V. Conclusion

In this correspondence, we proposed a novel DCT-based algorithm for the fast computation of the MCLT. The method is based on two DCTs, two stages of butterfly operations. We also gave the detailed signal flow graph for the inverse MCLT. It is shown that the multiplicative complexity of the proposed algorithm is superior to any other previous algorithm, and there is no penalty on the total number of multiplications and additions.

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Application of Randomization Techniques to Space-Time Convolutional Codes

Hamid R. Sadjadpour, Kyungmin Kim, Hongyun Wang, Rick S. Blum, and Yong H. Lee

***Abstract*—This correspondence introduces a new approach to design space–time convolutional codes (STCCs) with large constellation size in systems with any number of transmit antennas. Our design procedure is based on utilizing quadrature phase-shift keying (QPSK) STCCs as component codes, and, consequently, unlike existing techniques, the search space does not grow exponentially with the constellation size. Our approach is further based on the fact that an $n \times m$ multiple-input multiple-output (MIMO) system is equivalent to n distinct $1 \times m$ systems. By employing a common design for each individual $1 \times m$ system, we arrive at an approach whose complexity does not grow with the number of transmit antennas. To describe our approach, we first demonstrate that a system employing an STCC can be implemented with only a single transmit antenna when there are multiple receive antennas. The idea is to transmit more than one symbol from a single transmit antenna during a symbol period by superimposing the encoded symbols on top of each other. This objective is achieved by inducing randomness into the system, that creates additional channel paths, called virtual paths. The design of the distributions of the induced random variables is studied for slow Rayleigh and Rician fading channels by utilizing an upper bound on the pairwise block error probability. Simulation results evaluate the performance of this technique for the case of two transmit antennas and several different number of receive antennas, a spectral efficiency of 4 b/s/Hz for slow Rayleigh and Rician fading channels.**

***Index Terms*—Multiple-input multiple-output (MIMO), space–time code.**

I. Introduction

Multiple antennas are very important for increasing capacity and reliability of wireless channels. It is a common belief that future wireless systems will have multiple antennas at both transmitter and receiver ends to be able to transmit high data rate video, data, and voice. A system with multiple-input multiple-output (MIMO) capability has much higher capacity than single-input single-output (SISO) and single-input multiple-output (SIMO) systems [1]. Recent research results have shown that not only is the channel capacity of MIMO systems very high [1], [2], but large fractions of this capacity can actually be achieved in implementations [3], [4]. Even with the extensive research on space–time convolutional codes (STCCs), STCC

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code designs are lacking for cases with large constellation size and/or a large number of transmit antennas. We will introduce a new approach to design STCCs for any arbitrary $n \times m$ MIMO system with a search space that does not increase exponentially with the constellation size and that does not increase at all with the number of transmit antennas.

Our approach is based on the fact that one can view an $n \times m$ MIMO system as an equivalent group of n distinct $1 \times m$ systems. Then, by producing a common design for each individual $1 \times m$ system, we arrive at an approach whose complexity does not grow with the number of transmit antennas. Further, we will build our STCCs from combining small [quadrature phase-shift keying (QPSK)] constellation size STCCs such that our search complexity grows slowly with constellation size. To describe our approach, we first demonstrate that a system employing a STCC can be implemented with only a single transmit antenna when there are multiple receive antennas. The idea is to transmit more than one symbol from a single transmit antenna during a symbol period by superimposing the encoded symbols on top of each other. This objective is achieved by inducing randomness into the system, that creates additional channel paths, called virtual paths.

Inducing randomness into a physical channel has been proposed by many authors [5]–[10]. The main objective of these techniques is to induce more fluctuations into the channel. In [5], the authors induce randomness into the downlink of a wireless communication channel to create more fluctuations into an environment where slow fading or little scattering may occur. The randomization concept has been also proposed for space–time code applications [8]–[10]. The main idea behind these works is to increase fluctuations in the channel. The work of [10] is the first approach that evaluates improvement of MIMO channels from an information theoretic perspective using outage probability. In this correspondence, we propose to induce randomness into the physical channel in what appears to be a new way. The goal is to explore the rich diversity capabilities of MIMO systems and to design STCC with high spectral efficiency for any number of transmit antennas using QPSK STCC as component codes. We will derive the conditions under which maximum coding and diversity gain can be attained for fading channels. Our optimization criterion is based on minimizing the upper bound on the pairwise block error probability when induced random variables are used. Our randomization approach does not attempt to induce more fluctuations into the channel.

