{}^H\mathbf{\Gamma}(f_0)\mathbf{B}\mathbf{\Gamma}(f_0)^H\mathbf{r}_\mu - \mathbf{r}_\mu{}^H\mathbf{r}_\mu \\ &= 2\mathrm{Re}\left\{\sum_{l=0}^{N_s-1} \rho_\mu(l) e^{-j2\pi f_0 T_s l}\right\} \\ &\quad - \rho_\mu(0) - \sum_{l=0}^{N_s-1} |r_{l+\mu}|^2 \end{aligned} \tag{21}$$

where $\mathbf{B} = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$,

$$\rho_\mu(l) = \sum_{m=l}^{N_S-1} [\mathbf{B}]_{l-m,m}\, r_{m+\mu} r^*_{m-l+\mu}, \quad (22)$$

and $[\mathbf{B}]_{i,j}$ is the (i, j)th-entry for $\mathbf{B}$. After dropping the terms independent of f_o from (21), the ML frequency estimate can be expressed as

$$\hat{f}_0(\mu) = \arg\max_{f_0} \left[2\mathrm{Re}\left\{ \sum_{l=0}^{N_s-1} \rho_\mu(l) e^{-j2\pi f_0 T_s l} \right\} - \rho_\mu(0) \right]. \quad (23)$$

For a given μ, (23) is the same as the frequency estimator in [9]. This estimate is obtained by exhaustively searching the range $|f_0| \leq 0.5$. Using (23) in (21), the proposed frame sync rule for frequency selective channels is obtained:

$$\hat{\mu} = \arg\max_\mu \left\{ \mathbf{r}_\mu{}^H \mathbf{\Gamma}(\hat{f}_0(\mu)) \mathbf{B} \mathbf{\Gamma}(\hat{f}_0(\mu))^H \mathbf{r}_\mu - \mathbf{r}_\mu{}^H \mathbf{r}_\mu \right\}. \quad (24)$$

This rule is identical to the one in [8], with the exception that the frequency estimator in (23) is employed (in [8], the frequency offset is estimated assuming a periodic sync pattern with period L). Due to (23), the proposed rule can employ an arbitrary sync pattern. If the sync pattern is chosen to satisfy $\mathbf{A}^H\mathbf{A} = N_s\mathbf{I}_L$, then the rule in (24) becomes

$$\begin{aligned} \hat{\mu} &= \arg\max_\mu \left\{ \frac{1}{N_s} \mathbf{r}_\mu{}^H \mathbf{\Gamma}(\hat{f}_0(\mu)) \mathbf{A}\mathbf{A}^H \mathbf{\Gamma}(\hat{f}_0(\mu))^H \mathbf{r}_\mu \right. \\ &\qquad \left. - \mathbf{r}_\mu{}^H \mathbf{r}_\mu \right\} & (25) \\ &= \arg\max_\mu \left[\frac{1}{N_s} \sum_{l=0}^{L-1} \left| \sum_{k=0}^{N_s-1} r_{k+\mu} s^*_{k-l} e^{-j2\pi \hat{f}_0(\mu) T_s k} \right|^2 \right. \\ &\qquad \left. - \sum_{k=0}^{N_s-1} |r_{k+\mu}|^2 \right]. & (26) \end{aligned}$$

The computational load for evaluating (26) is close to that for (24), because matrix $\mathbf{B}$ can be precalculated. Equation (26) is mainly useful for grasping some intuition about the behavior of the proposed scheme. In particular, when the channel is flat fading ($L = 1$), the rule in (26) becomes almost identical to the one in (13): the only difference is that the former evaluates "squares," while the latter computes "absolute" values. For frequency selective channels, the proposed rule detects the sync pattern start position of the 1st path by combining the correlations of all paths. Although the rules in (24) and (26) do not require explicit channel estimation, a frequency estimation is needed for each μ. Since (23) requires an exhaustive search, the major computational burden of the proposed rule comes from the frequency estimation. The frequency offset and channel estimates are given by $\hat{f}_0(\hat{\mu})$ and $\hat{\mathbf{h}}(\hat{\mu}, \hat{f}_0(\hat{\mu}))$, respectively, and can be obtained after finding $\hat{\mu}$.

