Use of Linear Programming for Dynamic Subcarrier and Bit Allocation in Multiuser OFDM

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***Abstract*—An adaptive subcarrier allocation and an adaptive modulation for multiuser orthogonal frequency-division multiplexing (OFDM) are considered. The optimal subcarrier and bit allocation problems, which are previously formulated as nonlinear optimizations, are reformulated into and solved by integer programming (IP). A suboptimal approach that performs subcarrier allocation and bit loading separately is proposed. It is shown that the subcarrier allocation in this approach can be optimized by the linear-programming (LP) relaxation of IP, while the bit loading can be performed in a manner similar to a single-user OFDM. In addition, a heuristic method for solving the LP problem is presented. The LP-based suboptimal and heuristic algorithms are considerably simpler to implement than the optimal IP, plus their performances are close to those of the optimal approach.**

***Index Terms*—Integer programming (IP), linear programming (LP), multiuser orthogonal frequency-division multiplexing (OFDM), subcarrier and bit allocation.**

I. Introduction

It HAS been suggested that multiuser orthogonal-frequency-division-multiplexing (OFDM) systems employ adaptive subcarrier allocation as well as adaptive bit loading. By adaptively assigning subcarriers depending on channel gains, multiuser OFDM can take advantage of the channel diversity among users in different locations, thereby enabling an efficient use of all subcarriers.

The optimal bit loading and subcarrier-allocation problems related to multiuser OFDM have already been formulated in [1] and [2]: specifically, minimization of the overall transmission power under a data rate constraint [1] and maximization of the data rate under a power constraint [2]. These are both nonlinear optimization problems with integer variables and referred to as margin adaptive (MA) and rate adaptive (RA) optimizations, respectively [3]. However, solving these problems is extremely difficult; they are only solved after relaxing the requirement regarding integer variables to allow real numbers. Consequently, even though this approach requires an intensive computation, it cannot yield an optimal solution. To reduce the computational load, some suboptimal algorithms have been proposed for MA and RA optimizations in [4]–[6]. However, the use of these algorithms is rather limited for the following reasons: In [4], it is assumed that the average signal-to-noise ratios (SNRs) of all users are identical; [5] is based on a modification of the RA cost; and [6] considers the case where the total transmission power is equally distributed to all subcarriers. Adaptive loading techniques for OFDM systems with multiple antennas have been developed in [7] and [8].

This paper shows that the above nonlinear optimization problems [1] and [2] can be reformulated into integer programming (IP), thereby allowing the optimal subcarrier and bit allocation to be achieved. Based on the observation that the optimal approach tends to assign constant bits to the subcarriers allocated to a user, a suboptimal approach that performs channel allocation and bit loading separately is proposed. This separate subcarrier allocation is shown to be a specific type of IP that is called a transportation problem, which significantly simplifies the subcarrier allocation problem, as a transportation problem can be solved through the linear-programming (LP) relaxation of IP [9]. Furthermore, there are already various efficient heuristic algorithms for solving transportation problems, and one such algorithm, which is called Vogel's method [10], is applied to the subcarrier allocation. The bit loading, which follows the subcarrier allocation, can be performed in a manner similar to a single-user OFDM. The application of the subcarrier and bit allocation algorithms to multiuser OFDM enables the performance of the proposed suboptimal approach to be close to that of the optimal approach.

The organization of this paper is as follows. Section II formulates the MA and RA optimizations and discloses the relation between these problems. Section III shows that the MA and RA problems can be converted into IP. A suboptimal approach based on the LP relaxation of IP is presented in Section IV, and an efficient heuristic algorithm for solving the LP problem is described in Section V. Finally, computer simulation results comparing the optimal and suboptimal algorithms and demonstrating the advantage of the proposed algorithms are presented in Section VI.

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II. MA and RA Optimizations: Formulation and Their Relation

The structure of the adaptive multiuser OFDM system under consideration is shown in Fig. 1. The system has K users and N subcarriers. The base station receives downlink channel

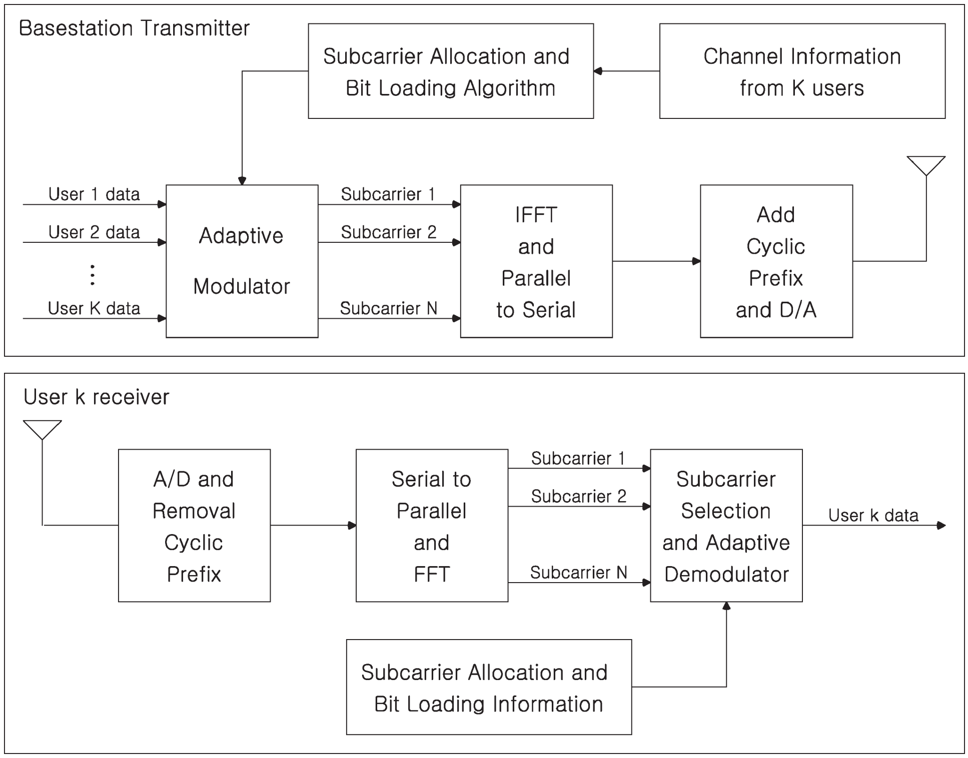

Fig. 1. Adaptive multiuser OFDM system.

information from all users, then using this information, it assigns a set of subcarriers to each user and determines the number of bits per OFDM symbol to be transmitted through each subcarrier. It is assumed that sharing a subcarrier by two or more users is not allowed. Depending on the number of bits assigned to the subcarriers, each user's data are distributed to the subcarriers allocated to the user, and adaptive modulation is performed at each subcarrier. The subcarrier and the bit allocation information is sent to the receivers via a separate control channel. At each receiver, the subcarriers assigned to the user are selected, and the signals associated with the subcarriers are demodulated.

To describe the optimization problems, various notations are introduced.[1] Denote the data rate (number of bits transmitted in an OFDM symbol) of the kth user by R_k and the number of bits of the kth user assigned to the nth subcarrier by $c_{k,n}$. It is assumed that $c_{k,n} \in \mathbf{D}$, where $\mathbf{D}$ is a set of nonnegative integers that are less than or equal to M, and M is the maximum number of bits/symbol that can be transmitted by each subcarrier. The data rate R_k can be expressed as $R_k = \sum_{n=1}^{N} c_{k,n}$, or equivalently

$$R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \tag{1}$$

where $\rho_{k,n}$ is an indicator variable defined as

$$\rho_{k,n} = \begin{cases} 0, & \text{if } c_{k,n} = 0 \\ 1, & \text{otherwise} \end{cases} \tag{2}$$

[1]The notations in [1] will be adopted throughout this paper.

and $\sum_{k=1}^{K} \rho_{k,n} = 1$ because a subcarrier can be occupied by at most one user. The transmission power allocated to user k's subcarrier n is expressed as

$$P_{k,n} = \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \tag{3}$$

where $f_k(c_{k,n})$ is the required received power in the nth subcarrier for a reliable reception of $c_{k,n}$ bits/symbol when the channel gain is equal to unity, and $\alpha_{k,n}^2$ is the channel gain of user k's subcarrier n. For the kth user, the average of all subcarrier gains α_k^2 and the average of assigned subcarrier gains $\bar{\alpha}_k^2$ are defined as follows:

$$\alpha_k^2 = \frac{1}{N}\sum_{n=1}^{N} \alpha_{k,n}^2 \tag{4}$$

and

$$\bar{\alpha}_k^2 = \frac{1}{n_k}\sum_{n\in S_k} \alpha_{k,n}^2 \tag{5}$$

where S_k is the set of indexes of subcarriers assigned to the kth user, and n_k represents the number of elements in S_k. Using these notations, the MA and RA optimizations are stated as follows.