In Section II, we will review the system model and STCC design [12] for slow fading wireless channels. The proposed algorithm utilizing a STCC for the single transmit antenna case is formulated in Section III. The optimum induced randomization for slow Rayleigh and Rician fading channels is described in Section IV. It is shown in this section, that this optimum randomization depends on the exact error pattern. In Section V, we design an alternative randomization approach. Simulation results on the performance of the proposed algorithm is given in Section VI. Section VII contains the conclusion.

II. System Model

We consider a wireless communication system utilizing n transmit and m receive antennas. The channel path gain from transmit antenna i to receive antenna j is denoted by $h_{i,j}$ and is a complex Gaussian random variable with mean $m_1/\sqrt{2}$ (m_1 is zero for Rayleigh and nonzero value for Rician fading channels) and variance 0.5 per complex dimension (real–imaginary parts). We assume that different channel path gains are statistically independent. We also assume that the channel coefficients are constant during one block of data and change independently from one block to another. The received data r_t^j at antenna j and time t (slow fading channel) can be written as

$$r_t^j = \sum_{i=1}^{n} h_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m \tag{1}$$

where c_t^i is the complex transmit symbol with unit average power sent from antenna i at time t, n_t^j is the additive white Gaussian noise sample with zero mean and variance $N_o/2$ per dimension, and E_s is the contraction factor of the signal constellation. A block error occurs when the decoded data sequence

$$\underline{\mathbf{E}} = e_1^1 \ldots e_1^n \ldots e_N^1 \ldots e_N^n$$

is different from the transmit sequence

$$\underline{\mathbf{C}} = c_1^1 \ldots c_1^n \ldots c_N^1 \ldots c_N^n$$

where N is the number of symbols in one block. It is shown in [12] that for a maximum-likelihood receiver, an upper bound on the conditional pairwise block error probability (slow fading channel) is

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} | h_{i,j}, 1 \le i \le n, 1 \le j \le m) \le \prod_{j=1}^{m} \exp\left(-\Omega_j B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) \Omega_j^* \frac{E_s}{4N_o}\right) \tag{2}$$

where $\Omega_j = (h_{1,j}, h_{2,j}, \ldots, h_{n,j})$, $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ is an $n \times n$ matrix whose elements are defined as $B_{s,mn}(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = \sum_{t=1}^{N} (c_t^m - e_t^m)(c_t^n - e_t^n)^*$ and $*$ denotes the conjugate transpose operation. It can be shown that [12] $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = V^* D V$ is a Hermitian matrix, V is a unitary matrix whose rows v_j, $1 \le j \le n$ are the eigenvectors of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ and $D = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$ where the λ_j's are the eigenvalues of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$. If a vector β_j is defined as $\beta_j = [\beta_{1,j} \ldots \beta_{n,j}] = \Omega_j V^*$, then

$$\Omega_j B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) \Omega_j^* = \sum_{i=1}^{n} \lambda_i |\beta_{i,j}|^2. \tag{3}$$

By substituting (3) into (2), one can average over $|\beta_{i,j}|$ for Rayleigh channels and arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left(\prod_{i=1}^{n} \frac{1}{\left(1 + \frac{E_s}{4N_o}\lambda_i\right)}\right)^m. \tag{4}$$

For Rician channels, the $h_{i,j}$'s have nonzero mean and $|\beta_{i,j}|$ has the Rician distribution. In this case, it can be shown that the pairwise block error probability is upper bounded as [12]

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \prod_{j=1}^{m} \left(\prod_{i=1}^{n} \frac{1}{1 + \frac{E_s}{4N_o}\lambda_i} \exp\left(-\frac{K_{i,j}\frac{E_s}{4N_o}\lambda_i}{1 + \frac{E_s}{4N_o}\lambda_i}\right)\right) \tag{5}$$

where $K_{i,j} = |\langle \beta_{i,j} \rangle|^2$ and $\langle \cdot \rangle$ denotes the expected value.

