

A Two-Step Approach to Power Allocation for OFDM Signals Over Two-Way Amplify-and-Forward Relay

Yong-Up Jang, Eui-Rim Jeong, and Yong H. Lee

Abstract—A two-way relay channel (TWRC) in which two terminals T_1 and T_2 exchange orthogonal frequency division multiplexing (OFDM) signals with the help of an amplify-and-forward (AF) relay T_3 is considered here, and an efficient technique for allocating powers to N parallel tones of OFDM is developed. A sum rate maximization problem is formulated by replacing the individual power constraints of the conventional sum rate maximization problem, which limit the power of each terminal, with the total power constraint limiting the sum of powers of all terminals. The maximization problem with the total power constraint yields a more efficient power allocation policy than the conventional problem with individual power constraints. It is shown that the closed-form solution of the maximization problem under the total power constraint can be obtained for a single-tone system ($N = 1$). Based on this result, a two-step suboptimal approach is proposed in which the power is optimally assigned to each tone first, and then at each tone the assigned power is distributed to the three terminals. The proposed method is shown to assign 50% of the total power to relay T_3 irrespective of the channels. It is demonstrated that the proposed method is considerably simpler to implement than the conventional dual-decomposition method (DDM), yet the performance of the former is almost identical to that of the latter.

Index Terms—Amplify-and-forward, dual decomposition, orthogonal frequency-division multiplexing (OFDM), power allocation, two-way relay.

I. INTRODUCTION

The relay channel and the two-way channel are basic channels of network information theory [1]. The relay channel consists of one sender, one receiver, and one or more intermediate relays to facilitate the communication between the sender and the receiver. In a two-way channel, two terminals simultaneously send their signals to each other over a shared channel. Combining these channels, the two-way relay channel (TWRC) consisting of three terminals T_1 , T_2 , and T_3 that operate in half-duplex mode is proposed in a patent [2] (Fig. 1). In the TWRC, messages of T_1 and T_2 are exchanged with the help of a relay T_3 , following a protocol with multiple-access (MA) and broadcast (BC) phases. In the MA phase, T_1 and T_2 simultaneously transmit their signals to T_3 over a shared channel, and in the BC phase, T_3 broadcasts the received signals after some processing. Due to its bidirectional nature, the TWRC can increase the sum rate of the network, as compared with the half-duplex network that allows only unidirectional traffic [3]–[6]. Achievable rates of TWRCs are derived for some relaying protocols including amplify-and-forward (AF) relaying and decode-and-forward (DF) relaying with superposition coding in [3].

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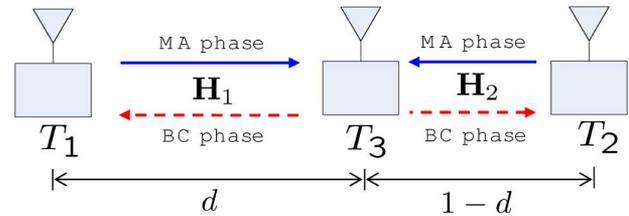


Fig. 1. The half-duplex two-way relay channel, where terminals T_1 and T_2 exchange their messages via a relay T_3 . The distance between T_1 and T_2 is normalized to 1 and the distance between T_1 and T_3 is denoted as d .

The capacity region is analyzed for DF relaying in [4] and [5], and throughput of a TWRC incorporating automatic-repeat-request (ARQ) is examined in [6]. Basic TWRCs are extended for multiple relays in [3] and [7], and for terminals with multiple antennas in [8].

