

# Antenna Selection with Superposition for $4 \times 2$ DSTTD Systems

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**Abstract**—From 1820  $4 \times 4$  binary matrices, 253 binary preprocessing matrices are designed for double space-time transmit diversity (DSTTD) systems with two  $2 \times 2$  space-time block code (STBC) encoders. Among them, six matrices yielding the highest average minimum-post-processing SNR are proposed from numerical experimentation under uncorrelated channel conditions. The proposed preprocessing performs a superposition of the first and second (second and first) STBC symbols and a selection of two transmit antennas. Simulation results show that the proposed method provides 1.7 dB (1.8 dB) SNR improvement in uncorrelated (correlated) channel environment at  $10^{-3}$  bit error rate over the conventional antenna shuffling method.

**Index Terms**—Double space-time transmit diversity (DSTTD), antenna shuffling, antenna selection, superposition.

## I. INTRODUCTION

TRANSMIT antenna shuffling (or grouping) has been proposed for the double space-time transmit diversity (DSTTD) systems to enhance system performance [1]–[3]. The DSTTD symbol matrix is generated by two space-time block code (STBC) encoders [4] at the transmitter. Shuffling is then performed by multiplying a DSTTD symbol matrix with a preprocessing matrix influenced by feedback from the receiver so that the system performance can be improved. For the preprocessing matrices, the systems in [2] and [3] use six permutation matrices to shuffle four DSTTD symbols with four transmit antennas. Thus, 3-bit feedback information is required to distinguish among the shuffling matrices.

In this paper, we introduce a general binary preprocessing matrix set with 1820 matrices including six conventional antenna shuffling matrices. The number of preprocessing matrices can be reduced to 253 by exploiting certain properties. Since 253 candidates can be a burden to the system, we propose to use the six preprocessing matrices that yield the highest average minimum-post-processing signal-to-noise ratio (SNR) by a computer simulation under uncorrelated channel conditions. Interestingly, the proposed binary preprocessing performs a superposition of the first and second (second and first) symbols coded by the different STBC encoders and a selection of two transmit antennas among four. From bit-error-rate (BER) simulations, it is seen that the proposed binary processing method provides SNR gain

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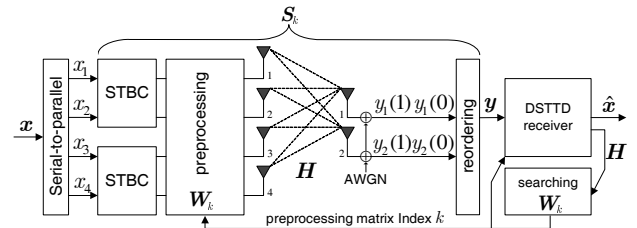


Fig. 1. Transceiver model for a preprocessed DSTTD system.

compared to the conventional antenna shuffling DSTTD system, regardless of the spatial correlation. At the  $10^{-3}$  BER, 1.7 dB and 1.8 dB SNR gains are provided in uncorrelated and correlated channel environments, respectively, with the same 3-bit feedback quantity. Furthermore, it is also seen that 0.5 dB and 1.1 dB gains in uncorrelated and correlated channel environments, respectively, can be still obtained with only 2-bit feedback information.

**Notation.** The superscripts  $T$  and  $*$  denote transposition and complex conjugate transposition for any scalar, vector or matrix, respectively;  $\mathbf{A}^{-1}$  and  $[\mathbf{A}]_{ll}$  denote matrix inversion and the  $l$ th diagonal element of  $\mathbf{A}$ , respectively; and  $\mathbf{I}_N$  and  $\mathbf{0}_N$  represent that  $N$ -dimensional identity and zero matrices, respectively.

## II. SYSTEM MODEL

The transceiver model of the DSTTD system with four transmit and two receive antennas is shown in Fig. 1. The  $2 \times 4$  MIMO channel matrix is denoted by  $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2]^T$ , where  $\mathbf{h}_n \in \mathbb{C}^{4 \times 1}$  is a vector channel for the  $n$ th receive antenna. An information symbol vector  $\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T$  yields two  $2 \times 2$  STBC matrices

$$\begin{bmatrix} \mathbf{c}_1^* \\ \mathbf{c}_2^* \end{bmatrix} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathbf{c}_3^* \\ \mathbf{c}_4^* \end{bmatrix} = \begin{bmatrix} x_3 & -x_4^* \\ x_4 & x_3^* \end{bmatrix}.$$

