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Regularized Channel Diagonalization for Multiuser MIMO Downlink Using a Modified MMSE Criterion

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Abstract—We propose a regularized channel diagonalization method for a joint transmit-receive linear optimization in the downlink of a multiuser multiple-input multiple-output (MIMO) communication system. This method is based on the use of a modified minimum mean-square error (MMSE) criterion, which employs a weighted information symbol vector for the target and signal scaling. The weights for the target are the equivalent channel gain resulting from a zero-forcing (ZF)-based MIMO channel diagonalization. A joint iterative algorithm for minimizing the mean-square error (MSE) under a total transmit power constraint is derived, and its convergence is proved. The signal-to-interference-plus-noise ratio (SINR) is analyzed and the sum rates evaluated in a computer simulation. The results demonstrate that the proposed method outperforms the existing ZF- and MMSE-based methods.

Index Terms—Downlink, minimum mean-square error (MMSE), multiuser multiple-input multiple-output (MIMO), spatial multiplexing, zeroforcing (ZF).

I. INTRODUCTION

Spatial multiplexing for multiple-input multiple-output (MIMO) radio systems, employing multiple transmit and receive antennas, has been recognized as an effective way to improve the spectral efficiency of wireless links [1]–[3]. This is realized by transmitting multiple data substreams in parallel and performing some transmit–receive processing which assists data recovery. In a multiuser MIMO downlink, where the base station communicates simultaneously with multiple users, the data substreams are generated by combining the signals of different users (Fig. 1). To mitigate the cochannel interference (CCI) caused by the spatial multiplexing in a multiuser downlink, a channel inversion and its modifications [4]–[7] have been introduced in the

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form of transmit processing algorithms. These are mainly useful for systems with single-antenna receivers. For systems with multiple receiver antennas, coordinated transmit–receive processors have been developed based on a joint-channel diagonalization¹ [8]–[11] and a minimum mean-square error (MMSE) criterion [12]–[17]. The channel diagonalization methods are zero-forcing (ZF) algorithms that attempt to eliminate the CCI, while ignoring noise. On the other hand, the MMSE algorithms control the degree of CCI suppression depending on the signal-to-noise ratio (SNR). Accordingly, the MMSE schemes outperform the channel diagonalization methods in low SNR environment [15].

In this paper, an alternative MMSE scheme is proposed that modifies the total-MMSE (T-MMSE) algorithm in [15]. It is observed that the T-MMSE performs a regularized channel inversion and acts like a channel inverter in high SNR environment. Consequently, the sum rate performance of the T-MMSE tends to become worse than that of the channel diagonalization methods as the SNR increases. To avoid this performance degradation, the MMSE criterion is modified so that the resulting MMSE scheme performs regularized channel diagonalization. Specifically, the MMSE criterion employs a target vector which is given by a weighted information vector, where the weights are the equivalent channel gain resulting from a joint-channel diagonalization, and signal scaling. Due to the use of the weights, the proposed approach can assign more power to stronger subchannels. The signal scaling enables us to derive a closed-form expression for the Lagrange multiplier, which is employed to consider a transmit power constraint [18]. An iterative algorithm for minimizing the MSE under a total transmit power constraint is derived, and its characteristics are analyzed. It will be shown that the proposed algorithm, termed the modified T-MMSE (MT-MMSE), can outperform the existing methods irrespective of the SNR.

The organization of this paper is as follows. Section II describes the multiuser MIMO system model. The proposed method is then derived and analyzed in Section III. Section IV presents computer simulation results to demonstrate the advantage of the proposed processors. Finally, Section V presents the conclusion.

II. MULTIUSER MIMO SYSTEM MODEL

The system configuration of a multiuser MIMO downlink with K users, N_T transmit antennas, and $N_{R,k}$ receive antennas, $k \in \{1, 2, \ldots, K\}$, is shown in Fig. 1. The MIMO channel is represented as $\mathbf{H}_k \in \mathbb{C}^{N_{R,k} \times N_T}$, where the entries are independently identically distributed (i.i.d.) zero-mean complex Gaussian random variables with a unit variance. The (m, n)th entry represents the complex gain from the *n*th transmit antenna to the *m*th receive antenna. It is assumed that all $\{\mathbf{H}_k\}$ are known at the transmitter, while the *k*th receiver only knows its own MIMO channel. This assumption indicates that the multiuser interference should be suppressed at the transmitter via preprocessing. The spatial multiplexing is performed by forming a vector signal \mathbf{x}_k with L_k symbols, $\mathbf{x}_k \in \mathbb{C}^{L_k \times 1}$, preprocessing matrix, and combining the preprocessed vectors from the users to yield $\sum_{k=1}^{K} \mathbf{T}_k \mathbf{x}_k$. Following this, the elements of $\sum_{k=1}^{K} \mathbf{T}_k \mathbf{x}_k$ are transmitted through different antennas. At the *k*th receiver signal $\mathbf{H}_k \sum_{j=1}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k$, where $\mathbf{n}_k \in \mathbb{C}^{N_{R,k} \times 1}$ is a

¹After the channel diagonalization, the equivalent channel gain matrix, which describes the cascade of transmit–receive processing and the physical channel, becomes a diagonal matrix. Channel inversion is a special case of channel diagonalization in which the equivalent channel gain is given by a normalized identity matrix.