In [9], a TWRC exchanging orthogonal frequency division multiplexing (OFDM) signals via an AF relay is considered and a method for optimally allocating powers to N parallel tones of an OFDM system is proposed. Specifically, the problem for maximizing the sum rate under an individual power constraint at each terminal is formulated and solved by a dual-decomposition method (DDM) [10]–[12]. In this correspondence, the problem of [9] is slightly modified, and a simpler and more efficient power allocation technique for TWRCs with OFDM signaling is derived under the assumption of channel reciprocity.¹ The problem in the present study is identical to the optimization problem of [9], with the exceptions that individual power constraints are replaced with the total power constraint that limits the sum of the powers of all three terminals and that the channels are assumed to be reciprocal. It is shown that a closed-form solution for this optimization problem can be obtained for the single-tone system ($N = 1$). Using this result, a two-step suboptimal method is developed which first performs an optimal power allocation across the tones and then determines optimal powers of the three terminals at each tone. It is observed that the proposed method always assigns 50% of the total power to relay T_3 irrespective of the channel characteristics and allocates the rest to terminals T_1 and T_2 so that the received SNRs at these terminals become identical. Due to this power allocation policy, the proposed scheme can perform better than the method in [9], where the sum of the powers of all tones at each terminal is pre-determined via individual power constraints. Furthermore, the proposed two-step optimization is considerably simpler to implement than the DDM used in [9]: The former requires only a one-dimensional search, while the latter needs a three-dimensional search over the space of allowable powers. The proposed method is also compared with the DDM for solving the sum rate maximization problem under the total power constraint. The results indicate that the two methods exhibit almost identical performance, while the former requires much less computation.

The organization of this correspondence is as follows. Section II shows the OFDM based two-way AF relaying system model. The problem formulation and asymptotically optimal solution are given in Section III. In Section IV, the proposed two-step power allocation is presented. The achievable rates are examined through computer simulation in Section V. Finally, Section VI concludes this correspondence.

¹In [9], it is also shown that the sum rate can be increased by permuting the tones at relay T_3 , and successive use of power allocation followed by tone permutation is suggested. Unfortunately, due to the assumption of channel reciprocity, the proposed power allocation is not compatible with tone permutation, whereas the conventional DDM-based power allocations are compatible. In this correspondence, we focus on power allocation and the use of tone permutation is briefly discussed in conjunction with the simulation.

The following notations are used throughout this correspondence: \mathbb{C}^N and \mathbb{R}^N denote a N -by-1 vector with all complex and real elements, respectively. $E[\cdot]$ represents the expectation. $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ denotes that \mathbf{x} is a zero-mean circularly symmetric complex Gaussian vector with covariance matrix \mathbf{I}_N .

II. SYSTEM MODEL

Suppose that terminals T_1 and T_2 exchange messages through terminal T_3 which acts as an AF relay (Fig. 1). It is assumed that there is no direct path between T_1 and T_2 , and that all terminals operate in half-duplex mode. Let $\mathbf{x}_k \triangleq [x_k(1), \dots, x_k(N)]^T$, $k \in \{1, 2\}$, denote the OFDM symbol to be transmitted by terminal T_k , where $\mathbf{x}_k \in \mathbb{C}^N$, N is the number of subcarriers, and $E[|x_k(n)|^2] = 1$. In a time slot for the MA phase, T_1 and T_2 transmit $\sqrt{\mathbf{P}_1}\mathbf{x}_1$ and $\sqrt{\mathbf{P}_2}\mathbf{x}_2$, respectively, to relay T_3 where $\mathbf{P}_k \triangleq \text{diag}\{p_k(1), \dots, p_k(N)\}$ is an N -by- N diagonal matrix that controls the transmission power and $\sqrt{\mathbf{P}_k} \triangleq \text{diag}\{\sqrt{p_k(1)}, \dots, \sqrt{p_k(N)}\}$. The relay receives

$$\mathbf{y}_3 = \mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \mathbf{w}_3 \quad (1)$$

where $\mathbf{H}_k \triangleq \text{diag}\{h_k(1), \dots, h_k(N)\}$ is an N -by- N diagonal matrix representing the channel between terminal T_k and relay T_3 ($\mathbf{H}_k \in \mathbb{C}^{N \times N}$) and $\mathbf{w}_3 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is additive white Gaussian noise at the relay. The relay amplifies each subcarrier of the received signal by multiplying \mathbf{y}_3 with $\boldsymbol{\Gamma} \triangleq \text{diag}\{\gamma(1), \dots, \gamma(N)\}$ where $\gamma(n) = \sqrt{p_3(n)/(p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + 1)}$ and $p_3(n)$ is the transmission power of the relay for the n th subcarrier. Then, in the next time slot the relay broadcasts the signal to both destinations (BC phase). Assuming the channel reciprocity for \mathbf{H}_1 and \mathbf{H}_2 , the received signals at terminals T_1 and T_2 are written as