III. Problem Formulation

STCCs were originally designed to achieve diversity and coding gain in wireless fading channels utilizing multiple transmit antennas. The search space for these codes increases exponentially with the constellation size. In this correspondence, we will first show that one can apply STCCs in systems with only a single transmit antenna by using randomization techniques as long as we have multiple receive antennas. Then, we use this approach to design STCCs with high spectral efficiencies using smaller constellation size STCCs such as QPSK STCC. Therefore, we assume for now that the number of transmit antennas is equal to 1, i.e., $n = 1$. Using this assumption, (1) can be written as

$$r_t^j = h_{1,j} C_t \sqrt{E_s} + n_t^j, \quad 1 \le j \le m. \tag{6}$$

Here again, the physical channel path gains, the $h_{1,j}$'s, are independent complex normal random variables with mean $m_1/\sqrt{2}$ and variance 0.5

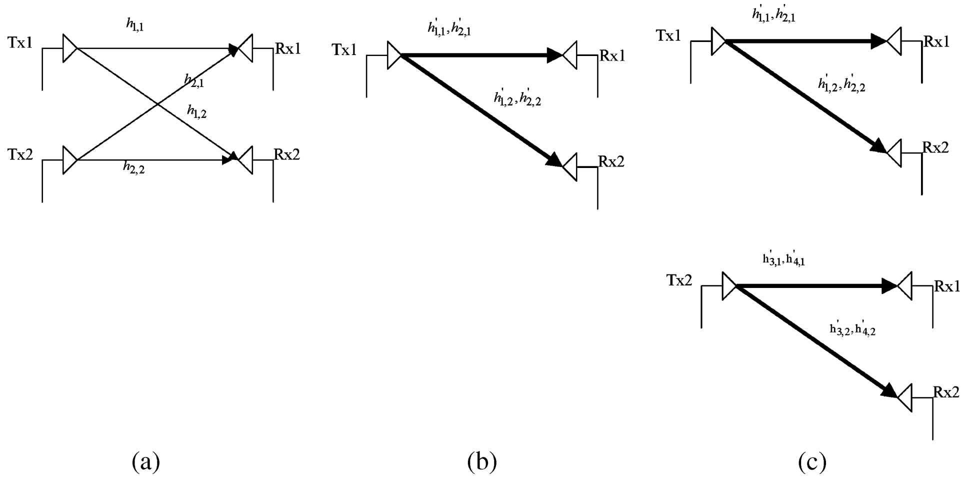


Fig. 1. Comparison of (a) a 2 × 2 MIMO system with (b) a 1 × 2 SIMO system using the proposed algorithm (PA) and (c) a 2 × 2 MIMO system using the PA.

per dimension. How can we modify the transmit signal (C_t) such that the system can use a STCC? We propose to use as the transmitted signal

$$C_t = A_1 c_t^1 + A_2 c_t^2 + \ldots + A_n c_t^n \tag{7}$$

where the induced random variables, the A_i's, are induced random variables and the c_t^i's are from the output of a STCC encoder. The A_i's are also independent of the physical channel path gains, the $h_{1,j}$'s. Combining (6) and (7) leads to

$$r_t^j = \sum_{i=1}^{n} h'_{i,j} c_t^i \sqrt{E_s} + n_t^j, \quad 1 \le j \le m \tag{8}$$

where $h'_{i,j} = h_{1,j} A_i$ is called a virtual path gain. We call this the virtual path gain because only m physical paths exist in this system and by introducing random data at the transmitter, we have created $n \times m$ virtual paths. Of course, some of these virtual paths are statistically dependent, but this approach will allow us to employ a STCC in a setting with a single transmit antenna and numerical results to be presented will demonstrate the gains that can be achieved. One way to interpret these gains is to recall that STCCs can provide gains in channels with correlated path gains, provided the correlation is not too close to unity.

One immediate application of the approach outlined above is to model an $n \times m$ MIMO system as an equivalent group of n distinct $1 \times m$ systems. Fig. 1 compares a 2×2 MIMO system, a 1×2 SIMO system, and a group of two 1×2 systems that illustrates our approach. Note that we do not claim that such an approach can change a SIMO channel into a MIMO channel with independent path gains, nor do we claim that the rank of the new channel matrix changes. The induced random variables (A_i's) can either change from symbol to symbol or they can be constant during one data frame. Our intention is to derive conditions under which one can obtain the minimum upper bound on the pairwise block error probability.

Applying (2) and (3) to the virtual paths ($h'_{i,j}$'s), we see that the conditional pairwise upper bound on the block error probability is

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} | A_i, h_{1,j}, 1 \le i \le n, 1 \le j \le m) \le \prod_{j=1}^{m} \exp\left(-|h_{1,j}|^2 A B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) A^* \frac{E_s}{4N_o}\right) \tag{9}$$

where $A = [A_1, A_2, \ldots, A_n]$ is the vector whose elements are the induced random variables.