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Fig. 1. Transmit and receive processing for multiuser MIMO downlink with K users.

noise vector whose elements are i.i.d. with zero mean and variance σ_k^2 , is postprocessed by $\mathbf{R}_k^H \in \mathbb{C}^{L_k \times N_{R,k}}$ to yield

$$\tilde{\mathbf{x}}_{k} = \mathbf{R}_{k}^{H} \mathbf{H}_{k} \sum_{j=1}^{K} \mathbf{T}_{j} \mathbf{x}_{j} + \mathbf{R}_{k}^{H} \mathbf{n}_{k}$$
(1a)

$$= \mathbf{R}_{k}^{H} \mathbf{H}_{k} \mathbf{T}_{k} \mathbf{x}_{k} + \mathbf{R}_{k}^{H} \mathbf{H}_{k} \sum_{\substack{j=1\\j\neq k}}^{K} \mathbf{T}_{j} \mathbf{x}_{j} + \mathbf{R}_{k}^{H} \mathbf{n}_{k}$$
(1b)

where the superscript H denotes the Hermitian transpose, and the second term in (1b) represents the residual multiuser interferences.

It is worthwhile to define the multiuser channel matrix $\mathbf{H} = [\mathbf{H}_1^H \cdots \mathbf{H}_K^H]^H$ and the multiuser transmit–receive weight matrices $\mathbf{T} = [\mathbf{T}_1 \cdots \mathbf{T}_K]$ and $\mathbf{R}^H = \text{blockdiag}[\mathbf{R}_1^H, \dots, \mathbf{R}_K^H]$. Collecting all $\tilde{\mathbf{x}}_k$ in (1a), the multiuser signal model is written as

$$\tilde{\mathbf{x}} = \mathbf{R}^H \mathbf{H} \mathbf{T} \mathbf{x} + \mathbf{R}^H \mathbf{n}$$
(2)

where $\tilde{\mathbf{x}} = [\tilde{\mathbf{x}}_1^H \cdots \tilde{\mathbf{x}}_K^H]^H$, $\mathbf{x} = [\mathbf{x}_1^H \cdots \mathbf{x}_K^H]^H$, and $\mathbf{n} = [\mathbf{n}_1^H \cdots \mathbf{n}_K^H]^H$. It is assumed that $\mathbf{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_L$, where $\mathbf{E}[\cdot]$ denotes the expectation, \mathbf{I}_L is the *L*-dimensional identity matrix, and $L = \sum_{k=1}^K L_k$. The transmit-matrix \mathbf{T} satisfies the power constraint $\mathbf{E}[\|\mathbf{T}\mathbf{x}\|^2] = \operatorname{tr}(\mathbf{T}^H\mathbf{T}) = P_T$, where $\|\cdot\|$ and $\operatorname{tr}(\cdot)$ denote the vector 2-norm and trace operation of a matrix, respectively, and P_T is the total transmission power.

III. DERIVATION OF PROPOSED PROCESSING

Suppose for the time being that a joint-channel diagonalization technique [8]–[11] is used for designing the transmit–receive processing matrices { \mathbf{T}_k , \mathbf{R}_k }. Then, in (1b), the channel \mathbf{H}_k is fully diagonalized and all CCI terms consisting of residual multiuser- and self-interferences are cancelled out. In this case, (1a) is rewritten as

$$\tilde{\mathbf{x}}_k = \mathbf{\Lambda}_k \mathbf{x}_k + \mathbf{R}_k^H \mathbf{n}_k$$

where $\Lambda_k \in \mathbb{R}^{L_k \times L_k}$ is a diagonal matrix with nonnegative entries representing the equivalent channel gain. This equation suggests the use of $\Lambda_k \mathbf{x}_k$ as the target vector of an MMSE criterion, as the present objective is to design $\{\mathbf{T}_k, \mathbf{R}_k\}$ so that the resulting coordinated transmit–receive processor acts like a joint-channel diagonalization method for high SNRs. The proposed MSE for the *k*th user is represented as²

$$\mathbf{E}[\|\mathbf{\Lambda}_k \mathbf{x}_k - \tilde{\mathbf{x}}_k\|^2] \tag{3}$$

where $\tilde{\mathbf{x}}_k$ is given by (1a). Now, to help simplify the derivation, we introduce a scaling parameter β^{-1} and represent \mathbf{R}_k by

$$\mathbf{R}_k = \beta^{-1} \mathbf{R}'_k \tag{4}$$

where β is a positive real number. Then, the MSE in (3) is rewritten as

$$\mathbf{E}[\|\mathbf{\Lambda}_k \mathbf{x}_k - \beta^{-1} \tilde{\mathbf{x}}_k'\|^2]$$
(5)

where $\tilde{\mathbf{x}}'_k = \mathbf{R}'^H_k \mathbf{H}_k \sum_{j=1}^K \mathbf{T}_j \mathbf{x}_j + \mathbf{R}'^H_k \mathbf{n}_k$, and finding the optimal \mathbf{T}_k , \mathbf{R}'_k and β becomes our objective. The problem for minimizing the multiuser MSE under the total power constraint is written as (6), shown at the bottom of the page. The total transmit power constraint in (6b) provides the flexibility of dynamic transmit power allocation to each user. Due to the use of $\mathbf{\Lambda}_k$ in the cost function, this approach tends to assign more power to stronger subchannels having larger equivalent channel gain. This power allocation assists stronger subchannels to support higher data rates [19]. When $\mathbf{\Lambda}_k = \mathbf{I}_{L_k}$ and $\beta = 1$, the optimization in (6) becomes identical to that of the T-MMSE. The total MSE in (6a) is convex over the transmit (receive) matrix when the receive (transmit) matrix and β are given, and strictly quasi-convex [22] with respect to β if both transmit and receive matrices are fixed. However,