$$\mathbf{y}_1 = \boldsymbol{\Gamma}\mathbf{H}_1\mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \boldsymbol{\Gamma}\mathbf{H}_1\mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \boldsymbol{\Gamma}\mathbf{H}_1\mathbf{w}_3 + \mathbf{w}_1 \quad (2)$$

and

$$\mathbf{y}_2 = \boldsymbol{\Gamma}\mathbf{H}_2\mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \boldsymbol{\Gamma}\mathbf{H}_2\mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \boldsymbol{\Gamma}\mathbf{H}_2\mathbf{w}_3 + \mathbf{w}_2 \quad (3)$$

where $\mathbf{w}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ for $k = 1$ and 2 is additive white Gaussian noise at terminal T_k . The first term in the right-hand-side (RHS) of (2) and the second term in the RHS of (3) are self-interferences that can be suppressed if \mathbf{H}_k is known at the terminal T_k . After suppressing the self-interferences, the received signals are rewritten as

$$\mathbf{y}'_1 = \boldsymbol{\Gamma}\mathbf{H}_1\mathbf{H}_2\sqrt{\mathbf{P}_2}\mathbf{x}_2 + \boldsymbol{\Gamma}\mathbf{H}_1\mathbf{w}_3 + \mathbf{w}_1 \quad (4)$$

and

$$\mathbf{y}'_2 = \boldsymbol{\Gamma}\mathbf{H}_2\mathbf{H}_1\sqrt{\mathbf{P}_1}\mathbf{x}_1 + \boldsymbol{\Gamma}\mathbf{H}_2\mathbf{w}_3 + \mathbf{w}_2. \quad (5)$$

Throughout the correspondence, it is assumed that all terminals T_1 , T_2 , and T_3 have the knowledge of both channels \mathbf{H}_1 and \mathbf{H}_2 , and that perfect self-interference suppression is possible.

III. PROBLEM FORMULATION AND LAGRANGE DUAL SOLUTION

The objective here is to optimize the transmission powers $\{p_k(n) | 1 \leq k \leq 3, 1 \leq n \leq N\}$ under the total power constraint

$$\sum_{k=1}^3 \sum_{n=1}^N p_k(n) \leq P_T \quad (6)$$

so that the communication rates between T_1 and T_2 are maximized. Let R_1 and R_2 denote the achievable rates for the $T_2 \rightarrow T_3 \rightarrow T_1$

and $T_1 \rightarrow T_3 \rightarrow T_2$ directions, respectively. Then, from (4) and (5), $\{R_k | k = 1, 2\}$ are given by

$$R_k = \frac{1}{2} \sum_{n=1}^N \log_2(1 + \text{SNR}_k(n)) \quad (7)$$

where

$$\text{SNR}_1(n) = \frac{p_2(n)p_3(n)|h_1(n)|^2|h_2(n)|^2}{p_3(n)|h_1(n)|^2 + p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + 1} \quad (8)$$

and

$$\text{SNR}_2(n) = \frac{p_1(n)p_3(n)|h_1(n)|^2|h_2(n)|^2}{p_3(n)|h_2(n)|^2 + p_1(n)|h_1(n)|^2 + p_2(n)|h_2(n)|^2 + 1}. \quad (9)$$

The optimization problem maximizing the sum rate is described as follows:

$$\begin{aligned} & \text{maximize } R_1 + R_2 \\ & \text{subject to } p_k(n), \forall k, n \\ & \text{subject to (6), and } p_k(n) \geq 0 \text{ for all } n \text{ and } k. \end{aligned} \quad (10)$$