After multiplying the two STBC matrices (DSTTD matrix) with a preprocessing matrix  $\mathbf{W}_k \in \mathbb{C}^{4 \times 4}$  according to the feedback index  $k$ , it is transmitted by four transmit antennas for two consecutive symbol periods. Denoting  $y_n(t)$  as a received symbol, where  $t$  is a time index, the received signal can be written as

$$\begin{bmatrix} y_1(0) & y_1(1) \\ y_2(0) & y_2(1) \end{bmatrix} = \underbrace{\mathbf{H} \mathbf{W}_k}_{\text{DSTTD}} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \\ x_3 & -x_4^* \\ x_4 & x_3^* \end{bmatrix} + \mathbf{V} \quad (1)$$

where  $\mathbf{V} \in \mathbb{C}^{2 \times 2}$  is a noise matrix whose elements are additive white Gaussian noise (AWGN) with variance  $N_0$ .

The linearized form expression of the signal model in (1) is given by

$$\mathbf{y} \triangleq [y_1(0), y_1^*(1), y_2(0), y_2^*(1)]^T = \mathbf{S}_k \mathbf{x} + \mathbf{v} \quad (2)$$

where  $\mathbf{v} \in \mathbb{C}^{4 \times 1}$  is the vector representation of the AWGN matrix  $\mathbf{V}$ , and  $\mathbf{S}_k$  is the  $4 \times 4$  equivalent channel matrix:

$$\mathbf{S}_k = \begin{bmatrix} \mathbf{s}_{k,1}^* \\ \mathbf{s}_{k,2}^* \\ \mathbf{s}_{k,3}^* \\ \mathbf{s}_{k,4}^* \end{bmatrix}^* = \begin{bmatrix} \mathbf{h}_1^T \mathbf{w}_{k,1} & \mathbf{h}_1^T \mathbf{w}_{k,2} & \mathbf{h}_1^T \mathbf{w}_{k,3} & \mathbf{h}_1^T \mathbf{w}_{k,4} \\ \mathbf{h}_1^* \mathbf{w}_{k,2} & -\mathbf{h}_1^* \mathbf{w}_{k,1} & \mathbf{h}_1^* \mathbf{w}_{k,4} & -\mathbf{h}_1^* \mathbf{w}_{k,3} \\ \mathbf{h}_2^T \mathbf{w}_{k,1} & \mathbf{h}_2^T \mathbf{w}_{k,2} & \mathbf{h}_2^T \mathbf{w}_{k,3} & \mathbf{h}_2^T \mathbf{w}_{k,4} \\ \mathbf{h}_2^* \mathbf{w}_{k,2} & -\mathbf{h}_2^* \mathbf{w}_{k,1} & \mathbf{h}_2^* \mathbf{w}_{k,4} & -\mathbf{h}_2^* \mathbf{w}_{k,3} \end{bmatrix}. \quad (3)$$

Here,  $\mathbf{s}_{k,m}$  and  $\mathbf{w}_{k,m}$  are the  $m$ th column vectors of  $\mathbf{S}_k$  and  $\mathbf{W}_k$ , respectively. If the minimum mean square error (MMSE) detector, i.e.,  $\hat{\mathbf{x}} = (\mathbf{S}_k^* \mathbf{S}_k + \rho^{-1} \mathbf{I}_4)^{-1} \mathbf{S}_k^* \mathbf{y}$ , is assumed at the receiver, the post-processing SNR for  $x_l$  can be written as follows [5]:

$$\text{SNR}_{k,l} = ([\bar{\Phi}]_{ll})^{-1} - 1 \quad (4)$$

where  $\bar{\Phi} = (\rho \mathbf{S}_k^* \mathbf{S}_k + \mathbf{I}_4)^{-1}$ ;  $\rho = \frac{E_s}{N_0}$  is the system SNR; and  $E_s$  is the average transmission power per antenna. The receiver determines the  $\mathbf{W}_{k_{opt}}$  from a set  $\mathbf{W}_S = \{\mathbf{W}_1, \dots, \mathbf{W}_K\}$  by solving the min-max optimization:

$$\mathbf{W}_{k_{opt}} = \arg \max_{\mathbf{W}_k \in \mathbf{W}_S} \left( \min_l (\text{SNR}_{k,l}) \right) \quad (5)$$

and feeds back  $k_{opt}$ , represented by  $\lceil \log_2 K \rceil$  bits, to the transmitter, where  $\lceil \cdot \rceil$  is a ceil operation.