²As an alternative to the MSE in (5), we may consider $E[||\mathbf{\Lambda}_k(\mathbf{x}_k - \bar{\mathbf{x}}_k)||^2]$, which is a special case of the weighted MSE for single-user MIMO systems [19], [20]. However, this weighted MSE can be shown to yield a transmit–receive processor that approaches a channel inversion as SNR increases; thus, it is not appropriate for our purpose.

$$\min_{\{\mathbf{T}_k\};\{\mathbf{R}'_k\};\beta} \sum_{k=1}^{K} \mathbb{E}\left[\left\| \mathbf{\Lambda}_k \mathbf{x}_k - \beta^{-1} \mathbf{R}'^H_k \left(\mathbf{H}_k \sum_{j=1}^{K} \mathbf{T}_j \mathbf{x}_j + \mathbf{n}_k \right) \right\|^2 \right]$$
(6a)
subject to $\sum_{k=1}^{K} \operatorname{tr}\left(\mathbf{T}^H_k \mathbf{T}_k \right) = P_T.$ (6b)

for a given β , it is not convex on $\{\mathbf{T}_k, \mathbf{R}'_k\}$ [20], [21].³ The necessary conditions for optimal $\{\mathbf{T}_k\}$, $\{\mathbf{R}'_k\}$ and β are found by constructing the Lagrangian function

$$\mathcal{L}\left(\{\mathbf{T}_{k}\}, \{\mathbf{R}_{k}'\}, \beta, \boldsymbol{\lambda}\right)$$

$$= \sum_{k=1}^{K} \mathbb{E}\left[\left\| \mathbf{\Lambda}_{k} \mathbf{x}_{k} - \beta^{-1} \mathbf{R}_{k}'^{H} \left(\mathbf{H}_{k} \sum_{j=1}^{K} \mathbf{T}_{j} \mathbf{x}_{j} + \mathbf{n}_{k}\right) \right\|^{2}\right]$$

$$+ \boldsymbol{\lambda}\left(\sum_{k=1}^{K} \operatorname{tr}\left(\mathbf{T}_{k}^{H} \mathbf{T}_{k}\right) - P_{T}\right)$$
(7)

with the Lagrange multiplier $\lambda \in \mathbb{R}$, and setting its derivatives to zero. This results in

$$\mathbf{T}_{k} = \beta \left(\sum_{j=1}^{K} \mathbf{H}_{j}^{H} \mathbf{R}_{j}^{\prime} \mathbf{R}_{j}^{\prime H} \mathbf{H}_{j} + \boldsymbol{\lambda} \beta^{2} \mathbf{I}_{N_{T}} \right)^{-1} \mathbf{H}_{k}^{H} \mathbf{R}_{k}^{\prime} \boldsymbol{\Lambda}_{k}$$
(8)

$$\mathbf{R}_{k}^{\prime} = \beta \left(\mathbf{H}_{k} \left(\sum_{j=1}^{K} \mathbf{T}_{j} \mathbf{T}_{j}^{H} \right) \mathbf{H}_{k}^{H} + \sigma_{k}^{2} \mathbf{I}_{N_{R,k}} \right)^{-1} \mathbf{H}_{k} \mathbf{T}_{k} \mathbf{\Lambda}_{k}$$
(9)

$$\beta = \frac{\sum_{k=1}^{K} \operatorname{tr} \left(\mathbf{R}_{k}^{\prime H} \mathbf{H}_{k} \left(\sum_{j=1}^{K} \mathbf{T}_{j} \mathbf{T}_{j}^{H} \right) \mathbf{H}_{k}^{H} \mathbf{R}_{k}^{\prime} + \sigma_{k}^{2} \mathbf{I}_{L_{k}} \mathbf{R}_{k}^{\prime H} \mathbf{R}_{k}^{\prime} \right)}{\sum_{k=1}^{K} \operatorname{Re} \left[\operatorname{tr} \left(\mathbf{\Lambda}_{k} \mathbf{R}_{k}^{\prime H} \mathbf{H}_{k} \mathbf{T}_{k} \right) \right]}$$
(10)

where $\operatorname{Re}[\cdot]$ denotes the real part. Since the optimum transmit processors $\{\mathbf{T}_k\}$ are functions of the optimum receiver processors $\{\mathbf{R}'_k\}$ (or vice versa), we consider an iterative algorithm that alternatively designs the transmit and receive processors [15], [21]. At each iteration, the algorithm first evaluates $\{\mathbf{T}_k\}$ and β for a given set of $\{\mathbf{R}'_k\}$ and then updates $\{\mathbf{R}'_k\}$. It can be seen that the update of $\{\mathbf{R}'_k\}$ can be easily performed by (9) once $\{\mathbf{T}_k\}$ and β are given. However, directly evaluating $\{\mathbf{T}_k\}$ and β from (6b), (8) and (10) is formidable. To overcome this difficulty, the optimization procedure in [18] is followed. Let

$$\mathbf{\Gamma}_k = \beta \mathbf{T}'_k(\xi) \tag{11}$$

where $\xi = \lambda \beta^2$ and $\mathbf{T}'_k(\xi) = (\sum_{j=1}^K \mathbf{H}_j^H \mathbf{R}'_j \mathbf{R}'_j^H \mathbf{H}_j + \xi \mathbf{I}_{N_T})^{-1} \mathbf{H}_k^H \mathbf{R}'_k \mathbf{\Lambda}_k$. Using (11) in (6b) produces

$$\beta = P_T^{\frac{1}{2}} \left[\operatorname{tr} \left(\sum_{k=1}^K \mathbf{T}'_k(\xi) \mathbf{T}'^H_k(\xi) \right) \right]^{-\frac{1}{2}}.$$
 (12)