This problem becomes identical to the optimization problem in [9] if the total power constraint in (6) is replaced with the following individual power constraints:

$$\sum_{n=1}^N p_k(n) \leq P_k \text{ for } k = 1, 2, 3 \quad (11)$$

where P_k is the maximum power for terminal T_k . As a result, the problem in (10) can be solved by slightly modifying the optimization process in [9], which follows the Lagrange dual method, as in [11] and [13]. The Lagrange dual problem of (10) is defined as

$$\text{minimize}_{\boldsymbol{\lambda} \geq 0} g(\boldsymbol{\lambda}) \quad (12)$$

where $g(\boldsymbol{\lambda})$ is the Lagrange dual function given by

$$\begin{aligned} g(\boldsymbol{\lambda}) = \max_{p_k(n), \forall k, n} & \left[R_1 + R_2 - \lambda_0 \left(\sum_{k=1}^3 \sum_{n=1}^N p_k(n) - P_T \right) \right. \\ & \left. + \sum_{k=1}^3 \sum_{n=1}^N \lambda_{(k-1)N+n} p_k(n) \right] \end{aligned} \quad (13)$$

and $\boldsymbol{\lambda} \triangleq [\lambda_0, \lambda_1, \dots, \lambda_{3N}]$ is the Lagrange multiplier vector ($\boldsymbol{\lambda} \in \mathbb{R}^{3N+1}$). Following the approach in [11], it can be shown that the duality gap, which is the difference between the optimal solution of (10) and that of the dual problem (12), approaches zero as $N \rightarrow \infty$. The dual function can be rewritten as

$$\begin{aligned} g(\boldsymbol{\lambda}) = \sum_{n=1}^N \max_{p_k(n), \forall k} & \left[r_1(n) + r_2(n) - \lambda_0 \sum_{k=1}^3 p_k(n) \right. \\ & \left. + \sum_{k=1}^3 \lambda_{(k-1)N+n} p_k(n) \right] + \lambda_0 P_T \end{aligned} \quad (14)$$

where $r_k(n) = (1/2)\log_2(1 + \text{SNR}_k(n))$, and thus $g(\boldsymbol{\lambda})$ can be obtained by solving N per-subcarrier maximization problems. The solution to each per-subcarrier problem in (14) is obtained through an exhaustive search over the three-dimensional space of $(p_1(n), p_2(n), p_3(n))$, assuming that each $p_k(n)$ takes discrete

values and that continuous bit loading is possible [10],² because the per-subcarrier problems are nonconvex and finding their closed-form solutions is difficult. Such an exhaustive search has $O(B^3)$ computational complexity where B is the number of power levels that can be taken by $p_k(n)$. Therefore, evaluation of the dual function in (14) has $O(NB^3)$ complexity.

The Lagrange dual function is known to be convex, and $g(\boldsymbol{\lambda})$ in (12) can be minimized with respect to $\boldsymbol{\lambda}$ by a gradient-type search such as the subgradient method [11]. If $g(\boldsymbol{\lambda})$ is minimized after I iterations of a gradient-type search, the overall computational load for solving the Lagrange dual problem (12) is $O(INB^3)$. The optimization process via (12)–(14) is referred to as the DDM.

IV. PROPOSED APPROACH TO POWER ALLOCATION

The proposed method for solving (10) is based on an observation that closed-form solutions for per-subcarrier sum rate maximization problems can be derived if an equality power constraint is imposed on each subcarrier. The per-subcarrier optimization problem is stated as follows:

$$\underset{p_k(n), \forall k}{\text{maximize}} \quad r_1(n) + r_2(n) \quad (15a)$$

$$\text{subject to} \quad \sum_{k=1}^3 p_k(n) = P_T(n), \text{ and } p_k(n) \geq 0 \text{ for all } k \quad (15b)$$

where $P_T(n)$ denotes the total power that can be allocated to the n th subcarriers of the three terminals. In contrast to the per-subcarrier problem in (14), which is an unconstrained optimization problem, the problem in (15) has an equality constraint that limits $\{p_k(n)\}$ to those values over the plane defined by (15b). Due to this equality constraint, (15) can be solved to find the optimal $\{p_k(n)\}$, denoted by $\{p_k^o(n)\}$, in terms of $P_T(n)$. The solution to (15) is presented below.