1) MA Optimization: Suppose that the user data rates $\{R_1, \ldots, R_K\}$ are fixed and given. The MA procedure then minimizes the total transmission power required for transmitting the data with rate $\{R_1, \ldots, R_K\}$:

$$\min_{c_{k,n},\rho_{k,n}} P_{\mathrm{T}} = \min_{c_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \cdot \rho_{k,n} \quad \text{(6a)}$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n}, \quad \text{for all } k \quad \text{(6b)}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \text{for all } n \quad \text{(6c)}$$

where P_{T} denotes the total transmission power.

2) RA Optimization: Suppose that the available total transmission power is limited. The RA procedure then maximizes the minimum of the user's throughput subject to the power constraint[2]

$$\max_{c_{k,n},\rho_{k,n}} \min_k R_k = \max_{c_{k,n},\rho_{k,n}} \min_k \sum_{n=1}^{N} c_{k,n}\rho_{k,n} \quad \text{(7a)}$$

$$\text{subject to} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}\rho_{k,n} \le P_{\mathrm{T}}. \quad \text{(7b)}$$

and the constraint in (6c).

The MA and RA optimizations are both nonlinear because $f_k(c)$ in (3) is nonlinear. For example, in the case of quadrature amplitude modulation (QAM), $f_k(c)$ can be represented as

$$f_k(c) = \frac{N_o}{3}\left[Q^{-1}\left(\frac{p_e}{4}\right)\right]^2 (2^c - 1) \quad \text{(8)}$$

where p_e denotes the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and $Q(x)$ is the Q-function defined in [12].

The MA and RA optimizations are related with each other. In fact, as shown below, it is possible to view the RA problem as an iterative MA problem. To disclose the relation between the MA and RA problems, the RA optimization in (7a) is rewritten as

$$\max z \quad \text{(9a)}$$

$$\text{subject to } R_1 = R_2 = \cdots = R_K = z. \quad \text{(9b)}$$

The constrained optimization in (9) is identical to the optimization in (7a) because of the following reason: If we denote the solution of (7a) by z^* and the corresponding data rates by $\{R_1^*, R_2^*, \ldots, R_K^*\}$, then $R_k^* \ge z^*$ for all k and a possible $\{R_1^*, R_2^*, \ldots, R_K^*\}$ is given by $R_1^* = R_2^* = \cdots = R_K^* = z^*$. Suppose, for the time being, that the maximum value of z, say z^*, in (9a) is given. Then, $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$ exist, which yield z^* while satisfying the constraints in the RA problem. It is interesting to note that such optimum $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$ can also be obtained by solving the following MA problem:

$$\min_{c_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2}\rho_{k,n} = P_{\mathrm{T}}^* \quad \text{(10a)}$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} c_{k,n}\rho_{k,n} = z^*, \quad \text{for all } k \quad \text{(10b)}$$

and the constraint in (6c).

Obviously, $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$, which are attained from (10), and (6c) yield z^*. In addition, the minimum value of the power in (10a) P_{T}^* should be less than or equal to P_{T} due to the existence of $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$ corresponding to z^*. Therefore, $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$, which are from the MA optimization of (10), and (6c) are indeed a solution of the RA optimization. The relation between the RA and MA problems can be summarized as follows.

Proposition 1: When the maximum value of z, say z^*, in (9a) is given, the RA problem in (7) and (6c) can be solved via the MA optimization in (10) and (6c).

This proposition yields the following recursive algorithm for the RA optimization.

Solving RA problem via recursive MA optimization:
Initialization
$z^* \leftarrow 1$
Iteration
Loop start:
Find optimal $\{c_{k,n}\}$, $\{\rho_{k,n}\}$, and P_{T}^* by solving (10a) and (10b).
If $P_{\mathrm{T}}^* < P_{\mathrm{T}}$, increase z^* by 1 and go to Loop start.
Else, stop.

This algorithm increases z^* unless the power constraint in (7b) is violated. The maximum of z^* and corresponding $\{c_{k,n}\}$ and $\{\rho_{k,n}\}$ are the solution of the RA problem. In Section IV, Proposition 1 plays a key role in deriving a suboptimal algorithm for the RA optimization.

III. IP: Formulation and Its Performance

A. Conversion to IP

Suppose that $c_{k,n} \in \{0, 1, \ldots, M\} (= \mathbf{D})$. Then

$$f_k(c_{k,n}) \in \{0, f_k(1), \ldots, f_k(M)\} \quad \text{(11)}$$

where $f_k(0) = 0$ and $\{f_k(c)\}$ are constants that can be precalculated: For example, (8) can be used for QAM. A new indicator variable $\gamma_{k,n,c}$ is defined as follows:

$$\gamma_{k,n,c} = \begin{cases} 1, & \text{if } \rho_{k,n} = 1 \text{ and } c_{k,n} = c \\ 0, & \text{otherwise.} \end{cases} \quad \text{(12)}$$

for all $c \in \{0, 1, \ldots, M\}$. Using $\gamma_{k,n,c}$, (11) is rewritten as

$$f_k(c_{k,n}) = \sum_{c=0}^{M} \gamma_{k,n,c} f_k(c). \quad \text{(13)}$$

[2]In the single-user case, the maximization of the total data rate has been considered in [11]. However, this criterion will not be adopted in the multiuser case because it tends to exclude certain users with poor channel responses.

Since $f_k(c)$ are constants, (13) indicates that $f_k(c_{k,n})$ is a linear combination of the indicator variables $\{\gamma_{k,n,c}\}$. Using (13) in (6a), the MA cost function is rewritten as

$$P_{\mathrm{T}} = \sum_{k=1}^{K}\sum_{n=1}^{N}\left\{\sum_{c=0}^{M}\gamma_{k,n,c}\frac{f_k(c)}{\alpha_{k,n}^2}\right\}\rho_{k,n}. \tag{14}$$

The indicators $\rho_{k,n}$ and $\gamma_{k,n,c}$ are related as follows:

$$\rho_{k,n} = \sum_{c=0}^{M}\gamma_{k,n,c}. \tag{15}$$

From (15), it can be seen that

$$\gamma_{k,n,c}\cdot\rho_{k,n} = \gamma_{k,n,c}. \tag{16}$$

Therefore, (14) is rewritten by

$$P_{\mathrm{T}} = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=0}^{M}\gamma_{k,n,c}\frac{f_k(c)}{\alpha_{k,n}^2}. \tag{17}$$

This is a linear cost function. In a similar manner, using $\gamma_{k,n,c}$, the constraints in (6b) and (6c) can be rewritten. Summarizing these results, the MA and RA problems are redescribed as follows.

1) MA Optimization:

$$\min_{\gamma_{k,n,c}}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=0}^{M}\frac{f_k(c)}{\alpha_{k,n}^2}\gamma_{k,n,c}, \quad \text{for } \gamma_{k,n,c}\in\{0,1\} \tag{18a}$$

$$\text{subject to } R_k = \sum_{n=1}^{N}\sum_{c=0}^{M}c\cdot\gamma_{k,n,c}, \quad \text{for all } k \tag{18b}$$

$$\sum_{k=1}^{K}\sum_{c=0}^{M}\gamma_{k,n,c} = 1, \quad \text{for all } n. \tag{18c}$$

2) RA Optimization:

$$\max_{\gamma_{k,n,c}}\min_{k}\sum_{n=1}^{N}\sum_{c=0}^{M}c\cdot\gamma_{k,n,c}, \quad \text{for } \gamma_{k,n,c}\in\{0,1\} \tag{19a}$$

$$\text{subject to } \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{c=0}^{M}\frac{f_k(c)}{\alpha_{k,n}^2}\gamma_{k,n,c}\le P_{\mathrm{T}}. \tag{19b}$$

and the constraint in (18c).