Finally, in this section, it is worthwhile to derive the rule in [8] from (24). Suppose that the sync pattern is given by a repeated CAZAC (constant-amplitude, zero autocorrelation) sequence [5] with period L. Then $\mathbf{A}^H\mathbf{A} = N_s\mathbf{I}_L$, and due to the repetition of a CAZAC sequence, $\mathbf{B}$ becomes

$$\mathbf{B} = \frac{1}{P} \begin{bmatrix} \mathbf{I}_L & \mathbf{I}_L & \cdots & \mathbf{I}_L \\ \mathbf{I}_L & \mathbf{I}_L & \cdots & \mathbf{I}_L \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I}_L & \mathbf{I}_L & \cdots & \mathbf{I}_L \end{bmatrix} \quad (27)$$

where P is the number of repetitions in the sync pattern ($P = \frac{N_s}{L}$). For the matrix $\mathbf{B}$ in (27), the rule in (24) is simplified to

$$\begin{aligned} \hat{\mu} &= \arg\max_\mu \left\{ -\rho_\mu(0)\left(1 - \frac{1}{P}\right) + 2\sum_{m=1}^{P-1} |\rho_\mu(mL)| \right. \\ &\qquad \left. \cdot \cos\left[\arg[\rho_\mu(mL)] - 2\pi m \hat{f}_0(\mu) T_s\right] \right\} \quad (28) \end{aligned}$$

where

$$\rho_\mu(l) = \begin{cases} \frac{1}{N_s} \sum_{m=l}^{N_s-1} r_{m+\mu} r^*_{m-l+\mu} \\ \quad \text{where } l = 0, L, \cdots, (P-1)L \\ 0, \ \text{otherwise.} \end{cases} \quad (29)$$

In [8], (28) is used in conjunction with the following frequency estimate: $\hat{f}_0(\mu) = \frac{1}{2\pi L T_s} \arg[\rho_\mu(L)]$ for $\rho_\mu(l)$ defined in (29). The rule in (28) can be thought of as a special case of the rule in (26). Both of them are derived from (24), assuming $\mathbf{A}^H\mathbf{A} = N_s\mathbf{I}_L$, but the latter allows an arbitrary sync pattern. However, these rules also exhibit a notable difference. The rule in (26) is based on the correlation between the received signal and the sync pattern. In (28), such a correlation is not obtained; instead, the rule searches for a periodic pattern of duration L in a received signal by examining the similarity between the received samples r_k and r_{k-L} (see (29)). Consequently, the test function in (28) decays slowly as μ moves away from the start position of the sync pattern [13][†], thereby degrading the frame synchronization performance (this problem does not occur in (26)). In the next section, the behavior of the rules in (26) and (28) is examined through computer simulation.

4. Simulation Results

The false acquisition probabilities (FAPs)[††] for the proposed rules were investigated by simulations using the following parameters: $M = 4$ (QPSK), N=80, and P=20. A sync pattern of length N_s=16 was taken from the midamble of GSM, given by

[†]To relieve the difficulty, [13] proposes the use of an aperiodic training sequence in which CAZAC blocks are phase-shifted and concatenated. However, this approach requires a longer training sequence with a length of $(N_s + (L-1)P)$, because a guard interval of length $L-1$ is appended at the begining of each CAZAC sequence.

[††]Since the sync pattern starts at the μth position, $\mu \in \{0, 1, \ldots, P + L - 1\}$, of N observations, the synchronizer only has two stages: it either acquires or false locks. Therefore, the probability of false acquisition completely characterizes the synchronizer performance.

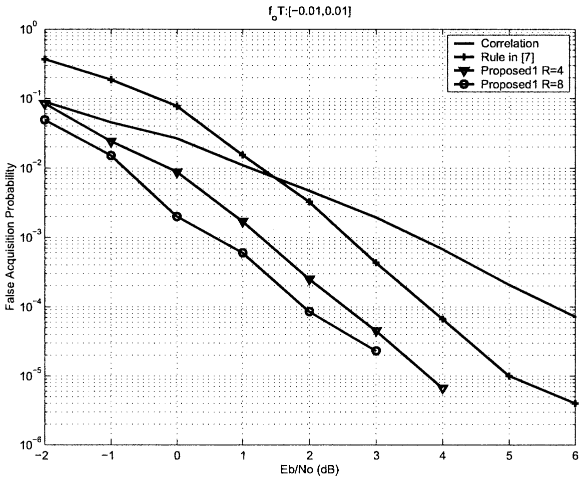


Fig. 4 False acquisition probability vs. signal to noise ratio for AWGN channels.