Observation 1: The optimal powers $\{p_k^o(n)\}$ are given by (16) and (17), shown at the bottom of the page, and

$$p_3^o(n) = \frac{P_T(n)}{2}. \quad (18)$$

The proof for this observation is outlined in the Appendix. It is interesting to note that in (18) 50% of the per-subcarrier total power, $P_T(n)$, is allocated to the relay irrespective of the channel gains. The optimal powers, $p_1^o(n)$ and $p_2^o(n)$, for terminals T_1 and T_2 are determined so that the received SNRs at these terminals become identical. This is stated as follows.

²Exhaustive search for discrete bit loading is also considered in [10].

Observation 2: Let $\text{SNR}_k^o(n)$, $k = 1$ and 2 , denote the received SNR values in (8) and (9) when $p_k(n) = p_k^o(n)$ for all k . Then

$$\text{SNR}_1^o(n) = \text{SNR}_2^o(n). \quad (19)$$

This observation is a direct consequence of Observation 1. The optimal powers $\{p_k^o(n)\}$ are functions of $P_T(n)$ and can be evaluated once $P_T(n)$ is given. It is natural to then ask how to determine $\{P_T(n), n = 1, \dots, N\}$. To answer this question, $r_k^o(n) \triangleq (1/2) \log_2(1 + \text{SNR}_k^o(n))$ and $R_k^o = \sum_{n=1}^N r_k^o(n)$ are defined. Then, due to (19), $r_1^o(n) = r_2^o(n) \triangleq r^o(n)$ and $R_1^o = R_2^o \triangleq R^o$. The optimization corresponding to the sum rate maximization problem in (10) is written as

$$\underset{P_T(n), \forall n}{\text{maximize}} \quad 2R^o \quad (20a)$$

$$\text{subject to} \quad \sum_{n=1}^N P_T(n) \leq P_T, \text{ and } P_T(n) \geq 0 \text{ for all } n. \quad (20b)$$

Observation 3: The problem in (20) is a convex optimization problem.

Proof: Since $\text{SNR}_k^o(n)$ is an increasing function of $P_T(n)$, then $r^o(n)$ is concave with respect to $P_T(n)$, and $R^o = \sum_{n=1}^N r^o(n)$ is also concave. ■

Because it is difficult to directly solve (20) even for moderate N , the optimization problem in (20) is solved using the Lagrange dual method. In this case, the duality gap is always zero because the problem in (20) is a convex optimization problem [13]. The Lagrange dual problem of (20) is defined as

$$\underset{\boldsymbol{\lambda} \geq 0}{\text{minimize}} \quad g'(\boldsymbol{\lambda}) \quad (21)$$

where $g'(\boldsymbol{\lambda})$ is the Lagrange dual function given by

$$g'(\boldsymbol{\lambda}) = \sum_{n=1}^N \max_{P_T(n)} [2r^o(n) - \lambda_0 P_T(n) + \lambda_n P_T(n)] + \lambda_0 P_T \quad (22)$$

and $\boldsymbol{\lambda} \triangleq [\lambda_0, \lambda_1, \dots, \lambda_N]$. Solving the per-subcarrier problems in (22) is not trivial and requires a numerical search. In this case, unlike (13), each per-subcarrier problem is a convex optimization problem and efficient search techniques such as the Golden Section search [14] can be employed. To minimize $g'(\boldsymbol{\lambda})$ with respect to $\boldsymbol{\lambda}$ in (21), a gradient-type search such as the subgradient method is employed. The computational load for solving the Lagrange dual problem in (21) is $O(I'NB')$, where I' is the number of iterations required for minimizing $g'(\boldsymbol{\lambda})$ by a gradient-type search and B' is that for solving the per-subcarrier problem. When the Golden Section search is used, B' is proportional to $\log(B_T)$ [14] where B_T is the number of power levels that can be