These optimization problems can be solved by IP when treating $\gamma_{k,n,c}$ as variables. However, in general, IP needs an exponential time algorithm whose complexity increases exponentially with the number of constraints and variables. In the following subsection, the behavior of the IP-based optimal algorithms is examined through computer simulation, and an observation is made regarding bit loading. The proposed suboptimal polynomial-time algorithms are then developed based on this observation.

B. Behavior of IP: Simulation Results

The IP-based algorithms were applied to an adaptive multiuser OFDM system with the following parameters: number of subcarriers $N = 64$; number of users $K = 4$; maximum number of loaded bits $M = 12$; and required BER $= 10^{-4}$. The channels were assumed to be fixed during one frame period and frequency selective Rayleigh fading channels with eight taps and exponentially decaying power profiles. The average channel magnitudes of the different users $\{\alpha_k^2\}$ [see (4)] were generally unequal, and the difference, which is denoted by γ, between the ensemble averages of the strongest and weakest channel magnitudes of the four users was either 0 or 30 dB. In what follows, the total transmission power P_{T} is represented in decibel scale, assuming that the noise power is 0 dB.

Figs. 2 and 3 show representative shapes of the channel magnitudes for the four users when γ was 0 and 30 dB, respectively. In Fig. 3, User 1 was 30 dB lower than User 4 on average, while the other users were between Users 1 and 4. MA and RA optimizations were performed for these channels.

Table I summarizes the results of the MA optimization when $R_1 = R_2 = R_3 = R_4 = 64$. For the channels in Fig. 2 ($\gamma = 0$ dB) with an identical average gain, the subcarriers were almost equally distributed to the users (15 to 17 subcarriers were assigned per user) and $c_{k,n} = 4$ for most subcarriers (54 out of 64 subcarriers were assigned 4 bits). When $\gamma = 30$ dB (channels in Fig. 3), almost half of the total subcarriers were assigned to User 1 who had the lowest average channel magnitude. This occurred because of the constraint $R_1 = R_2 = R_3 = R_4$. Note that 30 out of the 31 subcarriers of User 1 were assigned 2 bits. For User 4 with the highest average channel magnitude, only eight subcarriers were assigned, yet all were loaded with 8 bits. The other users were allocated 12 to 13 subcarriers. It is important to observe that the MA optimization tended to load the same number of bits (constellation size) to the subcarriers allocated to a user. If the dominant constellation size of the kth user is denoted by c_k, then $c_1 = c_2 = c_3 = c_4 = 4$ for $\gamma = 0$ dB; $c_1 = 2$, $c_2 = 6$, $c_3 = 4$, and $c_4 = 8$ for $\gamma = 30$ dB. The required total transmission powers (P_{T}) after the MA optimizations were 33.41 and 52.51 dB when $\gamma = 0$ and 30 dB, respectively. Table I also shows the ratio between the average subcarrier gains α_k^2 and $\bar{\alpha}_k^2$ defined in (4) and (5). The ratios were between 0.60 and 0.73 with only a small variation, which will be useful for deriving the proposed suboptimal algorithm.

The results of the RA optimization, which are shown in Table II, show that the arguments regarding MA optimization also held for the RA case. The dominant constellations were $c_1 = c_2 = c_3 = c_4 = 8$ for $\gamma = 0$ dB; $c_1 = 2$, $c_2 = 4$, $c_3 = 2$, and $c_4 = 6$ for $\gamma = 30$ dB. The minimum user throughput after the RA optimizations was 134 and 48 bits/symbol when $\gamma = 0$ and 30 dB, respectively. The ratios between α_k^2 and $\bar{\alpha}_k^2$ were in between 0.61 and 0.73.

To further examine the dominant constellations of the users, additional experiments were performed. For $\gamma = 0$ and 30 dB, the channels of the four users were generated 1000 times. The MA and RA optimizations were performed, and the probability that $c_{k,n} = c_k$, assuming $\rho_{k,n} = 1$, $\Pr(c_{k,n} = c_k|\rho_{k,n} = 1)$, was empirically evaluated, where c_k was the dominant

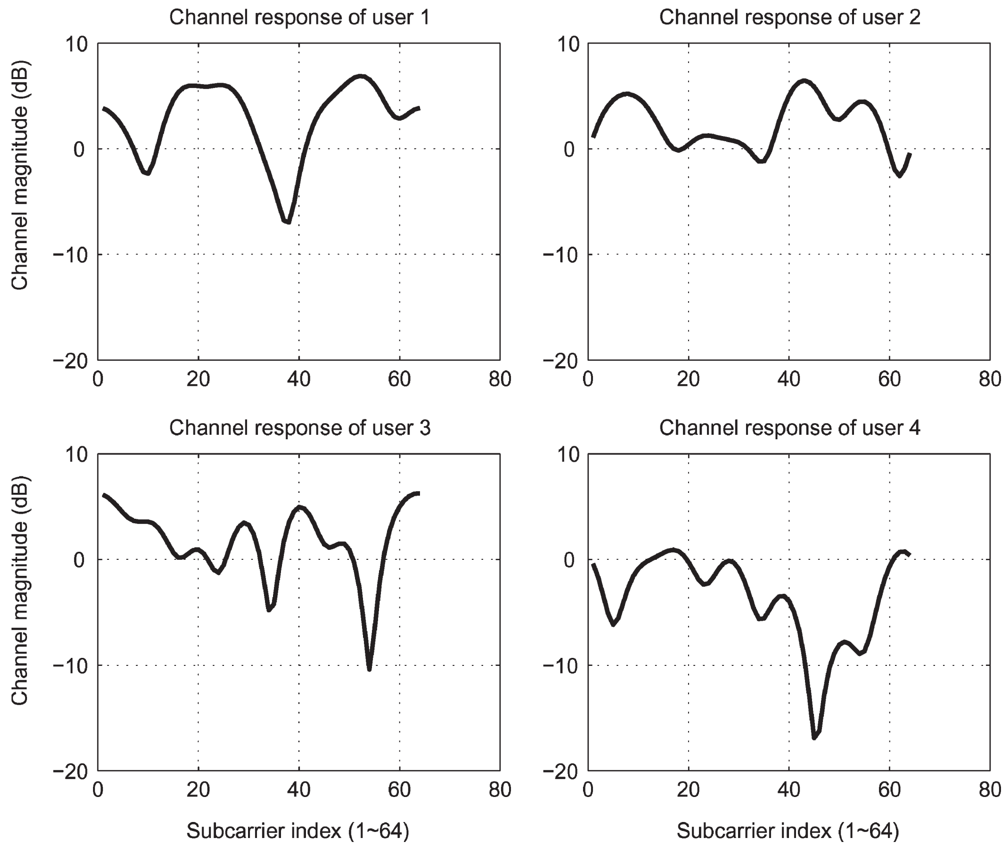

Fig. 2. Representative channel gains of four users when γ is 0 dB.

