Performance Analysis of User Selection for Multiuser Two-Way Amplify-and-Forward Relay

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***Abstract*—This paper considers a multiuser two-way relay channel (MU-TWRC) in which a base-station and a selected mobile-station (MS) among K number of MSs exchange messages during two time slots with the help of a half-duplex amplify-and-forward relay. We analyze the performances of a MU-TWRC. The closed-form expressions of the lower bound of the average rates are derived. The asymptotic expressions of the average rates are also analyzed in the limit of large K. Analysis and simulation results indicate that the former matches well to the latter, and the performance of a MU-TWRC can increase as K increases or be bounded to some constant according to the channel conditions.**

***Index Terms*—Two-way relay, amplify-and-forward, multiuser scheduling.**

I. Introduction

THE two-way relay channel (TWRC), which is a combination of the relay channel and the two-way channel introduced in network information theory [1], has been proposed in a patent [2]. In the TWRC, two nodes exchange messages during two time slots with the help of a relay. In first time slot, two nodes simultaneously transmit their signals to a relay, and in second time slot, a relay broadcasts the received signal after some processing such as amplify-and-forward (AF), decode-and-forward, or compress-and-forward. Due to its bidirectional nature, the TWRC enhances the sum rate compared to a one-way relay channel, which allows only unidirectional communication [3], [4]. The basic TWRC has been extended for a relay selection in multiple relays [5], a beamformer design at a relay equipping multiple antennas [6], the multiple node pairs with a relay [7], and the power allocation in orthogonal frequency division multiplexing [8].

In this paper, we extend the basic TWRC to the multiuser TWRC (MU-TWRC) in a cellular network where a base-station (BS) and a selected mobile-station (MS) among K number of MSs exchange messages via a half-duplex AF relay. We derive the closed-form expressions of the lower bound and the asymptotic expressions of the average rates when K is sufficiently large. Analysis and simulation results demonstrate that the derived closed-form expressions match well with simulation results over the various channel conditions. Furthermore, the asymptotic results indicate that the performance of a MU-TWRC either increases by $\log_2(1+\alpha\ln(K))$ where α is determined by the direction of traffic or is bounded by a constant.

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The following notations are used throughput this paper: $\mathbb{C}$ denotes a complex scalar. $\mathbb{E}_{x,y}[\cdot]$ represents the expectation with respect to the random variables x and y. $x \sim \mathcal{CN}(0,y)$ denotes that x is a zero-mean circularly symmetric complex Gaussian random variable with variance y. $\bar{E}_1(x) \triangleq e^x E_1(x)$ and $E_1(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}dt$ is the exponential integral function.

II. System Model and Rates

The system configuration considered in this paper consists of a BS, a fixed relay, and K number of MSs. It is assumed that the BS, relay, and all MSs operate in half-duplex mode and that there are no direct paths between the BS and K number of MSs. If the MU scheduling selects an MS (e.g., the k-th MS), MS_k among K number of MSs is performed. Then, the BS and MS_k exchange their messages with the help of an AF relay. Let x_1 and x_2 denote the transmitted message by the BS and MS_k, respectively, where $x_l \in \mathbb{C}$ and $E[|x_l|^2] = 1$ for $l \in \{1,2\}$. In first time slot, the BS and MS_k transmit signals $\sqrt{p_1}x_1$ and $\sqrt{p_2}x_2$, respectively, where p_1 and p_2 are the transmitting power of the BS and MS_k, respectively. Then, the received signal at the relay is represented as

$$y = h_1\sqrt{p_1}x_1 + h_{2,k}\sqrt{p_2}x_2 + w \quad (1)$$

where $h_1 \sim \mathcal{CN}(0,\gamma_1)$ and $h_{2,k} \sim \mathcal{CN}(0,\gamma_2)$, $k \in \{1,\cdots,K\}$, denote the channel between BS and relay, and relay and MS_k, respectively, and $w \sim \mathcal{CN}(0,1)$ is additive white Gaussian noise (AWGN) at the relay. In second time slot, the relay amplifies the received signal by multiplying y of (1) with β where $\beta = \sqrt{\frac{p_3}{|h_1|^2p_1+|h_{2,k}|^2p_2+1}}$ and broadcasts it. Assuming the perfect channel reciprocity[1], the BS and MS_k receive the signals

$$y_{1,k} = \beta h_1 h_1\sqrt{p_1}x_1 + \beta h_1 h_{2,k}\sqrt{p_2}x_2 + \beta h_1 w + w_1 \quad (2)$$

and

$$y_{2,k} = \beta h_{2,k} h_1\sqrt{p_1}x_1 + \beta h_{2,k} h_{2,k}\sqrt{p_2}x_2 + \beta h_{2,k} w + w_2, \quad (3)$$

respectively, where $w_1 \sim \mathcal{CN}(0,1)$ and $w_2 \sim \mathcal{CN}(0,1)$ are AWGN at the BS and MS_k, respectively. The self-interferences $\beta h_1 h_1\sqrt{p_1}x_1$ and $\beta h_{2,k} h_{2,k}\sqrt{p_2}x_2$ in the right-hand-side (RHS) of (2) and (3) can be suppressed if those are known at the BS and MS_k, respectively, and the corresponding received signals are rewritten as

$$y'_{1,k} = \beta h_1 h_{2,k}\sqrt{p_2}x_2 + \beta h_1 w + w_1 \quad (4)$$

and

$$y'_{2,k} = \beta h_{2,k} h_1\sqrt{p_1}x_1 + \beta h_{2,k} w + w_2, \quad (5)$$

[1]This assumption is valid when the local oscillators of the BS, relay, and MS are perfectly synchronized. The imperfect channel reciprocity and calibration [9] remain for future work.