$$p_1^o(n) = \frac{P_T(n) (1 + |h_2(n)|^2 P_T(n))}{2 \left(1 + |h_2(n)|^2 P_T(n) + \sqrt{(1 + |h_1(n)|^2 P_T(n)) (1 + |h_2(n)|^2 P_T(n))} \right)} \quad (16)$$

$$p_2^o(n) = \frac{P_T(n) \sqrt{(1 + |h_1(n)|^2 P_T(n)) (1 + |h_2(n)|^2 P_T(n))}}{2 \left(1 + |h_2(n)|^2 P_T(n) + \sqrt{(1 + |h_1(n)|^2 P_T(n)) (1 + |h_2(n)|^2 P_T(n))} \right)}, \quad (17)$$

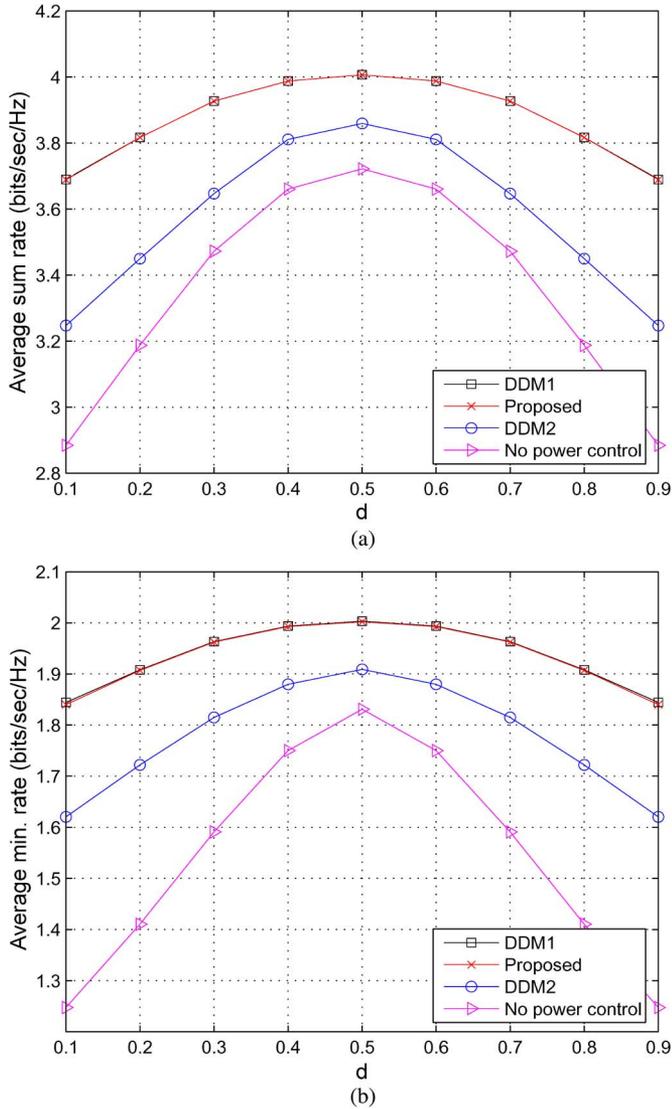


Fig. 2. Average rates against d where d is the distance between terminal T_1 and relay T_3 . (a) Average sum rate ($R_1 + R_2$). (b) Average minimum rate ($\min(R_1, R_2)$).

taken by $P_T(n)$. (A natural choice for B_T is $B_T = 3B$ where B is the number of power levels for $p_k(n)$.) On the other hand, I' is close to I because both I' and I represent the number of iterations for a gradient-type search. Obviously, the computational load for solving (21), $O(I'NB')$, is considerably less than that for solving (12), $O(INB^3)$.

In summary, the proposed method decomposes the sum rate maximization problem in (10) into two optimizations in (15) and (20). Because of this decomposition, the proposed power allocation becomes a two-step process providing a suboptimal solution to the problem in (10). The optimization in (15) allocates $P_T/2$ to the relay and balances the transmission rates R_1 and R_2 such that $R_1 = R_2$, while the optimization in (20) performs power allocation in the frequency domain. The powers $\{p_k(n)\}$ are determined through a two step process: first, $\{P_T(n)\}$ are obtained via (20); second, $\{p_k(n)\}$ are evaluated using (16)–(18).