Let $[T_1, \ldots, T_n] = AV^*$ and $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}}) = V^* D V$, then (9) can be written as

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} | A_i, h_{1,j}, 1 \le i \le n, 1 \le j \le m) \le \prod_{j=1}^{m} \exp\left(-|h_{1,j}|^2 \sum_{i=1}^{n} \lambda_i |T_i|^2 \frac{E_s}{4N_o}\right). \tag{10}$$

By taking the average over (10) with respect to channel coefficients and induced random variables assuming Rayleigh fading, we arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left\langle \left(\frac{1}{\left(1 + \sum_{i=1}^{n} \lambda_i |T_i|^2 \frac{E_s}{4N_o}\right)} \right)^m \right\rangle \tag{11}$$

where $\langle \cdot \rangle$ denotes the expected value. In (11), the expected value is with respect to $T_1, \ldots, T_n$, where each T_i depends on induced random variables.

For the slow Rician fading channel, following a similar approach, the pairwise upper bound on the block error probability is given by

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \le \left\langle \left(\frac{1}{\left(1 + \sum_{i=1}^{n} \lambda_i |T_i|^2 \frac{E_s}{4N_o}\right)} \right)^m \times \exp\left(-m \frac{\sum_{i=1}^{n} \lambda_i |T_i|^2 \frac{E_s}{4N_o} K_{i,1}}{1 + \sum_{i=1}^{n} \lambda_i |T_i|^2 \frac{E_s}{4N_o}}\right) \right\rangle. \tag{12}$$

IV. Optimum Solution for Slow Rayleigh and Rician Fading Channels

In this section, we first find the optimum solution for Rayleigh fading channels. It is straightforward to show that it is also the optimum solution for Rician channels. For Rayleigh channels, we want to find the joint probability density function (pdf) of $(|T_1|, |T_2|, \ldots, |T_n|)$ such that the pairwise upper bound on the block error probability given in (11) is minimized. Mathematically, the optimization problem is defined as

$$\min_{\langle |T_i|^2 \rangle = 1,\ i=1,2,\ldots,n} \left\langle \left(1 + \sum_{i=1}^{n} R_i |T_i|^2 \right)^{-m} \right\rangle \tag{13}$$

where $R_i = \lambda_i (E_s / 4N_o)$. The condition $\langle |T_i|^2 \rangle = 1$ is set to normalize the power of induced random variables to one (increasing power would clearly allow better performance so this is important).

Theorem IV.1: The minimum of (13) is attained when the pdf of $(|T_1|, |T_1|, \ldots, |T_n|)$ satisfies $f_{|T_1|,|T_1|,\ldots,|T_n|}(x_1, x_2, \ldots, x_n) = \delta(x_1 - 1)\delta(x_2 - 1)\cdots\delta(x_n - 1)$.

To prove this theorem, we first need these two lemmas.

Lemma IV.1 (slight generalization of Jensen's inequality): If the function $g(x_1, x_2, \ldots, x_n)$ is convex, that is, its Hessian matrix is positive semidefinite, then we have

$$\left\langle g(\vec{X}) \right\rangle \geq g\left(\langle \vec{X} \rangle\right).$$

Proof: The proof of this lemma is simple and for brevity of the correspondence, we omit the proof.

Lemma IV.2: The function $g(\vec{x}) = g(x_1, x_2, \ldots, x_n) = (1 + \sum_{i=1}^{n} R_i x_i)^{-m}$ is convex for $x_i \geq 0$.

Proof: It is straightforward to show that the Hessian matrix of $g(\vec{x})$ is a positive semi definite.

Proof of Theorem IV.1: First, we know that when $f_{|T_1|,|T_1|,\ldots,|T_n|}(x_1, x_2, \ldots, x_n) = \delta(x_1 - 1)\delta(x_2 - 1)\cdots\delta(x_n - 1)$, we have

$$\left\langle \left(1 + \sum_{i=1}^{n} R_i |T_i|^2\right)^{-m} \right\rangle = \left(1 + \sum_{i=1}^{n} R_i\right)^{-m}.$$