### III. PREPROCESSING MATRIX DESIGN

For the conventional DSTTD system with antenna shuffling in [2] and [3], the preprocessing matrix set was defined as

$$\mathbf{W}_{shuffling} \triangleq \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right\} \quad (6)$$

In this case, the antenna shuffling matrix is a permutation matrix, i.e., exactly one entry in each row and column is equal to 1, and all other entries are 0. This construction of the preprocessing matrix can be interpreted to mean that there are neither superposition nor repetition of  $\mathbf{c}_m$ 's and that every transmit antenna is used for every transmission. In contrast, to allow the superposition and repetition of  $\mathbf{c}_m$ 's with transmit antenna selection, any 4-by-4 matrix having the binary elements 0 and 1 can be a candidate for a preprocessing matrix  $\mathbf{W}_k$ . Meanwhile, to keep the average transmit power of the proposed system to the same level of the conventional shuffling system, we assume that the total number of elements 1 is limited to four, and we can then choose the candidates of 4-combinations from a set with 16 elements, i.e.,  $\binom{16}{4} = 1820$ . As a result, 11-bit ( $= \lceil \log_2(1820) \rceil$ ) feedback information is required. To reduce the feedback information, we examine some interesting properties. Using these properties, we can reduce the feedback information by 3 bits without performance degradation.

#### A. Properties of Preprocessing Matrices

The number of candidates for the binary preprocessing matrices can be reduced by the following properties without any performance degradation.

*Property 1.* The minimum-post-processing SNR in (5) from  $\mathbf{W}_k$  is identical to that from  $\mathbf{W}_k \mathbf{P}$ , where  $\mathbf{P}$  is the permutation matrix  $\mathbf{P}_1 = \begin{bmatrix} \mathbf{0}_2 & \mathbf{I}_2 \\ \mathbf{I}_2 & \mathbf{0}_2 \end{bmatrix}$  or  $\mathbf{P}_2 = \begin{bmatrix} \mathbf{Q} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{Q} \end{bmatrix}$ . Here,  $\mathbf{Q} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ .

*Proof:* The effective channel matrix that corresponds to  $\mathbf{W}_k \mathbf{P}$  can be represented as  $\bar{\mathbf{S}}_k = \mathbf{F} \mathbf{S}_k \mathbf{P}$ , where  $\mathbf{F} = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$ . Here,  $\mathbf{B} = \mathbf{I}_2$  when  $\mathbf{P} = \mathbf{P}_1$  or  $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$  when  $\mathbf{P} = \mathbf{P}_2$ . As a result,  $\bar{\Phi}$  in (4) is replaced by  $\bar{\Phi} = (\rho \bar{\mathbf{S}}_k^* \bar{\mathbf{S}}_k + \mathbf{I}_4)^{-1}$ . The new  $\bar{\Phi}$  can be seen to be:

$$\bar{\Phi} = (\mathbf{P}^T (\rho \mathbf{S}_k^* \mathbf{F}^T \mathbf{F} \mathbf{S}_k + \mathbf{I}_4) \mathbf{P})^{-1} = \mathbf{P}^T \bar{\Phi} \mathbf{P} \quad (7)$$

using  $\mathbf{P}^T = \mathbf{P}^{-1}$  and the orthonormality of the column vectors of  $\mathbf{F}$  and  $\mathbf{P}$ . Due to the column and row permutations with the same permutation matrix  $\mathbf{P}$ , the positions of the diagonal elements of  $\bar{\Phi}$  in (7) are permuted. Therefore, the minimum-post-processing SNR does not vary. ■

*Property 2.* When one  $\mathbf{w}_{k,m}$  is zero, the minimum-post-processing SNR in (4) from  $\mathbf{W}_k$  is identical to that from  $\mathbf{W}_k \mathbf{P}$ , where  $\mathbf{P}$  is a permutation matrix  $\mathbf{P}_3 = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{Q} \end{bmatrix}$ .

*Proof:* From the *Property 1*, without loss of generality, we assume that  $\mathbf{w}_{k,1}$  is zero. The effective channel matrix for  $\mathbf{W}_k \mathbf{P}_3$  can be represented as  $\bar{\mathbf{S}}_k = \mathbf{F} \mathbf{S}_k \mathbf{P}_3 \mathbf{G}$ , where  $\mathbf{G} = \begin{bmatrix} \mathbf{C} & \mathbf{0}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix}$  and  $\mathbf{C} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ . The new  $\bar{\Phi}$  can be then derived as

$$\bar{\Phi} = (\mathbf{G}^T \mathbf{P}_3^T (\rho \mathbf{S}_k^* \mathbf{F}^T \mathbf{F} \mathbf{S}_k + \mathbf{I}_4) \mathbf{P}_3 \mathbf{G})^{-1} = \mathbf{G}^T \mathbf{P}_3^T \bar{\Phi} \mathbf{P}_3 \mathbf{G} \quad (8)$$

using the orthonormality of the column vectors of  $\mathbf{F}$ ,  $\mathbf{G}$ , and  $\mathbf{P}_3$ . Noting that  $\mathbf{P}_3$  permutes the positions of the diagonal elements of  $\bar{\Phi}$ , and that  $\mathbf{G}$  does not change of diagonal elements of  $\mathbf{P}_3^T \bar{\Phi} \mathbf{P}_3$  in (8), the minimum-post-processing SNR does not vary. ■