Before concluding this section, the problem for maximizing $\min(R_1, R_2)$ that is given by (10) after replacing $(R_1 + R_2)$ with $\min(R_1, R_2)$ is briefly considered. It can be seen that the solution

to the max-min problem is identical to the sum rate maximization problem in (10), if the proposed method is used for solving both the problems. This is because the max-min problem can be converted into the sum rate maximization problem with an additional constraint, $R_1 = R_2$ [15], and the proposed method balances the rate ($R_1 = R_2$) when maximizing the sum rate $R_1 + R_2$.

V. SIMULATION RESULTS

The average rates of the proposed two-step method are examined through computer simulation. For comparison, the average rates of two dual-decomposition methods (DDMs) which are referred to as DDM1 and DDM2 are also obtained. DDM1 maximizes either $(R_1 + R_2)$ or $\min(R_1, R_2)$ under the total power constraint in (6) (see (10)), and DDM2 maximizes the same under the individual power constraint in (11). As in [9], DDM2 assumes that $\{P_k\}$ in (11) are identical to each other. In addition, the average rates are obtained for the case when powers are equally rather than optimally allocated to all subcarriers of the three terminals—this case will be referred to as “no power control.”

A linear one-dimensional network geometry is assumed, as shown in Fig. 1, where the distance between terminals T_1 and T_2 is normalized to one and the distance between terminal T_1 and relay T_3 is denoted by “ d .” The time-domain channel between terminals T_k and relay T_3 is a frequency-selective channel with L taps and is denoted by $\{h_{k,l} | 1 \leq l \leq L\}$, where $k = 1$ and 2. It is assumed that $h_{1,l} \sim \mathcal{CN}(0, 1/(Ld^\gamma))$ and $h_{2,l} \sim \mathcal{CN}(0, 1/(L(1-d)^\gamma))$ where γ is the path loss exponent. In the simulation, we set $L = 8$, $\gamma = 3$, $N = 64$, $P_T = 30N$ and $P_k = 10N$. During the numerical searches, discrete powers of $p_k(n)$ are taken from $\{0, 0.5, 1, 1.5, \dots, 30\}$ and those of $P_T(n)$ are from $\{0, 0.5, 1, 1.5, \dots, 90\}$.

Fig. 2 shows the average rates, representing $R_1 + R_2$ and $\min(R_1, R_2)$, against d . As expected, DDM1 performs the best and “no power control” performs the worst. The performance of the proposed scheme is almost identical to that of DDM1. Due to the total power constraint, DDM1 and the proposed scheme behave better than DDM2. The performance gain increases as either d or $1 - d$ decreases: the gain is about 5% when $d = 1 - d = 0.5$ and increases to 12% when either $d = 0.1$ or $1 - d = 0.1$. This happens because when $d = 1 - d = 0.5$, SNRs for the $T_2 \rightarrow T_3 \rightarrow T_1$ and $T_1 \rightarrow T_3 \rightarrow T_2$ directions are balanced, and thus DDM2 achieves its maximum average rates; such SNR balancing is violated more and more as either d or $1 - d$ decreases. In summary, the proposed scheme performs like DDM1 and outperforms DDM2, while requiring considerably less computational complexity.³

VI. CONCLUSION

An efficient two-step power allocation method was developed for a TWRC exchanging OFDM signals via an AF relay by maximizing the sum rate under the total power constraint and the assumption of channel reciprocity. It was shown that the proposed method needs a one-dimensional search over the space of allowable powers and is much simpler to implement than the conventional DDM requiring a three-dimensional search, yet the two methods exhibit comparable performances. Extending the proposed two-step method for AF TWRCs with various relay gains (Γ) [16] remains for future work.