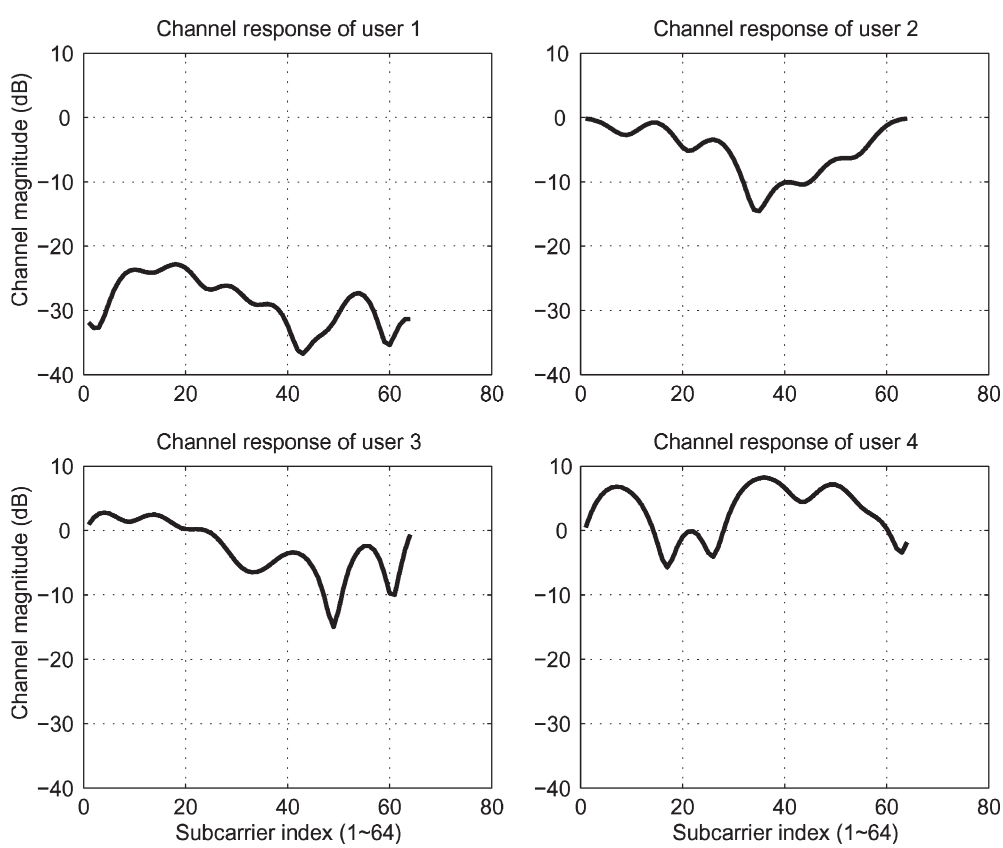

Fig. 3. Representative channel gains of four users when γ is 30 dB.

TABLE I
NUMBER OF ASSIGNED SUBCARRIERS FOR EACH USER AND NUMBER OF LOADED BITS FOR EACH SUBCARRIER IN MA PROBLEM WHEN $R_1 = R_2 = R_3 = R_4 = 64$

Channel type	User index	Number of assigned subcarriers	# of subcarriers with $c_{k,n} = i$				Ratio $\alpha_k^2/\bar{\alpha}_k^2$
			$i=2$	$i=4$	$i=6$	$i=8$	
$\gamma = 0$ dB (channels in Fig. 2)	1	15	0	13	2	0	0.65
	2	17	3	13	1	0	0.65
	3	15	0	13	2	0	0.67
	4	17	2	15	0	0	0.64
	Total	64	5	54	5	0	
$\gamma = 30$ dB (channels in Fig. 3)	1	31	30	1	0	0	0.60
	2	12	0	4	8	0	0.65
	3	13	0	8	4	1	0.70
	4	8	0	0	0	8	0.73
	Total	64	30	13	12	9	

TABLE II
NUMBER OF ASSIGNED SUBCARRIERS FOR EACH USER AND NUMBER OF LOADED BITS FOR EACH SUBCARRIER IN RA PROBLEM WHEN $P_{\mathrm{T}} = 50$ dB

Channel type	User index	Number of assigned subcarriers	# of subcarriers with $c_{k,n} = i$					Ratio $\alpha_k^2/\bar{\alpha}_k^2$
			$i=2$	$i=4$	$i=6$	$i=8$	$i=10$	
$\gamma = 0$ dB (channels in Fig. 2)	1	15	0	0	0	8	7	0.65
	2	16	0	0	1	11	4	0.65
	3	16	0	0	0	13	3	0.67
	4	17	0	0	1	16	0	0.64
	Total	64	0	0	2	48	14	
$\gamma = 30$ dB (channels in Fig. 3)	1	24	24	0	0	0	0	0.61
	2	15	6	9	0	0	0	0.64
	3	16	9	6	1	0	0	0.71
	4	9	1	1	7	0	0	0.73
	Total	64	40	16	8	0	0	

TABLE III
PROBABILITY OF DOMINANT CONSTELLATION ($c_{k,n} = c_k$) AND PROBABILITY THAT $c_{k,n}$ IS IN THE NEIGHBORHOOD OF c_k

	MA		RA	
	$\gamma = 0$dB	$\gamma = 30$dB	$\gamma = 0$dB	$\gamma = 30$dB
$\Pr(c_{k,n} = c_k \mid \rho_{k,n} = 1)$	0.84	0.79	0.85	0.80
$\Pr(c_{k,n} = c_k \mid \rho_{k,n} = 1) + \Pr(c_{k,n} = c_k \pm 2 \mid \rho_{k,n} = 1)$	1.00	0.99	1.00	0.99

constellation size. The results are summarized in Table III (in the table, the probability that $c_{k,n}$ is in the neighborhood of c_k is also shown). When $\rho_{k,n} = 1$, about 80% of $\{c_{k,n}\}$ was equal to the dominant constellation size c_k, and almost all $\{c_{k,n}\}$ were in the neighborhood of c_k. Next, suboptimal algorithms are derived based on these observations regarding dominant constellations.

IV. LP-Based Suboptimal Algorithms

The fact that the bits loaded to the subcarriers allocated to a given user tend to be identical yields the following two-step approach. In the first step, subcarriers are assigned under the following assumption of a constant bit loading.

Assumption 1:

$$c_{k,n} = \begin{cases} c_k, & \text{if } \rho_{k,n} = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

In the second step, bits are loaded to the subcarriers assigned in the first step. Of course, Assumption 1 is not assumed in this step. The subcarrier allocation in the first step can be performed via the LP relaxation of IP, while the bit assignment in the second step can adopt the bit loading algorithm derived for the single-user case. By performing subcarrier allocation and bit loading separately, this yields a suboptimal algorithm that is considerably simpler to implement than the IP-based approach.

A. Subcarrier Allocation for MA Problem

Using (20) in (6) yields

$$\min_{c_k, \rho_{k,n}} P_{\mathrm{T}} = \min_{c_k, \rho_{k,n}} \sum_{k=1}^{K} f_k(c_k) \sum_{n=1}^{N} \frac{\rho_{k,n}}{\alpha_{k,n}^2} \tag{21a}$$

$$\text{subject to } R_k = c_k \sum_{n=1}^{N} \rho_{k,n}, \quad \text{for all } k \tag{21b}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \text{for all } n. \tag{21c}$$

Suppose, for the time being, that $\{c_k\}$ are known. Then, $f_k(c_k)/\alpha_{k,n}^2$ becomes known, and (21) is rewritten as

$$\min_{\rho_{k,n}} P_{\mathrm{T}} = \min_{\rho_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} r_{k,n} \rho_{k,n} \tag{22a}$$

$$\text{subject to } \sum_{n=1}^{N} \rho_{k,n} = \frac{Rk}{c_k}, \quad \text{for all } k \tag{22b}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \text{for all } n \tag{22c}$$

where $r_{k,n} = f_k(c_k)/\alpha_{k,n}^2$. Because $\rho_{k,n}$ takes either zero or one, the optimization problem described by (22) is an IP

TABLE IV
$\{\rho'_{k,n}\}$ IN (29) WHEN THERE ARE THREE USERS ($K = 3$). HERE $n_l = R_l/c'_l$ FOR $1 \le l \le 3$

n	1	2	$\cdots$	n_1	n_1+1	n_1+2	$\cdots$	n_1+n_2	n_1+n_2+1	$\cdots$	N
$\rho'_{1,n}$	1	1	$\cdots$	1	0	0	$\cdots$	0	0	$\cdots$	0
$\rho'_{2,n}$	0	0	$\cdots$	0	1	1	$\cdots$	1	0	$\cdots$	0
$\rho'_{3,n}$	0	0	$\cdots$	0	0	0	$\cdots$	0	1	$\cdots$	1

problem. However, as observed below, this IP can be relaxed to LP because (22) takes the form of a specific IP that is called a transportation problem [10]. Indeed, in (22), N subcarriers are supplied to K users depending on the costs $\{r_{k,n}\}$. The parameter $\rho_{k,n}$ is the number of units shipped from the nth subcarrier to the kth user. The constraints in (22b) and (22c) are the demand and supply constraints, respectively, of the transportation problem. In the case of transportation, it is straightforward to show that IP is relaxed to LP [9, p. 39]. In summary, under Assumption 1, the MA optimization in (6) can be simplified to the transportation problem in (22), which can be solved via LP after dropping the integer constraints from $\rho_{k,n}$.