Due to the above properties, i.e.,  $\mathbf{W}_k \equiv \mathbf{W}_k \mathbf{P}$  (a  $\mathbf{w}_{k,m}$  is zero for *Property 2*), we can reduce the number of preprocessing candidates from 1820 to 307 without any sacrifice in the post-processing SNR: from 1820 to  $\frac{1820-28}{2} + 28 = 924$  where 28 among 1820 is the number of matrices excluded from being divided by two according to *Property 1* since  $\mathbf{w}_{k,1} = \mathbf{w}_{k,3}$  and  $\mathbf{w}_{k,2} = \mathbf{w}_{k,4}$ ; from 924 to  $\frac{924-28}{2} + 28 = 476$  where 28 among 924 is the number of matrices excluded from being divided by two according to *Property 1* since  $\mathbf{w}_{k,1} = \mathbf{w}_{k,2}$  and  $\mathbf{w}_{k,3} = \mathbf{w}_{k,4}$ ; and from 476 to  $476 - \frac{338}{2} = 307$  where 338 among 476 is the number of matrices that  $\mathbf{W}_k = \mathbf{W}_k \mathbf{P}_3$  in *Property 2*.

#### B. Conditions of Preprocessing Matrices

For the linear detection of  $\mathbf{x}$  in (2) without ambiguity, the effective channel matrix  $\mathbf{S}_k$  in (3) should be a full rank matrix satisfying the following conditions.

*Condition 1.* Both  $\mathbf{w}_{k,1}$  and  $\mathbf{w}_{k,2}$  are not zero; otherwise,  $\mathbf{s}_{k,1}$  and  $\mathbf{s}_{k,2}$  become zero and the rank of  $\mathbf{S}_k$  is then two. Similarly, both  $\mathbf{w}_{k,3}$  and  $\mathbf{w}_{k,4}$  are not zero.

*Condition 2.*  $\mathbf{w}_{k,1} \neq \mathbf{w}_{k,3}$  or  $\mathbf{w}_{k,2} \neq \mathbf{w}_{k,4}$ ; otherwise,  $\mathbf{s}_{k,1} = \mathbf{s}_{k,3}$  and  $\mathbf{s}_{k,2} = \mathbf{s}_{k,4}$  and the rank of  $\mathbf{S}_k$  is then two.

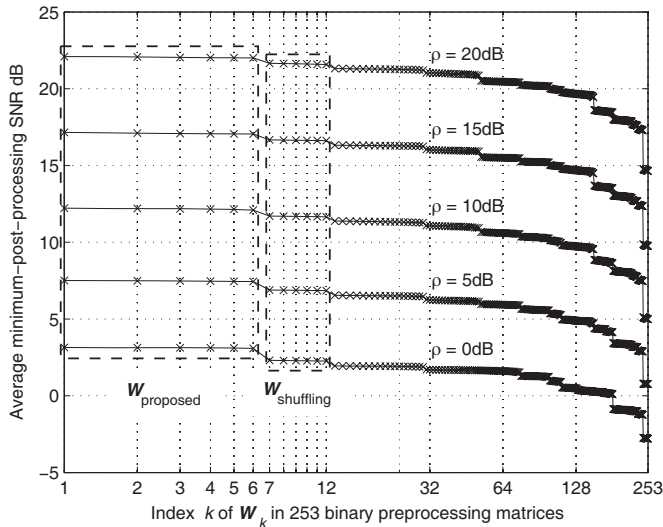
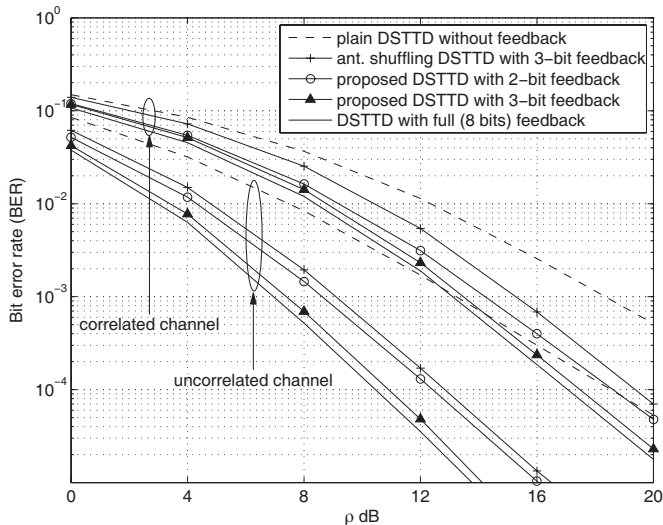


Fig. 2. Frequency of use of the designed 253 preprocessing matrices.