Then, consider function $g(\vec{x}) = (1 + \sum_{i=1}^{n} R_i x_i)^{-m}$ and random variable

$$\vec{X} = \left(|T_1|^2, |T_2|^2, \ldots, |T_n|^2\right).$$

Using Lemmas IV.1 and IV.2 above, we obtain

$$\begin{aligned}\left\langle \left(1 + \sum_{i=1}^{n} R_i |T_i|^2\right)^{-m} \right\rangle &= \left\langle g(\vec{X}) \right\rangle \geq g\left(\langle \vec{X} \rangle\right) \\ &= \left(1 + \sum_{i=1}^{n} R_i \left\langle |T_i|^2 \right\rangle\right)^{-m} \\ &= \left(1 + \sum_{i=1}^{n} R_i\right)^{-m}.\end{aligned}$$

For the Rician fading channels, we can prove Theorem IV.1 using a similar approach. Theorem IV.1 indicates that the amplitude of T_i should be deterministic and equal to 1. However, the phase of T_i is a random variable.

V. Selection of Induced Random Variables

In the previous section, we demonstrated that if the magnitude of each random variable, T_i for $1 \leq i \leq n$, is equal to 1, then the pairwise upper bound on the block error probability will be minimized. However, this derivation does not describe how the induced random variables (A_i's) should behave. In general, it is not possible to select the random variables $A_1, \ldots, A_n$ to achieve $|T_i| = 1$ for all possible choices of v_i^*. Since v_i is an eigenvector of $B_s(\underline{\mathbf{C}}, \underline{\mathbf{E}})$ which is constructed from the difference between the transmitted codeword and the decoded error data sequence, there are many values of v_i to consider. To address this difficulty, in the following, we provide a suboptimal solution. This solution is based on designing dependent induced random variables to improve the distant properties of the enlarged constellation STCC.

A. Design of the Induced Random Variables Based on Minimum Euclidean Distance of the New Code

Our objective in this section is to design a set of discrete random variables that statistically depend on the STCC encoder output. The objective is twofold, first, to generate a M-PSK signal constellation for the transmit signal C_t that was defined in (7), and second, to maximize the minimum Euclidean distance of the code $(d_{\min})$. Maximizing $d_{\min}$ will be a design criterion for these codes.

The conditional upper bound on the pairwise block error probability in this case can be derived as

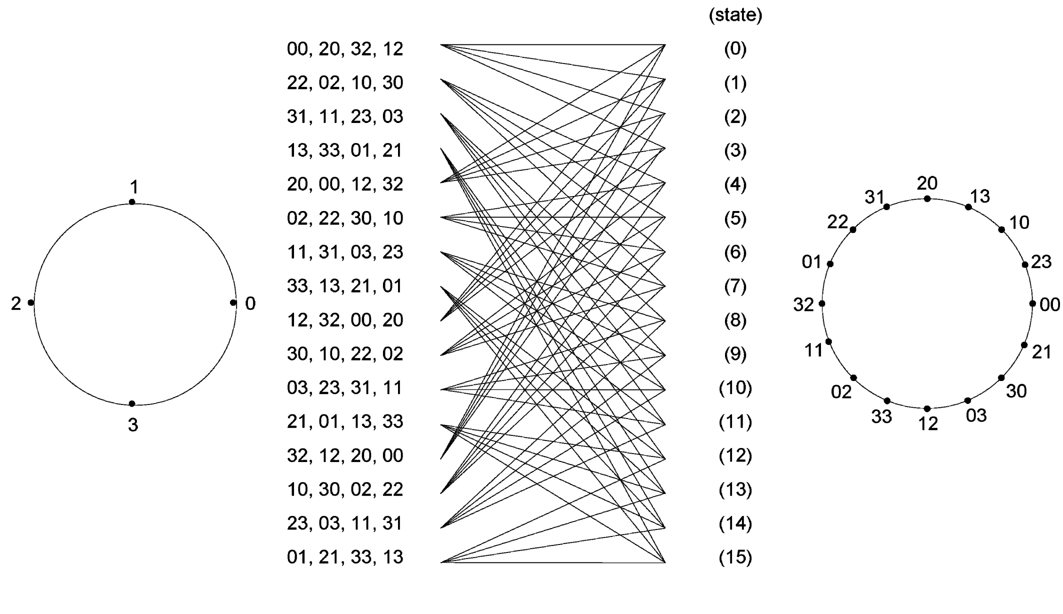
$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}} | h_{1,j}, 1 \leq j \leq m) \leq \prod_{j=1}^{m} \exp\left(-|h_{1,j}|^2 \sum_{t=1}^{N} |C_t - E_t|^2 \frac{E_s}{4N_o}\right) \quad (14)$$