Next, a procedure for obtaining $\{c_k\}$ is presented. From (22c)

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\rho_{k,n} = N. \tag{23}$$

Due to (22b), this equation is rewritten as

$$\sum_{k=1}^{K}\frac{R_k}{c_k} = N. \tag{24}$$

In the special case where $c_k = c$ for all k (this can happen when the distances between the base station and the K users are identical), c is directly obtained from (24):

$$c = \frac{1}{N}\sum_{k=1}^{K}R_k. \tag{25}$$

To obtain $\{c_k\}$ when they are not equal, an additional assumption on the channel response is made.

Assumption 2:

$$\alpha_{k,n}^2, = \begin{cases} \alpha_k^2, & \text{if } \rho_{k,n} = 1 \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

where α_k^2 is the average subcarrier gain defined in (4).[3] This assumption is closely connected to Assumption 1: (20) holds when (26) is true. Therefore, Assumption 2 is reasonably valid in practical situations. Under Assumption 2, the cost in (21a) can be simplified as

$$\min_{c_k} P_{\mathrm{T}} = \min_{c_k}\sum_{k=1}^{K}\frac{f_k(c_k)}{\alpha_k^2}\frac{R_k}{c_k} \tag{27}$$

where $\sum_{n=1}^{N}\rho_{k,n}$ is replaced with R_k/c_k due to (21b). The next objective is to find $\{c_k\}$ minimizing (27) under the constraint in (21c). This optimal $\{c_k\}$ can be obtained after replacing (21c) with (24). This is shown in the following proposition.

Proposition 2: Suppose that for all k, $f_k(c_k)$ is a convex function of c_k. Then, the optimal $\{c_k\}$, say $\{c_k^*\}$, of (27) and (21c) can be obtained through the following constrained minimization:

$$\min_{c_k}\sum_{k=1}^{K}\frac{f_k(c_k)}{\alpha_k^2}\cdot\frac{R_k}{c_k} \tag{28a}$$

$$\text{subject to}\sum_{k=1}^{K}\frac{R_k}{c_k} = N. \tag{28b}$$

Proof: Let $\{c'_k\}$ be the optimal solution of (28). Then, it is sufficient to show that $c'_k = c_k^*$ for all k. Define $\{\rho'_{k,n}\}$ that correspond to c'_k as follows:

$$\rho'_{k,n} = \begin{cases} 1, & \text{if } \sum_{l=1}^{k-1}R_l/c'_l + 1 \le n \le \sum_{l=1}^{k}R_l/c'_l \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

where $\sum_{l=1}^{k-1}R_l/c'_l = 0$ when $k = 1$, $R_l/c'_l \ge 1$, and $\sum_{l=1}^{K}R_l/c'_l = N$ [see (22b) and (24)]. $\rho'_{k,n}$ are illustrated in Table IV. Note that $\{c'_k, \rho'_{k,n}\}$ satisfies (21b) and (21c) and becomes a feasible solution of the MA optimization in (27) and (21c). Since $\{c_k^*\}$ is the optimal solution of (27) and (21c), then

$$\sum_{k=1}^{K}\frac{f_k(c_k^*)}{\alpha_k^2}\cdot\frac{R_k}{c_k^*} \le \sum_{k=1}^{K}\frac{f_k(c'_k)}{\alpha_k^2}\cdot\frac{R_k}{c'_k}. \tag{30}$$

Considering (28), $\{c'_k\}$ is the optimal solution, and $\{c_k^*\}$ is a feasible solution. Therefore

$$\sum_{k=1}^{K}\frac{f_k(c'_k)}{\alpha_k^2}\cdot\frac{R_k}{c'_k} \le \sum_{k=1}^{K}\frac{f_k(c_k^*)}{\alpha_k^2}\cdot\frac{R_k}{c_k^*}. \tag{31}$$

Equations (30) and (31) indicate that

$$\sum_{k=1}^{K}\frac{f_k(c_k^*)}{\alpha_k^2}\cdot\frac{R_k}{c_k^*} = \sum_{k=1}^{K}\frac{f_k(c'_k)}{\alpha_k^2}\cdot\frac{R_k}{c'_k}. \tag{32}$$

Hence, both c_k^* and c'_k are the solution of (28). Now, the proof is completed by showing that the solution of (28) is unique.

[3] In (26), it is reasonable to use $\bar{\alpha}_k^2$ in (5) instead of α_k^2. However, $\bar{\alpha}_k^2$ is not available when obtaining $\{c_k\}$. Fortunately, $\{c_k\}$ can be accurately calculated using $\{\alpha_k^2\}$ if $\alpha_k^2 = b\bar{\alpha}_k^2$ for all k, where b is a constant, and the condition is roughly satisfied as shown in Tables I and II.

Define $b_k = 1/c_k$. Then, (28) can be expressed as

$$\min_{b_k} \sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \cdot \frac{R_k}{\alpha_k^2} \tag{33a}$$

$$\text{subject to} \sum_{k=1}^{K} b_k R_k = N. \tag{33b}$$

Note that the region specified by (33b) is a convex subspace of the K-dimensional space specified by $\{b_k\}$. In addition, $f_k(1/b_k)b_k$ is strictly convex because $f_k(c_k)$ is strictly convex. Therefore, the solution of (33) is unique. This completes the proof. ■

The optimization in (28) is considerably simpler than the one in (27) and (21c): The former has K variables, while the latter has $K + KN$ variables. Equations (28) can be used to obtain the optimal $\{c_k\}$ in most practical cases because $f_k(c_k)$ is generally convex [1].

Suppose, for the time being, that $\{c_k\}$ is not an integer but a real number. Then, the solution $\{c_k^*\}$ of (28) can be calculated by $\{b_k^*\}$, which can be obtained from a convex optimization problem in (33) using the method of Lagrange multipliers. Let λ be a Lagrange multiplier. The optimization in (33) can be rewritten as

$$\min_{b_k} L(b_k, \lambda) = \sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \frac{R_k}{\alpha_k^2} - \lambda\left(\sum_{k=1}^{k} R_k b_k - N\right). \tag{34}$$

Differentiating $L(b_k, \lambda)$ with respect to b_k and setting the result equal to zero yield

$$f_k\left(\frac{1}{b_k}\right) - f_k'\left(\frac{1}{b_k}\right)\frac{1}{b_k} - \lambda\alpha_k^2 = 0 \quad \text{for all } k. \tag{35}$$

The system of $K + 1$ simultaneous equations, consisting of (35) and the original constraint given in (33b), defines the optimum solutions $\{b_k^* | k = 1, \ldots, K\}$ and λ^*. However, deriving closed-form expressions of the optimum solutions is impossible due to the nonlinear function $f_k(1/b_k)$. Therefore, the use of a numerical technique is suggested, such as the vector-form Newton's method [13], for evaluating $\{b_k^*\}$. Due to the uniqueness of $\{b_k^*\}$ and convexity of $f_k(1/b_k)b_k$, such a method yields a reasonably accurate $\{b_k^*\}$ after only a few iterations. Appendix A summarizes the vector-form Newton's method for calculating $\{b_k^*\}$. In particular, it is shown that each iteration of Newton's method only requires $O(K)$ operations.

Although the obtained $\{c_k^*\}$ via (34) is not an integer, it can still be used for the MA optimization in (22). The proposed suboptimal approach to MA subcarrier allocation is summarized as follows:

Algorithm 1 (MA subcarrier allocation).

Step 1) Evaluate real-valued $\{c_k^*\}$ from (33b) and (35) using the vector-form Newton's method.

Step 2) Substitute $\{c_k^*\}$ for $\{c_k\}$ in (22), and solve the transportation problem in (22) using LP to obtain the optimal $\{\rho_{k,n}\}$.

B. Subcarrier Allocation in RA Problem

The proposed algorithm for RA subcarrier allocation is developed based on Proposition 1: z^* is obtained, then the problem in (10) and (6c) is solved. The following shows that z^* can be evaluated under Assumptions 1 and 2.