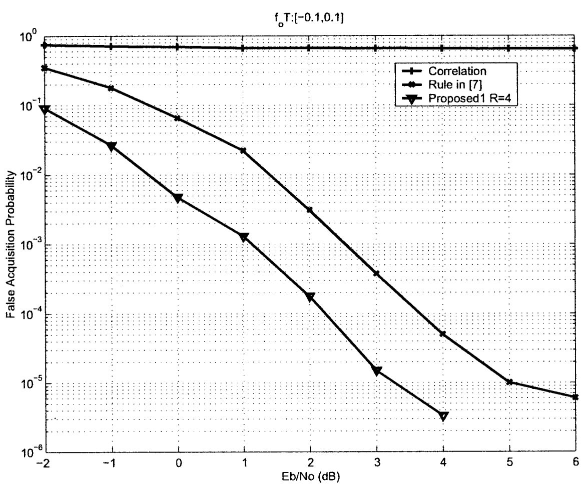


Fig. 5 False acquisition probability vs. signal to noise ratio for AWGN channels.

$$\{1, -j, 1, j, 1, -j, -1, -j, -1, j, -1, -j, -1, j, -1, -j\}. \tag{30}$$

This pattern satisfies $\mathbf{A}^H\mathbf{A} = N_s\mathbf{I}_L$ when $L < 8$. The frequency selective channel was modeled as:

$$h(t) = \sum_{l=0}^{L-1} h_l \delta(t - lT_s) \tag{31}$$

where $\{h_l\}$ are independently identically distributed complex Gaussian random variables with a zero mean and variance of $1/L$. The simulations used 500,000 independent frames and the FAP was empirically estimated by counting the number of frame synchronization failures[†]. For comparison, the conventional correlation rule, the rule in [7] for AWGN channels, and the rule in [8], as defined in (28) for dispersive channels, were also considered. It was observed that all these rules could be directly applied to the frame structure shown in Fig. 1. The rule in [7] is expressed as

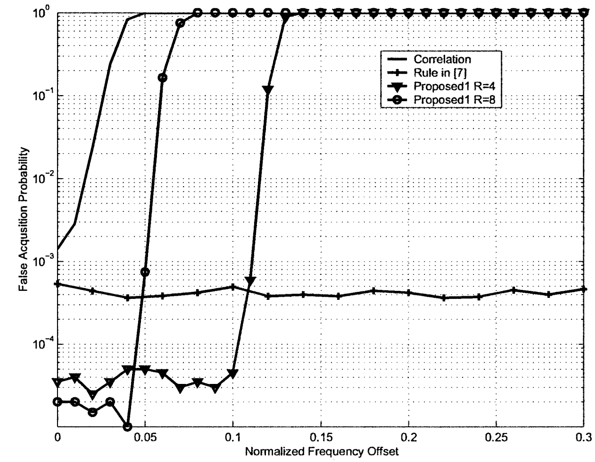


Fig. 6 False acquisition probability vs. normalized frequency offset for AWGN channels when E_b/N_o is 3 dB.

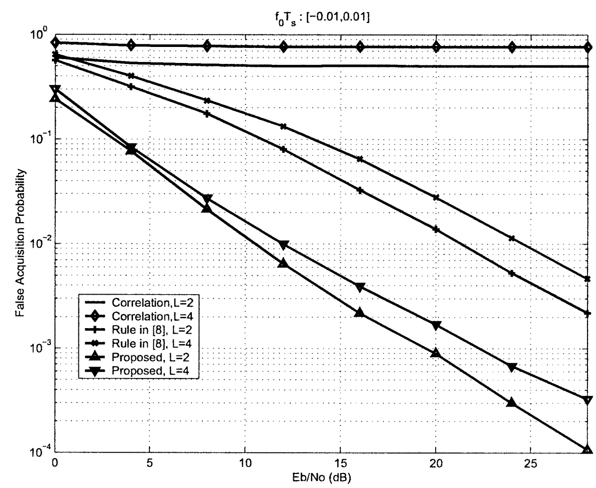


Fig. 7 Probability of false acquisition for frequency selective channels of length L. The normalized frequency offset was generated randomly over $[-0.01, 0.01]$.