where E_t is the error signal defined similar to (7). Note that, in this case, the induced random variables, the A_i's, are embedded in C_t and E_t and for that reason, it is not feasible to separate them in this equation. Averaging over (14) with respect to the channel coefficients, we arrive at

$$P(\underline{\mathbf{C}} \to \underline{\mathbf{E}}) \leq \left(\frac{1}{\left(1 + \sum_{t=1}^{N} |C_t - E_t|^2 \frac{E_s}{4N_o}\right)}\right)^m. \quad (15)$$

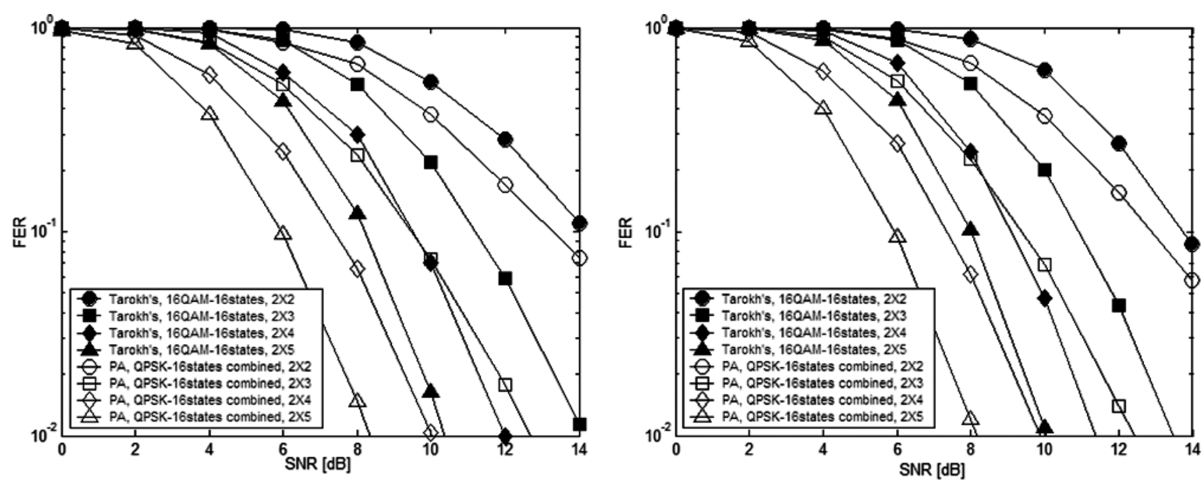
This equation suggests that in order to minimize the upper bound on the pairwise block error probability, we need to maximize the minimum Euclidean distance of the modified codeword $(C_t, t = 1, \ldots, N)$. Therefore, we design the induced random variables such that this minimum Euclidean distance is maximized.

There are several different designs of QPSK STCC that consider maximizing the minimum Euclidean distance between the pair of codewords [13]–[15] while providing the maximum coding and diversity gains. In this correspondence, we use the QPSK STCC design of [13] with 16 states for simulation and code design. Fig. 2(a) and (b) demonstrates the signal constellation and trellis diagram of QPSK 16 states STCC design of [13]. This code is designed to have the maximum diversity and coding gains as well as maximizing the minimum Euclidean distance between the codeword pairs. Our objective is to design induced random variables such that when we combine two QPSK signals, the new transmitted signal has a finite number of constellation points, i.e., equivalent to a 16-PSK constellation in this example, and at the same time maximizes the minimum Euclidean distance between codewords of the new transmitted signal (C_t). Fig. 2(c) illustrates this mapping for two QPSK signals. In this construction and mapping, we have chosen to use only induced random variables that have unit amplitude and who have statistically dependent random phases. These random phases depend on the signals output by the STCC encoder that will result in such a construction Fig. 2(c). The construction of transmitted signals (C_t) are carried in two steps. First, since all different combinations of two QPSK signals yield 16 possible choices, we use the trellis diagram of Fig. 2(b) to assign these 16 combinations such that the minimum Euclidean distance of the new code is maximized. This objective is achieved by assigning every two QPSK symbols to a point in the 16-PSK constellation for all 16 possible choices and then computing the distance properties of the resulting code. The signal assignment that maximizes the minimum Euclidean distance of the resulting code words will be selected. In the second step, we compute the A_i's such that the linear combination of two QPSK symbols using (7) will create the appropriate C_t as shown in Fig. 2(c). Utilizing this construction, for any two QPSK signals at the output of the STCC encoder of [13], the values of these induced random variables are known, however, since the signals from the encoder output are random and unknown at the receiver, the induced random variables are also unknown and random at the receiver. Note that for this particular construction of the signal, we no longer need to keep any table of induced random variables since there is a one-to-one mapping between any two QPSK signals from the output of STCC encoder and A_i's. The design of these dependent induced random variables are based on the desire to maximize the minimum Euclidean distance between any two codewords to improve (15).