Continuing from (11) and (12), the problem in (6a) can be rewritten as

$$\begin{array}{l} \underset{\xi}{\operatorname{minimize}} \sum_{k=1}^{K} \mathbb{E} \left[\left\| \mathbf{\Lambda}_{k} \mathbf{x} - \mathbf{R}_{k}^{\prime H} \mathbf{H}_{k} \sum_{j=1}^{K} \mathbf{T}_{j}^{\prime}(\xi) \mathbf{x}_{j} \right. \\ \left. - P_{T}^{-\frac{1}{2}} \left[\operatorname{tr} \left(\sum_{j=1}^{K} \mathbf{T}_{j}^{\prime}(\xi) \mathbf{T}_{j}^{\prime H}(\xi) \right) \right]^{\frac{1}{2}} \cdot \mathbf{R}_{k}^{\prime H} \mathbf{n}_{k} \right\|^{2} \right]$$
(13)

for fixed $\{\mathbf{R}'_k\}$. The cost in (13) is strictly quasi-convex with respect to ξ (see the Appendix). Then the optimal ξ , which is obtained by differentiating the cost, is given by

$$\xi = P_T^{-1} \left[\operatorname{tr} \left(\sum_{k=1}^K \sigma_k^2 \mathbf{R}_k^{\prime H} \mathbf{R}_k^{\prime} \right) \right].$$
(14)

Summarizing these results, the transmit matrices $\{\mathbf{T}_k\}$ and β are evaluated as follows: for given $\{\mathbf{R}'_k\}$, $\xi = \lambda \beta^2$ is first evaluated using (14), then $\mathbf{T}'_k(\xi)$ is obtained from (11) and (14). Finally, β and \mathbf{T}_k are calculated from (12) and (11), respectively. The result for β satisfies (10) because of the uniqueness of β for given $\{\mathbf{T}_k, \mathbf{R}'_k\}$.

 $^{3}\mbox{In}$ [21], a T-MMSE-type processor is developed for multiuser MIMO uplinks.

To initiate the iteration, the algorithm needs the equivalent channel gain matrix Λ_k and the initial receive-processing matrices. These matrices can be obtained by applying a joint-channel diagonalization technique. If $\{\mathbf{T}_{zf,k}, \mathbf{R}_{zf,k}\}$ denote the transmit- and receive-matrices of such a diagonalization technique which is employed for the initialization, then

$$\mathbf{\Lambda}_{k} = \mathbf{R}_{zf,k}^{H} \mathbf{H}_{k} \mathbf{T}_{zf,k} \tag{15}$$

and $\{\mathbf{T}_{zf,k}\}$ satisfies the power constraint

$$\sum_{k=1}^{K} \operatorname{tr}\left(\mathbf{T}_{zf,k}^{H} \mathbf{T}_{zf,k}\right) = P_{T}.$$

Denoting the coordinated transmit–receive processor of the kth user at the *i*th iteration by $\{\mathbf{T}_{(i),k}(\xi), \mathbf{R}'_{(i),k}, \beta_{(i)}\}\)$, the proposed algorithm is described as follows.

MT-MMSE Algorithm

Step 1) Initialization
$$(i = 0)$$
:
Evaluate Λ_k and $\mathbf{R}'_{(0),k} = \mathbf{R}_{zf,k}, \quad \forall k$
Step 2) $i = i + 1$

$$\mathbf{T}_{(i),k} = \beta_{(i)} \mathbf{T}'_{(i),k}(\xi), \quad \forall k$$

where

$$\mathbf{T}'_{(i),k}(\xi) = \left(\sum_{j=1}^{K} \mathbf{H}_{j}^{H} \mathbf{R}'_{(i-1),j} \mathbf{R}'^{H}_{(i-1),j} \mathbf{H}_{j} + \xi \mathbf{I}_{N_{T}}\right)^{-1}$$
$$\times \mathbf{H}_{k}^{H} \mathbf{R}'_{(i-1),k} \mathbf{\Lambda}_{k}$$
$$\beta_{(i)} = P_{T}^{1/2} \left[\operatorname{tr} \left(\sum_{k=1}^{K} \mathbf{T}'_{(i),k}(\xi) \mathbf{T}'^{H}_{(i),k}(\xi) \right) \right]^{-1/2}$$

and

$$\xi = P_T^{-1} \operatorname{tr} \left(\sum_{k=1}^K \sigma_k^2 \mathbf{R}_{(i-1),k}^{\prime H} \mathbf{R}_{(i-1),k}^{\prime} \right)$$

Update Receive Processing: $\mathbf{R}'_{(i),k}$

$$= \beta_{(i)} \left(\mathbf{H}_k \left(\sum_{j=1}^K \mathbf{T}_{(i),j} \mathbf{T}_{(i),j}^H \right) \mathbf{H}_k^H + \sigma_k^2 \mathbf{I}_{N_{R,k}} \right)^{-1} \\ \times \mathbf{H}_k \mathbf{T}_{(i),k} \mathbf{\Lambda}_k, \quad \forall k.$$

Step 3) Stop, if $\|\mathbf{R}'_{(i-1),k} - \mathbf{R}'_{(i),k}\|_F^2 < \epsilon \ \forall k$. MT-MMSE is given by $\mathbf{T}_k = \mathbf{T}_{(i),k}$ and $\mathbf{R}_k = \beta_{(i)}^{-1} \mathbf{R}'_{(i),k}$. Otherwise, go to Step 2). Here, $\|\cdot\|_F$ is the Frobenius norm. (In our simulation, we use $\epsilon = 0.001$.)