³Some additional simulation results, which are not reported here, indicate that tone permutation by the greedy algorithm in [9] can provide some additional gain to DDMs. The additional gain was less than 0.1 bits/sec/Hz and 0.05 bits/sec/Hz for $(R_1 + R_2)$ and $\min(R_1, R_2)$, respectively, and DDM2 with tone permutation still exhibited worse performance than the proposed method performing only power allocation.

$$\text{SNR}(n) = \frac{|h_1(n)|^2 |h_2(n)|^2 P_T^2(n)}{2 + |h_1(n)|^2 P_T(n) + |h_2(n)|^2 P_T(n) + 2\sqrt{(1 + |h_1(n)|^2 P_T(n))(1 + |h_2(n)|^2 P_T(n))}}$$

APPENDIX

A. Proof for Observation 1

The Lagrangian function of (15) is given by

$$L(p_1(n), p_2(n), p_3(n), \lambda_0, \dots, \lambda_3) = r_1(n) + r_2(n) - \lambda_0 \left(\sum_{k=1}^3 p_k(n) - P_T(n) \right) + \sum_{k=1}^3 \lambda_k p_k(n) \quad (23)$$

where λ_k , $k = 0, 1, 2, 3$, are Lagrangian multipliers. Then, the Karush–Khun–Tucker (KKT) conditions are written as

$$\frac{\partial r_1(n)}{\partial p_k(n)} + \frac{\partial r_2(n)}{\partial p_k(n)} - \lambda_0 + \lambda_k = 0, \quad \forall k \quad (24a)$$

$$\lambda_k p_k(n) = 0, \text{ and } \lambda_k \geq 0, \quad \forall k \quad (24b)$$

$$\sum_{k=1}^3 p_k(n) = P_T(n), \text{ and } p_k(n) \geq 0, \quad \forall k. \quad (24c)$$

Note that $r_1(n) + r_2(n) = 0$ if $p_3(n) = 0$. Since the objective is to maximize $r_1(n) + r_2(n)$, the cases associated with $p_3(n) = 0$ are avoided in favor of considering those with $p_3(n) > 0$. There are three cases that satisfy (24b), (24c), and $p_3(n) > 0$:

Case 1) $p_k(n) > 0, \forall k, \lambda_1 = \lambda_2 = \lambda_3 = 0$, and $\sum_{k=1}^3 p_k(n) = P_T(n)$;

Case 2) $p_1(n) = 0, p_2(n) > 0, p_3(n) > 0, \lambda_1 > 0, \lambda_2 = \lambda_3 = 0$, and $p_2(n) + p_3(n) = P_T(n)$;

Case 3) $p_2(n) = 0, p_1(n) > 0, p_3(n) > 0, \lambda_2 > 0, \lambda_1 = \lambda_3 = 0$, and $p_1(n) + p_3(n) = P_T(n)$.

The solution that satisfies both (24a) and the conditions in case i), $i \in \{1, 2, 3\}$, is denoted as $\mathbf{p}^{(i)}$. After some calculation, it can be shown that $\mathbf{p}^{(1)} = [p_1^o(n), p_2^o(n), p_3^o(n)]$, $\mathbf{p}^{(2)} = [0, 2p_2^o(n), 2p_1^o(n)]$, and $\mathbf{p}^{(3)} = [2p_1^o(n), 0, 2p_2^o(n)]$ where $\{p_k^o(n)\}$ are defined in (16)–(18). The sum rates corresponding to $\{\mathbf{p}^{(i)}\}$, which are denoted as $\{r_1^{(i)}(n) + r_2^{(i)}(n)\}$, are given by $r_1^{(1)}(n) + r_2^{(1)}(n) = \log_2(1 + \text{SNR}(n)/2)$, and $r_1^{(2)}(n) + r_2^{(2)}(n) = r_1^{(3)}(n) + r_2^{(3)}(n) = \log_2(1 + \text{SNR}(n))/2$, where $\text{SNR}(n)$ is shown at the top of this page. Due to the fact that $\log(1 + x)/2 \leq \log(1 + x/2)$ for $x \geq 0$, $r_1^{(1)}(n) + r_2^{(1)}(n)$ is larger than the others and $\mathbf{p}^{(1)}$ is the optimal solution.

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