(a) (b) (c)

Fig. 2. Description of (a) mapping of QPSK constellation, (b) vucetic 16-states QPSK STCC, and (c) mapping of two QPSK signals into 16-PSK constellation.



(a) (b)

Fig. 3. Frame error rate comparison between 16-QAM STCC of [12] and the PA for 2 × 2, 2 × 3, 2 × 4, and 2 × 5 systems in (a) Rayleigh and (b) Rician fading channels with 260 bits for each block.

We do not claim that such construction is necessarily the optimum solution; however, our simulation results will show that this construction outperforms 16-QAM STCC of [12].

It is noteworthy to mention that another solution would be to choose a uniform distribution for the random phase. Our simulation results show that such approach performs poorly. This result implies that our randomization technique is different from the phase sweeping approach [6] or its extensions [5], [7]–[10].

VI. Simulation Results

In the simulations presented in this section, coherent detection is assumed along with perfect knowledge of the channel coefficients at the receiver. We apply our approach for MIMO systems where each transmit antenna can employ a STCC. Therefore, for a $n \times m$ system, we can model it as an equivalent group of n distinct $1 \times m$ systems, each one transmitting a STCC with small constellation size. Fig. 1(c) demonstrates this concept for a 2×2 system. We have used this approach to design a STCC with spectral efficiency of 4 b/s/Hz, using a QPSK STCC for each antenna, and compare it with the 16-QAM STCC of [12]. Simulation results, given in Fig. 3(a) and (b), clearly show that our approach can perform better than that of [12] for $2 \times m$ MIMO systems with $2 \leq m \leq 5$ and block lengths of 260 bits for Rayleigh and Rician slow fading channels. For the Rician channel, the parameter $k = 0$ dB is used. The parameter k is defined as the ratio between the deterministic signal power and the variance of the multipath, i.e., $k(\mathrm{dB}) = 10\log(m_1^2/1)$ for this example.

The implementation of the Viterbi algorithm (VA) for the new scheme is slightly different from the original VA. In the proposed algorithm (PA), at each time interval the equivalent of n data symbols are transmitted simultaneously from n different transmit antennas. In this case, each path metric for the VA using the PA will compute the equivalent of n path metrics from the original trellis diagram [see

Fig. 2(b)]. Accordingly, we need to incorporate this feature when we compute the survivor paths for each STCC block.

VII. Conclusion

In this correspondence, we present a new approach to design STCC for MIMO systems with arbitrary constellation size and any number of transmit antennas. Our approach is based on modeling an $n \times m$ MIMO system as an equivalent of n distinct $1 \times m$ SIMO systems. Then, we demonstrate that a STCC can be implemented for a SIMO system by inducing randomness into the channel. Therefore, this design can be applied to any arbitrary number of transmit antennas. The search space for this code does not increase with the constellation size but rather remains the same regardless of the constellation size and only depends on the constellation size of the component codes, i.e., QPSK. One advantage of this approach is the fact that the minimum number of states of convolutional code for this approach depends on the constellation size of the component codes rather than the entire code, allowing us to have a better tradeoff between complexity and performance. The characteristics of these induced random variables are defined based on minimizing an upper bound on the pairwise block error probability. Simulation results for Rayleigh and Rician slow fading channels show that our approach performs better than one of the best existing 16-QAM STCC designs when we use two QPSK STCCs as our component codes. This is a powerful result since it allows us to design STCCs of any constellation size by utilizing QPSK STCC as component codes. This simplifies the design of STCCs for cases with high constellation sizes and will no longer require exhaustive search for such codes. Our preliminary results also show that using this approach allows us to reduce the computational complexity of the receiver significantly. Due to space limitations, we will present these results in another manuscript. Another potential application for this approach is in the design of space–time block codes (STBC).

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