Some remarks of interest are as follows:

- Due to the strict quasi-convexity of the MSE in (13) with respect to ξ and the convexity of the MSE in (6a) over {T_k} for the given {R_k', β}, the total MSE is reduced by the transmit process update. Similarly, updating the receive process by (9) also reduces the total MSE. Therefore, in each iteration of the MT-MMSE algorithm, the total MSE, which is lower bounded by zero, decreases monotonically. This algorithm guarantees at least the convergence to a local minimum.
- 2) In Step 3), the receive matrix \mathbf{R}_k is given by $\beta_{(i)}^{-1} \mathbf{R}'_{(i),k}$ due to (4). Consequently, \mathbf{R}_k becomes $\mathbf{R}_k = (\mathbf{H}_k (\sum_{j=1}^K \mathbf{T}_j \mathbf{T}_j^H) \mathbf{H}_k^H + \sigma_k^2 \mathbf{I}_{N_{R,k}})^{-1} \mathbf{H}_k \mathbf{T}_k \mathbf{\Lambda}_k, \forall k$. Note that \mathbf{R}_k implicitly depends on

 β , because $\mathbf{T}_k = \beta \mathbf{T}'_k(\xi)$. In this step, convergence may be defined in some other fashion, such as in terms of the average MSE. We employed the stopping criterion based on $\mathbf{R}'_{(i),k}$, because the criterion is simple to implement and $\mathbf{R}'_{(i),k}$ is updated last in each iteration.

- 3) In contrast to the T-MMSE algorithm, which needs a numerical search to obtain the Lagrange multiplier λ, the proposed MT-MMSE algorithm does not explicitly obtain the Lagrange multiplier λ but evaluates ξ = λβ² using the closed-form expression in (14). This fact, which is attributable to the use of β in (6a), simplifies the derivation of the transmit and receive processors.
- 4) Due to the use of Λ_k for the MSE in (6a), the MT-MMSE processor behaves like the corresponding joint-channel diagonalization technique when the SNR is high. This is shown as follows. Using (15) in the expression for $\mathbf{T}'_{(1),k}(\xi)$, it can be seen that $\mathbf{T}'_{(1),k}(\xi) \simeq \mathbf{T}_{zf,k}$ when $\xi \simeq 0$, which is true for high SNRs. In this case, $\beta \simeq 1$ and $\mathbf{T}_{(1),k} \simeq \mathbf{T}_{zf,k}$. Furthermore, $\mathbf{R}'_{(1),k} \simeq \mathbf{R}_{zf,k}$. In this manner, $\{\mathbf{T}_{(i),k}, \mathbf{R}'_{(i),k}\} \simeq \{\mathbf{T}_{zf,k}, \mathbf{R}_{zf,k}\}$ for all i, can be proved.
- 5) In order to evaluate $\{\mathbf{T}_{zf,k}\}\$ and $\{\mathbf{T}_k\}\$ of the MT-MMSE algorithm, the base station needs to know both $\{\mathbf{H}_k\}\$ and $\{\sigma_k^2\}\$, which can be estimated at each mobile station and delivered to the base station (in time duplex mode, $\{\mathbf{H}_k\}\$ may be estimated directly at the base station). On the other hand, evaluating \mathbf{R}_k at the *k*th mobile station requires a knowledge of $\{\mathbf{T}_k\}\$ and Λ_k . However, in practice, Λ_k can be dropped in the implementation of the receiver filter, because in (9) the diagonal matrix Λ_k is the last factor applied by the receiver (Λ_k should be kept in the iterative procedure performed at the transmitter). Therefore, the receiver needs not know Λ_k , and only $\{\mathbf{T}_k\}\$ may be broadcast by the base station one at a time so that no CCI occurs during the training period.
- 6) When the number of transmit antennas N_T is large, it is possible to analytically show that the signal-to-interference-plus-noise ratio (SINR) of { $\mathbf{T}_{(1),k}(\xi)$, $\beta_{(1)}^{-1}\mathbf{R}_{zf,k}$ } is greater than or equal to that of { $\mathbf{T}_{zf,k}$, $\mathbf{R}_{zf,k}$ }. This is shown based on the multiuser signal model in (2). For { $\mathbf{T}_{zf,k}$, $\mathbf{R}_{zf,k}$ }, (2) is rewritten as

$$\tilde{\mathbf{x}}_{zf} = \mathbf{R}_{zf}^{H} \mathbf{H} \mathbf{T}_{zf} \mathbf{x} + \mathbf{R}_{zf}^{H} \mathbf{n}.$$
 (16)