Consider the RA optimization described by (9), (7b), and (6c). Due to (1), the constraint in (9b) can be rewritten as

$$\sum_{n=1}^{N} c_{k,n}\rho_{k,n} = z, \quad \text{for all } k. \tag{36}$$

When Assumption 1 holds, (36) becomes

$$\sum_{n=1}^{N} \rho_{k,n} = z/c_k, \quad \text{for all } k. \tag{37}$$

In addition, from (23)

$$\sum_{k=1}^{K} z/c_k = N. \tag{38}$$

Using (37) in (7b), the RA optimization under Assumptions 1 and 2 can be represented as follows:

$$\max_{c_k, \rho_{k,n}} z \tag{39a}$$

$$\text{subject to} \sum_{k=1}^{K} \frac{f_k(c_k)}{\alpha_k^2} \cdot \frac{z}{c_k} \le P_{\mathrm{T}} \tag{39b}$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1, \quad \text{for all } n. \tag{39c}$$

As in the MA case (Proposition 2), the optimum z and $\{c_k\}$ can be obtained after relaxing the constraint in (38) to (39c).

Proposition 3: Suppose that for all k, $f_k(c_k)$ is a convex function of c_k. Then, the optimal $\{z, c_k\}$, say $\{z^*, c_k^*\}$, of (39) can be obtained through the following constrained maximization:

$$\max_{c_k} z \tag{40a}$$

$$\text{subject to} \quad \sum_{k=1}^{K} \frac{f_k(c_k)}{\alpha_k^2} \cdot \frac{z}{c_k} \le P_{\mathrm{T}} \tag{40b}$$

$$\sum_{k=1}^{K} \frac{z}{c_k} = N. \tag{40c}$$

The proof of this proposition is similar to that of Proposition 2 and, thus, is omitted.

Suppose, as in the MA case, that $\{z, c_k\}$ are real numbers. Then, the inequality in (40b) can be replaced with the equality

$$\sum_{k=1}^{K} \frac{f_k(c_k)}{\alpha_k^2} \cdot \frac{z}{c_k} = P_{\mathrm{T}}. \tag{41}$$

This is true because transmission with the maximum power P_{T} tends to increase the data rate and helps maximize z ($\{z, c_k\}$ satisfying (41) exist due to the assumption of real-valued $\{z, c_k\}$). The real-valued optimal solution of (40a), (40c), and (41) can be obtained using the method of Lagrange multipliers. Introducing two Lagrange multipliers λ and μ, the convex optimization problem for RA problem can be rewritten as

$$\max_{b_k} L(z, b_k, \lambda, \mu) = z - \lambda \left(\sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \frac{z}{\alpha_k^2} - P_{\mathrm{T}} \right) - \mu \left(\sum_{k=1}^{K} b_k z - N \right) \quad (42)$$

where $b_k = 1/c_k$. Differentiating $L(z, b_k, \lambda, \mu)$ with respect to z, b_k, λ, and μ, and setting the results equal to zero yield

$$1 - \lambda \sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \frac{1}{\alpha_k^2} - \mu \sum_{k=1}^{K} b_k = 0 \quad (43a)$$

$$f_k\left(\frac{1}{b_k}\right) - f_k'\left(\frac{1}{b_k}\right)\frac{1}{b_k} + \mu z/\lambda = 0, \quad \text{for all } k \quad (43b)$$

$$z \sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \frac{1}{\alpha_k^2} - P_{\mathrm{T}} = 0 \quad (43c)$$

$$z \sum_{k=1}^{K} b_k - N = 0. \quad (43d)$$

Note that $\{z, b_k\}$ and μ/λ can be obtained from (43b)–(43d) while ignoring (43a), which is only necessary for calculating μ and λ. Following the approach described in the previous section, the subsequent observations can be made.

1) The system of $K+2$ simultaneous equations of (43b)–(43d) has a unique solution for $\{b_k\}$ and z.
2) The solution of (43b)–(43d) can be obtained using the vector-form Newton's method, which is summarized in Appendix B for the RA case.
3) Each iteration of Newton's method only requires $O(K)$ operations.

After obtaining z^*, the RA subcarrier allocation can be performed via a single use of the MA optimization (Proposition 1). The proposed algorithm for the RA subcarrier allocation is stated as follows:

Algorithm 2 (RA subcarrier allocation).

Step 1) Evaluate $\{z^*, b_k^*\}$ from (43b)–(43d) using the vector-form Newton's method, and then obtain real-valued $\{z^*, c_k^*\}$.

Step 2) Substitute z^* and $\{c_k^*\}$ for R_k and $\{c_k\}$, respectively, in (22), and solve the transportation problem in (22) to obtain $\{\rho_{k,n}\}$.

Step 2) of this algorithm describes the procedure for the MA subcarrier allocation in Algorithm 1. Recall that in Step 1) of Algorithm 1, $\{c_k^*\}$ is obtained from (33b) and (35). In Algorithm 2, it is possible to evaluate $\{c_k^*\}$ from (43b)–(43d) because they are equivalent to $\{c_k^*\}$ from (33b) and (35), when $R_k = z^*$ for all k [this can be seen by comparing (43b)–(43d) with (33b) and (35)].

C. Bit Loading

After the subcarrier allocation, the MA optimization problem in (6) can be simplified as follows:

MA Bit Loading

$$\min_{c_{k,n}} P_{\mathrm{T}} = \min_{c_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \rho_{k,n} \quad (44a)$$

$$\text{subject to} \quad R_k = \sum_{n=1}^{N} c_{k,n} \rho_{k,n} \quad \text{for all } k. \quad (44b)$$

Equations in (44) reveal that the MA bit loading for each user can be independently performed: Minimizing $\sum_{n=1}^{N} f_k(c_{k,n})\rho_{k,n}/\alpha_{k,n}^2$ for each k under the constraint in (44b) eventually minimizes the total power P_{T}. This fact indicates that the MA bit loading for each user can be performed as in the case of single-user OFDM, as such it is possible to apply the greedy algorithm proposed for the single-user OFDM in [3] and [11]. The greedy algorithm for the MA bit loading with multiuser OFDM is described below.

Bit Loading To Given Subcarriers (MA-Type)

Definition

S_k is the set of indexes of subcarriers assigned to the kth user.

$$\Delta P_{k,n}(c) = \left[f_k(c+1) - f_k(c)\right]/\alpha_{k,n}^2, \quad \text{for } n \in S_k. \quad (45)$$

Initialization

Let $c_{k,n} = 0$ for all k and n.

Evaluate $\Delta P_{k,n}(0)$ for each k and $n \in S_k$.

Bit Assignment Iteration

For each k, repeat the following R_k times:

$\hat{n} = \arg\min_{n \in S_k} \Delta P_{k,n}(c_{k,n})$

$c_{k,\hat{n}} = c_{k,\hat{n}} + 1$

evaluate $\Delta P_{k,\hat{n}}(c_{k,\hat{n}})$.

In (45), $\Delta P_{k,n}(c)$ denotes the additional power needed for transmitting one additional bit through the nth subcarrier of the kth user. The greedy algorithm eventually minimizes P_{T} by assigning each bit to those subcarriers requiring the least additional power.

The RA optimization, after subcarrier allocation, is written as follows:

RA Bit Loading

$$\max_{c_{k,n}} \min_{k} R_k = \max_{c_{k,n}} \min_{k} \sum_{n=1}^{N} c_{k,n} \rho_{k,n} \quad (46a)$$

$$\text{subject to} \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{f_k(c_{k,n})}{\alpha_{k,n}^2} \rho_{k,n} \le P_{\mathrm{T}}. \quad (46b)$$

TABLE V
REQUIRED TRANSMISSION POWER IN MA PROBLEM WHEN $\gamma = 0$ dB

R_1	R_2	R_3	R_4 (bits/OFDM symbol)	Optimal IP	Suboptimal LP	Suboptimal Vogel's	Conventional in [1]
32	32	32	32	27.45 dB	27.59 dB	27.60 dB	27.54 dB
64	64	64	64	34.34 dB	34.45 dB	34.49 dB	34.41 dB
96	96	96	96	40.43 dB	40.52 dB	40.55 dB	40.50 dB
42	42	86	86	34.51 dB	34.63 dB	34.66 dB	34.60 dB
32	32	96	96	34.69 dB	34.81 dB	34.85 dB	34.78 dB
26	26	102	102	34.85 dB	34.97 dB	35.01 dB	34.93 dB

For this problem, unlike the MA case, an independent bit loading for each user is impossible because of the max–min cost in (46a). Accordingly, utilizing the relation between the MA and RA problems is suggested for RA bit loading instead of considering (46). Proposition 1 indicates that the greedy algorithm for MA bit loading can be used for the RA case after substituting z^* from Algorithm 2 for R_k. Therefore, the greedy algorithm can be employed for both MA and RA bit loading.