$$\hat{\mu} = \arg\max_{\mu} \sum_{l=1}^{N_s-1} \left\{ \left|\rho_\mu(l)\right| - \sum_{m=l}^{N_s-1} |r_{m+\mu}||r_{m-l+\mu}| \right\} \tag{32}$$

where $\rho_\mu(l)$ is given by (12). Repeated CAZAC sequences of $\{1, 1, -1, 1\}$ ($L = 4$) and $\{1, -1\}$ ($L = 2$) were employed in implementing the rule in (28), while all the other rules in the simulation used the training sequence in (30).

Figures 4 and 5 compare the performances of the rules when f_0T_s was uniformly distributed over $[-f_mT_s, f_mT_s]$, where f_mT_s=0.01 and 0.1. As expected, the correlation rule generally performed worst, whereas the proposed rule outperformed the others.

The robustness of the frame synchronizers was examined by estimating the FAPs for various normalized frequency offsets (f_0T_s) between 0 and 0.3, while fixing E_b/N_0

[†]For frequency selective channels, synchronization failure was declared whenever the synchronizer failed to detect the start of the sync pattern for the 1st path.

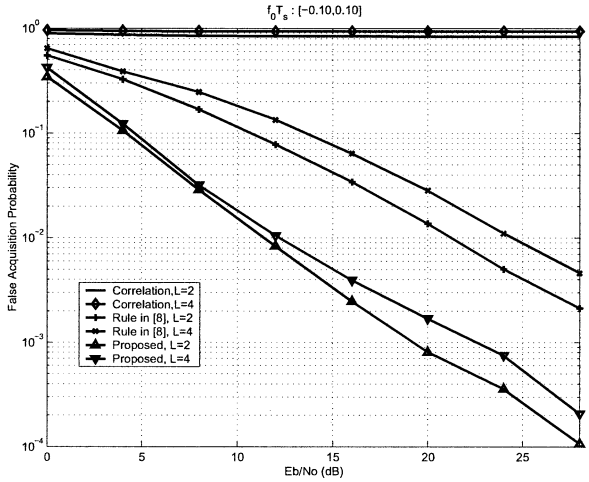


Fig. 8 Probability of false acquisition for frequency selective channels of length L. The normalized frequency offset was generated randomly over $[-0.10, 0.10]$.

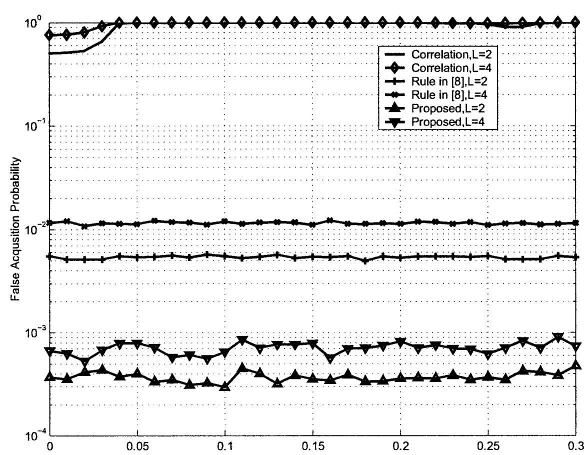


Fig. 9 Probability of false acquisition for frequency selective channels of length L. E_b/N_o was fixed at 24 dB.

at 3 dB. The results are shown in Fig. 6. Again, the correlation rule performed worst. The FAP of the proposed rule decreased as R increased, at the expense of a narrower frequency acquisition range. The rule in [7] was most robust to frequency offsets, yet its FAPs were larger than those of the proposed rule when $R = 4$ and 8.

Figures 7 and 8 show the FAPs for the frequency selective channels with $L = 2$ and $L = 4$. As in the case of the AWGN channels, the correlation rule performed worst, and the proposed rule outperformed the others. The performance degradation of the rule in [8] was due to the slow decaying characteristic of the test function, as explained at the end of Sect. 3.