Assuming that the rank of $\mathbf{R}_{zf,k}$ is L_k , which is true when $N_{R,k} \geq L_k$, the QR decomposition of \mathbf{R}_{zf}^H yields: $\mathbf{R}_{zf,k}^H = \mathbf{U}_{zf,k}^H \mathbf{S}_k^H$, where $\mathbf{U}_{zf,k} \in \mathbb{C}^{L_k \times L_k}$ and $\mathbf{S}_k \in \mathbb{C}^{N_{R,k} \times L_k}$. Let $\mathbf{U}_{zf} = \text{blockdiag}[\mathbf{U}_{zf,1}, \dots, \mathbf{U}_{zf,K}]$. Multiplying both sides of (16) by \mathbf{U}_{zf} produces

$$\tilde{\mathbf{z}}_{zf} = \mathbf{\underline{H}}\mathbf{\underline{H}}^{\dagger}\mathbf{z} + \mathbf{\underline{n}}$$
(17)

where $\tilde{\mathbf{z}}_{zf} = \mathbf{U}_{zf}\tilde{\mathbf{x}}_{zf}$, $\mathbf{z} = \mathbf{U}_{zf}\mathbf{R}_{zf}^{H}\mathbf{H}\mathbf{T}_{zf}\mathbf{x} = \underline{\mathbf{H}}\mathbf{T}_{zf}\mathbf{x}$, $\underline{\mathbf{H}} = \mathbf{S}^{H}\mathbf{H}$, $\mathbf{S} = \text{blockdiag}[\mathbf{S}_{1}, \dots, \mathbf{S}_{K}]$, $\underline{\mathbf{n}} = \mathbf{S}^{H}\mathbf{n}$, and $\underline{\mathbf{H}}^{\dagger} = \underline{\mathbf{H}}^{H}(\underline{\mathbf{H}}\mathbf{H}^{H})^{-1}$. Similarly, using $\{\mathbf{T}_{(1),k}, \beta_{(1)}^{-1}\mathbf{R}_{zf,k}\}$ the multiuser signal model in (2) can be rewritten as

$$\tilde{\mathbf{x}} = \mathbf{R}_{zf}^{H} \mathbf{H} \mathbf{T}_{(1)}^{\prime}(\xi) \mathbf{x} + \beta_{(1)}^{-1} \mathbf{R}_{zf}^{H} \mathbf{n}$$
(18)

and multiplying both sides of (18) by U_{zf} produces

$$\tilde{\mathbf{z}} = \beta_{(1)} \underline{\mathbf{H}} \underline{\mathbf{H}}^{H} \left(\underline{\mathbf{H}} \underline{\mathbf{H}}^{H} + P_{T}^{-1} \underline{\sigma}^{2} \mathbf{I}_{L} \right)^{-1} \mathbf{z} + \underline{\mathbf{n}}$$
(19)

where $\tilde{\mathbf{z}} = \beta_{(1)} \mathbf{U}_{zf} \tilde{\mathbf{x}}$ and $\underline{\sigma}^2 = \mathbf{E}[\underline{\mathbf{n}}^H \underline{\mathbf{n}}]$. $\underline{\mathbf{H}}^{\dagger}$ in (17) represents the ZF-based channel inversion for the input \mathbf{z} and $\beta_{(1)} \underline{\mathbf{H}}^H (\underline{\mathbf{H}} \underline{\mathbf{H}}^H + P_T^{-1} \underline{\sigma}^2 \mathbf{I}_{N_T})^{-1}$ in (19) is a regularized version of $\underline{\mathbf{H}}^{\dagger}$. Following the approach in [6], it can be shown that SINR($\tilde{\mathbf{z}}$) \geq SINR($\tilde{\mathbf{z}}_{zf}$), where SINR(\mathbf{x}) denotes the SINR of \mathbf{x} which is the sum of the signal powers divided by the sum of the interference-plus-noise powers. Then, since \mathbf{U}_{zf} is



Fig. 2. Average SINR against the number of iterations *i*. The MT-MMSE with i = 0 is identical to the Nu-SVD. The T-MMSE starts with either the identity or partial identity matrices $[I_{L_k} \ 0]$ where 0 is a L_k -by- $(N_{R,k} - L_k)$ zero matrix, and its SINRs for i = 0 are not shown. (a) $P_T/\sigma^2 = 5 \text{ dB}$. (b) $P_T/\sigma^2 = 20 \text{ dB}$.

unitary, SINR($\mathbf{U}_{zf}^{H}\tilde{\mathbf{z}}$) \geq SINR($\mathbf{U}_{zf}^{H}\tilde{\mathbf{z}}_{zf}$). This is equivalent to SINR($\beta_{(1)}\tilde{\mathbf{x}}$) \geq SINR($\tilde{\mathbf{x}}_{zf}$), because $\mathbf{U}_{zf}^{H}\tilde{\mathbf{z}} = \beta_{(1)}\tilde{\mathbf{x}}$ and $\mathbf{U}_{zf}^{H}\tilde{\mathbf{z}}_{zf} = \tilde{\mathbf{x}}_{zf}$ for $\tilde{\mathbf{x}}_{zf}$ and $\tilde{\mathbf{x}}$ in (16) and (18), respectively. Finally, due to the fact that SINR($\beta_{(1)}\tilde{\mathbf{x}}$) = SINR($\tilde{\mathbf{x}}$), the desired result SINR($\tilde{\mathbf{x}}$) \geq SINR($\tilde{\mathbf{x}}_{zf}$) is obtained.