V. Heuristic Subcarrier-Allocation Algorithm

It is well known that a transportation problem can be efficiently solved using a suboptimal algorithm such as Vogel's method [10]. This section briefly states the application of Vogel's method to Step 2) of Algorithms 1 and 2.

To describe Vogel's method, the following notations are necessary: S and U denote sets of subcarrier and user indexes, respectively. These sets meet

$$S \subseteq \{1, 2, \ldots, N\} \tag{47}$$

$$U \subseteq \{1, 2, \ldots, K\}. \tag{48}$$

$P(k)$ is the penalty defined as the difference between the smallest cost and the $(n_k + 1)$th smallest cost in $\{r_{k,n} | n \in S\}$, where $r_{k,n} = f_k(c_k^*)/\alpha_{k,n}^2$ as defined in (22a) and c_k^* can be obtained via Algorithms 1 and 2 in Section IV. Specifically

$$P(k) = r_k^{(n_k+1)} - r_k^{(1)} \tag{49}$$

where $r_k^{(i)}$ is the ith smallest value among $\{r_{k,n} | n \in S\}$.[4] At the beginning, $S = \{1, 2, \ldots, N\}$ and $U = \{1, 2, \ldots, K\}$. As such, Vogel's method is stated as follows:

Vogel's method for subcarrier allocation

Initialization

Set $\rho_{k,n}$ to zero for all k and n.
Set n_k to R_k/c_k^* for all k.
Subcarrier index set $S = \{1, \ldots, N\}$.
User index set $U = \{1, \ldots, K\}$.
Calculate initial penalties $\{P(k) | k = 1, \ldots, K\}$.

Iteration

Repeat the following operations until S becomes the null set ϕ.
$\hat{k} = \arg\max\{P(k) | k \in U\}$
$\hat{n} = \arg\min\{r_{\hat{k},n} | n \in S\}$
Set $\rho_{\hat{k},\hat{n}}$ to one.
$S = S - \{\hat{n}\}$ and $n_{\hat{k}} = n_{\hat{k}} - 1$
If $\sum_{n=1}^{N} \rho_{\hat{k},n} = R_{\hat{k}}/c_{\hat{k}}^*$, then $U = U - \{\hat{k}\}$.
Update the penalties for the users whose indexes are in the set U.

For each iteration of this algorithm, the size of the subcarrier index set S is reduced by one and the penalty $\{P(k)\}$ updated due to the supply constraint in (22c). The user index set U is reduced whenever the demand constraint in (22b) is satisfied for a certain k.

The computational load needed by Vogel's method is $O(N^2)$ while that for LP is $O(N^4)$. The overall suboptimal algorithm for subcarrier allocation and bit loading requires $O(K)$ operations for one iteration of the vector-form Newton's method, $O(N^2)$ for Vogel's method, and $O(NK)$ for bit loading. This algorithm is a polynomial-time algorithm that can be applied to real-time OFDM systems.

VI. Simulation Results

The proposed algorithms were applied to an adaptive multiuser OFDM system with the parameters explained in Section III-B. The whole simulation results presented in this section are the average of 1000 independent trials. Since Newton's method tends to converge within about three steps, the results after three iterations were used for the MA and RA optimizations. For comparison, the methods in [1] and [2] were also considered: The algorithm in [2][5] was applied to the subcarrier allocation, and the bit loading was performed as in Section IV-C.

Tables V and VI compare the total required transmission power in the MA problem when γ was 0 and 30 dB, respectively. The values of the required power obtained from the optimal and suboptimal algorithms were reasonably close to each other. The performance gap between the optimal IP and the suboptimal algorithm with Vogel's method did not exceed 0.16 dB when $\gamma = 0$ dB and 0.2 dB when $\gamma = 30$ dB. The performances of the suboptimal algorithms were comparable to each other: When $\gamma = 0$ dB, the algorithm with Vogel's method performed slightly worse than the LP-based algorithm and the conventional method in [1] (the performance gap was less than or equal to 0.08 dB), but this was reversed in most cases when $\gamma = 30$ dB. This happened due to the effect of performing the

[4]Penalties other than the one in (49) may be considered. For example, it can be defined as the difference between the two smallest costs in $\{r_{k,n} | k \in U\}$. Vogel's method employing (49) outperformed those with other penalties when user's average channel magnitudes are considerably different from each other.

[5]Reference [2] focuses on increasing capacity by dynamic subcarrier allocation, and does not consider practical bit loading.

TABLE VI
REQUIRED TRANSMISSION POWER IN MA PROBLEM WHEN $\gamma = 30$ dB

R_1	R_2	R_3	R_4 (bits/OFDM symbol)	Optimal IP	Suboptimal LP	Suboptimal Vogel's	Conventional in [1]
32	32	32	32	47.46 dB	47.63 dB	47.63 dB	47.65 dB
64	64	64	64	52.18 dB	52.40 dB	52.38 dB	52.41 dB
96	96	96	96	56.28 dB	56.38 dB	56.41 dB	56.35 dB
42	42	86	86	52.35 dB	52.54 dB	52.47 dB	52.57 dB
32	32	96	96	52.68 dB	52.88 dB	52.82 dB	52.86 dB
26	26	102	102	52.94 dB	53.13 dB	53.08 dB	53.10 dB

TABLE VII
MINIMUM DATA RATE IN RA PROBLEM WHEN $\gamma = 0$ dB

P_T (dB)	Optimal IP	Suboptimal LP	Suboptimal Vogel's	Conventional in [2]
40.0	93.22 bits	92.48 bits	92.37 bits	90.42 bits
45.0	120.26 bits	119.48 bits	119.34 bits	116.50 bits
50.0	135.38 bits	134.42 bits	134.13 bits	132.65 bits

TABLE VIII
MINIMUM DATA RATE IN RA PROBLEM WHEN $\gamma = 30$ dB

P_T (dB)	Optimal IP	Suboptimal LP	Suboptimal Vogel's	Conventional in [2]
40.0	7.52 bits	7.44 bits	7.45 bits	6.52 bits
45.0	20.52 bits	20.36 bits	20.36 bits	16.90 bits
50.0	47.10 bits	46.62 bits	46.69 bits	40.24 bits

subcarrier allocation and bit loading separately. The algorithm with Vogel's method would be preferred to the other algorithms because of its simplicity in implementation: For various values of N and K, it needed less than 2% of the computational complexity of the conventional method in [1]—see Appendix C for details.

The simulation results for the RA optimization are presented in Tables VII ($\gamma = 0$ dB) and VIII ($\gamma = 30$ dB). Again, in this case, the optimal IP and proposed suboptimal algorithms behaved in a similar manner. The performance degradation was less than 0.96 bits/OFDM symbol when LP was employed and 1.25 bits/OFDM symbol when Vogel's method was used. The algorithm in [2] performed somewhat worse than the others: The maximum loss was 6.86 bits/OFDM symbol.

VII. CONCLUSION

It was shown that subcarrier allocation and bit loading for multiuser OFDM can be optimized using IP. Various suboptimal algorithms requiring less computation than IP were developed via the LP relaxation of IP. In addition, an efficient heuristic algorithm called Vogel's method for solving the LP problem was presented. Simulation results indicated that performances of the proposed suboptimal methods were reasonably close to those of optimal IP.

As indicated in Section II, the proposed schemes assume perfect channel information from all users. However, this assumption may be violated due to channel estimation errors, quantization errors, and delay in the feedback information. The effect of the imperfect channel information on the proposed techniques needs further research. For time-varying channels, it is also necessary to analyze how often the subcarrier and bit allocations should be updated.