Finally, Fig. 9 compares the robustness of the frame synchronizers to frequency offset in dispersive channels. Both the proposed rule and the rule in [8] were robust to a frequency offset: the robustness of the former resulted from the fact that the acquisition range of the frequency estimator in (23) is given by $|f_0| \leq 0.5$. For all values of frequency offset, the proposed rule performed the best.

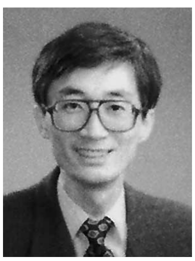
5. Conclusion

ML-type frame synchronizers that can jointly achieve frame sync, plus a frequency and channel estimation were derived and the advantages of the proposed techniques over existing ones demonstrated through computer simulation. Further work in this area will include the development of frame synchronizers for systems with transmitter and receiver antenna diversity.

References

[1] J.L. Massey, "Optimum frame synchronization," IEEE Trans. Commun., vol.20, no.2, pp.115–119, April 1972.
[2] P.T. Nielsen, "Some optimum and suboptimum frame synchronizers for binary data in Gaussian noise," IEEE Trans. Commun., vol.21, no.3, pp.770–772, June 1973.
[3] R.A. Scholtz, "Frame synchronization techniques," IEEE Trans. Commun., vol.28, no.8, pp.1204–1212, Aug. 1980.
[4] G.L. Lui and H.H. Tan, "Frame synchronization for Gaussian channels," IEEE Trans. Commun., vol.35, no.8, pp.1828–1841, Aug. 1987.
[5] H. Meyr, M. Moeneclaey, and S.A. Fechtel, Digital Communication Receivers, John Wiley & Sons, New York, 1998.
[6] J.M. Gansman, M.P. Fitz, and J.V. Krogmeier, "Optimum and suboptimum frame synchronization for pilot-symbol-assisted modulation," IEEE Trans. Commun., vol.45, no.10, pp.1327–1337, Oct. 1997.
[7] Z.Y. Choi and Y.H. Lee, "Frame synchronization in the presence of frequency offset," IEEE Trans. Commun., vol.50, no.7, pp.1062–1065, July 2002.
[8] S.A. Fechtel and H. Meyr, "Fast frame synchronization, frequency offset estimation and channel acquisition for spontaneous transmission over unknown frequency-selective radio channels," Proc. IEEE PIMRC'93, pp.229–233, 1993.
[9] M. Morelli and U. Mengali, "Carrier-frequency estimation for transmissions over selective channels," IEEE Trans. Commun., vol.48, no.9, pp.1580–1589, Sept. 2000.
[10] Radio Transmission and Reception, GSM Recommendation 0505, ETSI/PT 12, V. 3.11.0, Jan. 1990.
[11] M. Luise and R. Reggiannini, "Carrier frequency recovery in all-digital modems for burst-mode transmission," IEEE Trans. Commun., vol.43, no.2/3/4, pp.1169–1178, Feb./March/April 1995.
[12] S.M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Prentice-Hall, Englewood Cliffs, NJ, 1993.
[13] S.A. Fechtel and H. Meyr, "Improved frame synchronization for spontaneous packet transmission over frequency-selective radio channels," Proc. IEEE PIMRC'94, pp.353–357, 1994.

Young-Hoi Koo received the B.S. and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), Taejon, Korea, in 1990 and 1992, respectively. He is currently working toward the Ph.D. degree in electrical engineering at KAIST. Since 1992, he has been with LG Electronics Inc. in Korea as a Technical engineer. His research interests include synchronization, wireless digital communication, and Orthogonal Frequency Devision Multiplexing (OFDM) system.



Yong Hoon Lee (S'81-M'84-SM'98) was born in Seoul, Korea, on July 12, 1955. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in electrical engineering from the University of Pennsylvania, Philadelphia, in 1984. From 1984 to 1989, he was an Assistant Professor with the Department of Electrical and Computer Engineering, State University of New York, Buffalo. Since 1989, he has been with the Department of Electrical Engineering KAIST, where he is currently a Professor. His research activities are in the area of communication signal processing which includes synchronization, interference cancellation, and spcace-time signal processing for CDMA and OFDM systems.