IV. SIMULATION RESULTS

The average received SINR and the sum rate are evaluated through computer simulation. Under the assumption that the residual interferences are Gaussian and independent of the information symbol \mathbf{x}_k , the sum rate is given by $\sum_{k=1}^{K} \sum_{l=1}^{L_k} \log_2(1 + \text{SINR}_{(i)}(\tilde{x}_{k,l}))$ [5], [6], and obtained using a numerical estimate of the SINR, denoted by $\widehat{\text{SINR}}(\tilde{x}_{k,l})$, where $\tilde{x}_{k,l}$ is the *l*th element of $\tilde{\mathbf{x}}_k$. Specifically, $\widehat{\text{SINR}}(\tilde{x}_{k,l})$ is given by the average of $\widehat{\text{SINR}}(\tilde{x}_{k,l}|\mathbf{H}_k)$ over $\{\mathbf{H}_k\}$, where

$$\widehat{\operatorname{SINR}}(\tilde{x}_{k,l} | \mathbf{H}_{k}) = \frac{\left| \left[\mathbf{R}_{k}^{H} \mathbf{H}_{k} \mathbf{T}_{k} \right]_{(l,l)} \right|^{2}}{\sum_{\substack{i j = 1 \\ j = k, \text{ then } l' \neq l}} \sum_{l'=1}^{L_{j}} \left| \left[\mathbf{R}_{k}^{H} \mathbf{H}_{k} \mathbf{T}_{j} \right]_{(l,l')} \right|^{2} + \sigma_{k}^{2} L_{k}^{-1} \operatorname{tr}\left(\mathbf{R}_{k}^{H} \mathbf{R}_{k} \right)$$
(20)



Fig. 3. Sum rates versus P_T/σ^2 for various MIMO configurations. (a) $[N_T, N_{R,k}(L_k), K] = [4, 2(1), 4]$ and [6, 2(1), 6]. (b) $[N_T, N_{R,k}(L_k), K] = [4, 2(2), 2]$. (c) $[N_T, N_{R,k}(L_k), K] = [6, 2(2), 3]$.

and $[\cdot]_{(m,n)}$ denotes the (m,n)th entry of a matrix [14]. The nullspace-directed singular value decomposition (Nu-SVD) method [11], which is an efficient ZF-based channel diagonalization technique for MIMO systems with $N_T \geq L$ and $N_{R,k} \geq L_k$, is used for evaluating the initial processing matrices $\{\mathbf{T}_{zf,k}, \mathbf{R}_{zf,k}\}$ in (15). The proposed method is compared with the Nu-SVD, the T-MMSE and the MMSE method in [13] which will be referred to as the iterative transmit (IT)-MMSE. In the latter, the transmit matrix is obtained using an iterative procedure based on an MMSE criterion and per-user power constraint, and then the optimum receive matrices are derived. Its MMSE criterion is identical to that of the T-MMSE and equal power is assigned to individual users. Both SINR and sum rates of these methods are compared. In addition, the sum rate capacity of a MIMO Gaussian broadcasting channel [23] is shown for comparison. The MIMO channel is obtained by generating independent Gaussian random variables with zero mean, and the results shown below are the averages over 10 000 independent trials. The notation $[N_T, N_{R,k}(L_k), K]$ is used to represent various MIMO configurations.

Fig. 2 shows the SINR, which is given by (1/K) $\sum_{k=1}^{K} ((1/L_k) \sum_{l=1}^{L_k} SINR(\tilde{x}_{k,l}))$, against the number of iterations. Here, $SINR(\tilde{x}_{k,l})$ is obtained using (20), $\sigma_k^2 = \sigma^2$ for all k, and P_T is the total transmission power (recall that each element of \mathbf{H}_k has a unit variance). Note that all the iterative MMSE methods tend to converge after a fairly small number of iterations (when $\epsilon = 0.001$, the MT-MMSE required less than five iterations for convergence, while the others needed up to ten iterations⁴). Due to the fact that the MT-MMSE starts with the initial matrices that result from the zero-forcing based Nu-SVD, while the T-MMSE and IT-MMSE begin with the (partial) identity and zero matrices, respectively, the MT-MMSE tends to converge faster than the others [24]. This is particularly true for a high SNR where the performance gap

⁴The stopping criterion for IT-MMSE was as follows: stop if $\|\mathbf{T}_{(i-1),k} - \mathbf{T}_{(i),k}\|_{F}^{2} < \epsilon$, $\forall k$.

between the MT-MMSE and the Nu-SVD decreases. The MT-MMSE outperformed the others for both low and high SNRs.⁵ This is due to the use of Λ_k in the MMSE cost.

Fig. 3 shows the sum rate against P_T/σ^2 . The sum rates of MT-MMSE are always larger than the corresponding sum rates of T-MMSE, IT-MMSE and Nu-SVD. As expected, the performance gap between the MT-MMSE and the Nu-SVD decreases as P_T/σ^2 increases. The T-MMSE performs better than the Nu-SVD for a low SNR, but this is reversed for high SNR cases. In general, the sum rate performance of the IT-MMSE is worse than that of the T-MMSE because of the per-user power constraint of the IT-MMSE.⁶

Fig. 4 shows the rate regions for two users, where the average channel gain of user 1 (unit-variance) is 6 dB larger than that of user 2 (variance $10^{-0.6}$). The rate regions of MT-MMSE encompasses those of T-MMSE and Nu-SVD indicating that MT-MMSE can always achieve larger sum rates than the others for a given P_T/σ^2 .