 Fig. 3. BER performance comparison over the various  $\rho = E_s/N_0$ .

Due to *Conditions 1* and *2*, the number of candidates for  $\mathbf{W}_k$  can be reduced by 38 and 16, respectively. Consequently, 253 binary preprocessing matrices yield the different minimum-post-processing SNR, and 8-bit ( $= \lceil \log_2(253) \rceil$ ) feedback information is required for this optimal binary preprocessing DSTTD system.

#### IV. NUMERICAL RESULTS

To reduce further feedback information and system complexity with the least performance degradation, we propose to use the six binary preprocessing matrices that yield the highest average minimum-post-processing SNR among the 253.

##### A. Proposed Six Preprocessing Matrices

The simulation environment is as follows. An MMSE detector is employed; the elements of  $\mathbf{H}$  are i.i.d complex Gaussian random variables with zero mean and unit variance; the feedback channel is error free; and the information bits are modulated by QPSK with  $E_s = 1$ .

Fig. 2 shows the average minimum-post-processing SNR, i.e.,  $E(\min_l(\text{SNR}_{k,l}))$  over 10,000 channel realizations, in descending order. From this simulation, a six binary preprocessing matrix set yielding the highest average minimum-post-processing SNR can be obtained as

$$\mathbf{W}_{\text{proposed}} = \left\{ \left[ \begin{array}{cc} \underline{0110} \\ 0000 \\ 0000 \\ 1001 \end{array} \right], \left[ \begin{array}{cc} \underline{0110} \\ 0000 \\ 0000 \\ 0000 \end{array} \right], \left[ \begin{array}{cc} \underline{0110} \\ 0000 \\ 0000 \\ 0000 \end{array} \right], \left[ \begin{array}{cc} \underline{0110} \\ 1001 \\ 0000 \\ 0000 \end{array} \right], \left[ \begin{array}{cc} \underline{0000} \\ 0000 \\ 0000 \\ 1001 \end{array} \right], \left[ \begin{array}{cc} \underline{0000} \\ 0000 \\ 0000 \\ 0000 \end{array} \right] \right\} \quad (9)$$

Using  $\mathbf{W}_k \in \mathbf{W}_{\text{proposed}}$ ,  $\mathbf{c}_1^*$  and  $\mathbf{c}_2^*$  in (1) are superposed with  $\mathbf{c}_4^*$  and  $\mathbf{c}_3^*$ , respectively, and they are then transmitted through *two selected transmit antennas* (the underlined  $m$ th row in (9) means the selected  $m$ th transmit antenna). The conventional antenna shuffling matrices in (6) yield the second largest minimum-post-processing SNR as illustrated in Fig. 2.

##### B. BER Performance Comparison

In this subsection, we compare the uncoded BER performance of the conventional antenna shuffling DSTTD system with the proposed DSTTD system. For the sake of comparison, the plain DSTTD system without feedback and the DSTTD system with full (8 bits) feedback as an upper bound are included in our simulation. The BER is obtained by averaging over the independent transmission of 50,000 frames constructed by 100 QPSK symbols for a given  $\rho$ . Channel  $\mathbf{H}$  is fixed during a frame, but it varies independently over frames. A spatial correlation is considered as  $\mathbf{R}_R^{1/2} \mathbf{H} \mathbf{R}_T^{1/2}$ , where  $\mathbf{R}_R = \mathbf{I}_2$  and  $\mathbf{R}_T = \text{toeplitz}[1, 0.9, 0.81, 0.729]^T$  [3].

As expected, the performance of the proposed DSTTD system is the closest to the bound in Fig. 3. The proposed DSTTD system provides 1.7 and 1.8 dB SNR gains in uncorrelated and correlated channels, respectively, compared to the conventional antenna shuffling DSTTD system with the same 3-bit feedback quantity at the  $10^{-3}$  BER. Moreover, using only 2-bit feedback information, the proposed DSTTD system can obtain 0.5 and 1.1 dB SNR gains in uncorrelated and correlated channels, respectively.

#### V. CONCLUSION

For the DSTTS systems, six processing matrices yielding the highest average minimum-post-processing SNR were proposed and improved BER performance was achieved compared to the conventional antenna shuffling method.

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