APPENDIX A

The vector-form Newton's method for calculating $\{b_k^*\}$ from (35) and (33b) is summarized as follows. Define $(K+1)$-dimensional vectors $\mathbf{x}$ and $\mathbf{F}(\mathbf{x})$: $\mathbf{x} = [b_1, b_2, \ldots, b_K, \lambda]^{\mathrm{T}}$ and $\mathbf{F}(\mathbf{x}) = [F_1(\mathbf{x}), F_2(\mathbf{x}), \ldots, F_K(\mathbf{x}), G(\mathbf{x})]^{\mathrm{T}}$, where

$$F_k(\mathbf{x}) = f_k\left(\frac{1}{b_k}\right) - f_k'\left(\frac{1}{b_k}\right)\frac{1}{b_k} - \lambda\alpha_k^2, \quad \text{for all } k \tag{A1}$$

$$G(\mathbf{x}) = \sum_{k=1}^{K} R_k b_k - N. \tag{A2}$$

$F_k(\mathbf{x})$ and $G(\mathbf{x})$ correspond to (35) and (33b), respectively. The update equation for obtaining the optimal $\mathbf{x}$ is given by

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \boldsymbol{\delta}_i \tag{A3}$$

where $\mathbf{x}_i = [b_1(i), \ldots, b_K(i), \lambda(i)]^{\mathrm{T}}$ is the value of $\mathbf{x}$ at the ith iteration and $\boldsymbol{\delta}_i$ satisfies

$$-\mathbf{J}(\mathbf{x}_i)\boldsymbol{\delta}_i = \mathbf{F}(\mathbf{x}_i). \tag{A4}$$

Here, $\mathbf{J}(\mathbf{x}_i)$ is a Jacobian matrix of dimension $(K+1)$ by $(K+1)$ [13]. For $F_k(\mathbf{x})$ and $G(\mathbf{x})$ in (A1) and (A2), the Jacobian matrix is given by (A5), shown at the bottom of the page. Using (A5) in (A4), it is straightforward to derive the closed-form expression of $\boldsymbol{\delta}_i$ (this expression is rather tedious and thus not presented here). The result indicates that only $O(K)$ operations are needed for each iteration in (A3).

$$\mathbf{J}(\mathbf{x}) = \left[\begin{array}{cccccc|c} f_1''(1/b_1)/b_1^3 & 0 & 0 & \cdots & 0 & & -\alpha_1^2 \\ 0 & f_2''(1/b_2)/b_2^3 & 0 & \cdots & 0 & & -\alpha_2^2 \\ \vdots & \vdots & \ddots & & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & f_K''(1/b_K)/b_K^3 & & -\alpha_K^2 \\ \hline R_1 & R_2 & R_3 & \cdots & R_K & & 0 \end{array}\right] \tag{A5}$$

$$\mathbf{J}(\mathbf{x}) = \left[\begin{array}{cccccc|cc} f_1''(1/b_1)/b_1^3 & 0 & 0 & \cdots & 0 & & 0 & -\alpha_1^2 \\ 0 & f_2''(1/b_2)/b_2^3 & 0 & \cdots & 0 & & 0 & -\alpha_2^2 \\ \vdots & \vdots & \ddots & & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & f_K''(1/b_K)/b_K^3 & & 0 & -\alpha_K^2 \\ \hline w_1 & w_2 & w_3 & \vdots & w_K & & v & 0 \\ z & z & z & \cdots & z & & \sum_{k=1}^{K} b_k & 0 \end{array}\right] \tag{B4}$$

APPENDIX B

The vector-form Newton's method for calculating $\{z^*, b_k^*\}$ from (43b)–(43d) is summarized as follows. Define $(K+2)$-dimensional vectors $\mathbf{x}$ and $\mathbf{F}(\mathbf{x}) : \mathbf{x} = [b_1, b_2, \ldots, b_K, z, \lambda']$ and $\mathbf{F}(\mathbf{x}) = \{F_1(\mathbf{x}), F_2(\mathbf{x}), \ldots, F_K(\mathbf{x}), H(\mathbf{x}), G(\mathbf{x})]^{\mathrm{T}}$, where $\lambda' = \mu/\lambda$

$$F_k(\mathbf{x}) = f_k\left(\frac{1}{b_k}\right) - f_k'\left(\frac{1}{b_k}\right)\frac{1}{b_k} - \lambda'\alpha_k^2 \quad \text{for all } k \tag{B1}$$

$$H(\mathbf{x}) = \sum_{k=1}^{K} f_k\left(\frac{1}{b_k}\right) b_k \frac{z}{\alpha_k^2} - P_{\mathrm{T}} \tag{B2}$$

$$G(\mathbf{x}) = z\sum_{k=1}^{K} b_k - N. \tag{B3}$$

$F_k(\mathbf{x})$, $H(\mathbf{x})$, and $G(\mathbf{x})$ correspond to (43b)–(43d), respectively. The update equation for obtaining the optimal $\mathbf{x}$ is given by (A3), where $\mathbf{x}_i = [b_1(i), b_2(i), \ldots, b_K(i), z(i), \lambda'(i)]$ and $\boldsymbol{\delta}_i$ satisfy (A4). For $F_k(\mathbf{x})$, $H(\mathbf{x})$, and $G(\mathbf{x})$ in (B1)–(B3), the $(K+2)$ by $(K+2)$ Jacobian matrix is given by (B4), shown at the top of the page, where $w_k = [f_k(1/b_k) - f_k'(1/b_k)/b_k]z/\alpha_k^2$ for all k, and $v = \sum_{k=1}^{K} f_k(1/b_k)b_k/\alpha_k^2$.

APPENDIX C

As in the proposed suboptimal approach, the algorithm in [1] performs subcarrier allocation and bit loading separately. For MA subcarrier allocation, this algorithm iteratively searches for the set of power coefficients $\{\lambda_k | k = 1, \ldots, K\}$ such that the individual rate constraints are satisfied. Assuming that the nonlinear function inversion that is required at each iteration is implemented using a lookup table, its complexity is $O(I_1KN + I_2N)$, where I_1 and I_2 denote the number of iterations for outer and inner loops, respectively ($I_2 \gg I_1$). Since the parameters I_1 and I_2 cannot be predicted theoretically, these values and the number of overall operations (additions, comparisons, and multiplications) were counted through simulation and compared with the proposed algorithm employing Vogel's method. The parameters for the simulation were identical to those in Section III-B with the exception that various values of N and K were considered: Specifically, $N \in \{16, 32, 64, 128\}$, $K \in \{4, 8\}$, and $R_k = 4N/K$ for all k.

Table IX compares the number of operations required by the proposed algorithm with Vogel's method and the conventional method in [1] when $\gamma = 0$ dB (the results for $\gamma = 30$ dB were similar to those in Table IX, and thus, they are not presented). For the conventional method, I_2 increased in proportion to both K and N, while I_1 was mainly proportional to K. The computational load needed by the proposed method was about 2% of the complexity of the conventional method when $K = 4$, and it was reduced to less than 1% when $K = 8$.

TABLE IX
NUMBER OF OPERATIONS WHEN $\gamma = 0$ dB

		$N = 16$	$N = 32$	$N = 64$	$N = 128$
$K = 4$	Suboptimal Vogel's	444	1012	2916	9796
	Conventional in [1]	21648 $I_1 = 51$ $I_2 = 102$	53438 $I_1 = 57$ $I_2 = 155$	162123 $I_1 = 66$ $I_2 = 290$	476682 $I_1 = 66$ $I_2 = 501$
$K = 8$	Suboptimal Vogel's	768	1528	3816	11464
	Conventional in [1]	77328 $I_1 = 97$ $I_2 = 200$	238944 $I_1 = 129$ $I_2 = 404$	720694 $I_1 = 151$ $I_2 = 745$	2140912 $I_1 = 160$ $I_2 = 1286$

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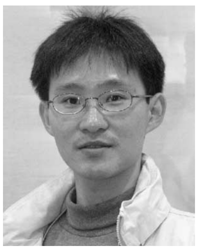

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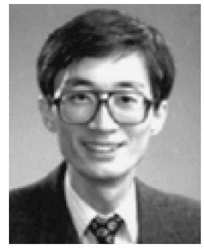

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