V. CONCLUSION

Based on the use of an MMSE criterion which employs a weighted information symbol vector as the target and signal scaling, a regularized channel diagonalization technique is developed for a multiuser MIMO downlink. The proposed MT-MMSE method is a joint transmit–receive processor that can outperform the current ZF-based channel diagonalization and MMSE-based methods. The implementation of the MT-MMSE requires channel state information at the base station. Examining the robustness of the MT-MMSE against channel estimation errors remains as a future study.

⁵This observation also holds for the average SINR of each user, $(1/L_k) \sum_{l=1}^{L_k} \widehat{SINR}(\bar{x}_{k,l})$.

⁶The IT-MMSE can be modified to consider the total transmit power constraint [13]. In this case, our simulation results, which are not reported here, indicate that the IT-MMSE acts like the T-MMSE method.



Fig. 4. Rate regions for users 1 and 2 with $[N_T, N_{R,k}(L_k), K] = [4, 2(2), 2]$ where $P_T/\sigma^2 = 10$ and 15 dB.

APPENDIX A

Quasiconvexity of (13) with respect to ξ : *Proof:* Using the multiuser signal model in (2) and the expression for β in (12), the multiuser MSE in (13) can be rewritten as

$$E\left[\left\|\mathbf{\Lambda x} - \mathbf{R}'^{H}\mathbf{H}\left(\mathbf{H}^{H}\mathbf{R}'\mathbf{R}'^{H}\mathbf{H} + \xi\mathbf{I}_{N_{T}}\right)^{-1} \times \mathbf{H}^{H}\mathbf{R}'\mathbf{\Lambda x} - \beta^{-1}\mathbf{R}'^{H}\mathbf{n}\right\|^{2}\right].$$
 (A1)

After some algebraic manipulations, (A1) becomes

$$\operatorname{tr}(\mathbf{\Lambda}^{2}) + \operatorname{tr}\left(\left(\mathbf{H}^{H}\mathbf{R}'\mathbf{R}'^{H}\mathbf{H} + \xi\mathbf{I}_{N_{T}}\right)^{-2} \times \left(P^{-1}\operatorname{tr}\left(\sum_{k=1}^{K}\sigma_{k}^{2}\mathbf{R}_{k}'\mathbf{R}_{k}'^{H}\right) - \xi\mathbf{I}_{N_{T}}\right)\mathbf{H}^{H}\mathbf{R}'\mathbf{\Lambda}^{2}\mathbf{R}'^{H}\mathbf{H}\right).$$
(A2)

When **H**, **R**' and **A** are given, (A2) is a function of ξ and denoted by $f(\xi)$. Then, it can be shown that

$$\begin{cases} f'(\xi) < 0 & \text{and} \quad f''(\xi) > 0, \quad \text{for} \quad 0 < \xi < \xi_1 \\ f'(\xi) = 0 & \text{and} \quad f''(\xi) > 0, \quad \text{for} \quad \xi = \xi_1 \\ f'(\xi) > 0 & \text{and} \quad f''(\xi) > 0, \quad \text{for} \quad \xi_1 < \xi < \frac{3}{2}\xi_1 \\ f'(\xi) > 0 & \text{and} \quad f''(\xi) \in \mathbb{R}, \quad \text{for} \quad \xi \ge \frac{3}{2}\xi_1 \end{cases}$$

where ξ_1 is equal to the right-hand-side of (14). Consequently, $f(\xi)$ is strictly quasi-convex with respect to ξ .

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Binaural Noise Reduction Algorithms for Hearing Aids That Preserve Interaural Time Delay Cues

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Abstract—Binaural hearing aids use microphone inputs from both the left and right hearing aid to generate an output for each ear. On the other hand, a monaural hearing aid generates an output by processing only its own microphone inputs. This correspondence presents a binaural extension of a monaural multichannel noise reduction algorithm for hearing aids based on Wiener filtering. In addition to significantly suppressing the noise interference, the algorithm preserves the interaural time delay (ITD) cues of the speech component, thus allowing the user to correctly localize the speech source. Unfortunately, binaural multichannel Wiener filtering distorts the ITD cues of the noise reduction performed by the algorithm can be controlled, and traded off for the preservation of the noise ITD cues.

Index Terms—Binaural hearing, hearing aids, noise reduction, Wiener filtering.

I. INTRODUCTION

Hearing impaired persons localize sounds better without their bilateral hearing aids than with them [1]. In addition, noise reduction algorithms currently used in hearing aids are not designed to preserve localization cues [2]. The inability to correctly localize sounds puts the hearing aid user at a disadvantage. The sooner the user can localize a speech signal, the sooner the user can begin to exploit visual cues. Generally, visual cues lead to large improvements in intelligibility for hearing impaired persons [3]. Furthermore, preserving the spatial separation between the target speech and the interfering signals leads to an improvement in speech understanding [4].

It is important to explain the difference between bilateral and binaural hearing aids. A hearing impaired person wearing a monaural hearing aid on each ear is said to be using bilateral hearing aids. Each monaural hearing aid uses its own microphone inputs to generate an output for its respective ear. No information is shared between the hearing aids. In contrast, binaural hearing aids use the microphone inputs from both the left and right hearing aid to generate an output for the left and right ear. Additional information regarding binaural hearing aids can be found in [5].

What are the benefits of a binaural algorithm? First, noise reduction performance of the binaural algorithm will be better than that of the monaural algorithm. Double the number of microphones are now at the

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