respectively. Using the received signals in (4) and (5), the transmission rates of the BS and MS$_k$ are given as

$$r_{1,k} = \frac{1}{2}C\left(\frac{p_1p_3g_1g_{2,k}}{p_1g_1 + (p_2+p_3)g_{2,k}+1}\right) \quad (6)$$

and

$$r_{2,k} = \frac{1}{2}C\left(\frac{p_2p_3g_1g_{2,k}}{(p_1+p_3)g_1 + p_2g_{2,k}+1}\right), \quad (7)$$

respectively, where $C(x) \triangleq \log_2(1+x)$, $g_1 \triangleq |h_1|^2$, and $g_{2,k} \triangleq |h_{2,k}|^2$.

III. Multiuser Scheduling and Performance Analysis

We consider the MU scheduling that selects an MS having the maximum instantaneous transmission rates $r_{1,k}$ and $r_{2,k}$ among K MSs, when the BS knows the channels h_1 and $h_{2,k}$ for $k = 1, \cdots, K$. Hence, the average rates are expressed as

$$\bar{r}_i \triangleq \mathbb{E}_{r_i}[r_i] \quad (8)$$

where $r_i \triangleq \max_{k=1,\cdots,K}\{r_{i,k}\}$ for $i = 1, 2$. The derivation of the exact closed-form expressions of (8) for $i = 1, 2$ is difficult. Hence, we derive the lower bounds of those.

Theorem 1: The closed-form expressions of the lower bounds of the average rates $\bar{r}_1$ and $\bar{r}_2$ in (8) are given as (9) at the top of next page for $i = 1, 2$ where $a_1 = 1 + \frac{p_2}{p_3}$, $a_2 = 1 + \frac{p_1}{p_3}$, $b_1 = p_1$, $b_2 = p_1 + p_3$, $c_1 = p_2 + p_3$, and $c_2 = p_2$.

Proof: The closed-form expression (9) for $i = 1$ is derived as follows:

$$\bar{r}_1 = \mathbb{E}_{g_1,g_2}\left[\frac{1}{2}\log_2\left(1 + \frac{p_1p_3g_1g_2}{p_1g_1+(p_2+p_3)g_2+1}\right)\right] \quad (10a)$$

$$> \mathbb{E}_{g_1,g_2}\left[\frac{1}{2}\log_2\left(\frac{1}{1+p_2/p_3} + \frac{p_1p_3g_1g_2}{p_1g_1+(p_2+p_3)g_2+1}\right)\right] \quad (10b)$$

$$= \frac{1}{2}\log_2\left(\frac{1}{1+p_2/p_3}\right) + \frac{1}{2\ln(2)} \times \underbrace{\int_{x=0}^{\infty}\int_{y=0}^{\infty}\ln\left(1+\frac{p_1x(p_2+p_3)y}{p_1x+(p_2+p_3)y+1}\right)f_{g_1}(x)f_{g_2}(y)dxdy}_{J} \quad (10c)$$

where $g_2 \triangleq \max_{k=1,\cdots,K}\{g_{2,k}\}$, $f_{g_1}(x) = \frac{1}{\gamma_1}e^{-x/\gamma_1}$, and $f_{g_2}(y) = \frac{K}{\gamma_2}\sum_{k=0}^{K-1}(-1)^k\binom{K-1}{k}e^{-(k+1)y/\gamma_2}$. Since $r_{1,k}$ is a strictly increasing function with respect to $g_{2,k}$,[2] and g_1 and g_2 are independent, the equality from (8) to (10a) is obtained[3]. By applying the relationship that $\log_2(x+y) = \log_2(x) + \log_2(1+y/x)$, (10c) is derived from (10b). The probability density function (pdf) of $f_{g_1}(x)$ comes from the fact that the random variable g_1 is exponentially distributed. The pdf of $f_{g_2}(x)$ is derived using the order statistics [10] of an exponential random variable $g_{2,k}$ and the binomial theorem. The closed-form expression of J in (10c) is derived in the Appendix A. The closed-form expression (9) for $i = 2$ can be derived by following the derivation procedure of (9) for $i = 1$. ■

[2]This can be verified by that $\frac{\partial r_{1,k}}{\partial g_{2,k}} = p_1p_3g_1(1+g_1p_1)/(2\ln(2)(1 + p_1g_1 + (p_2+p_3)g_{2,k})(1 + p_1g_1(1+p_3g_{2,k}) + (p_2+p_3)g_{2,k})) > 0$.

[3]Since $r_{2,k}$ is also a strictly increasing function with respect to $g_{2,k}$, an MS maximizing $r_{i,k}$ for $i = 1, 2$ is identical to selecting an MS having the largest instantaneous channel gain among $g_{2,k}$ for $k = 1, \cdots, K$.

In addition, we derive the asymptotic expressions of (8) for large K.

Theorem 2: When the K number of MSs is sufficiently many, the rates of $\bar{r}_1$ and $\bar{r}_2$ in (8) are asymptotically expressed as

$$\bar{r}_i \approx \begin{cases} \frac{1}{2}\log_2(1 + k_ip_3\gamma_2\ln(K)), & \text{if } m_i \ll 1, \quad (11a) \\ \frac{1}{2\ln(2)}\bar{E}_1\left(\frac{l_i}{p_3\gamma_1}\right), & \text{if } m_i \gg 1, \quad (11b) \end{cases}$$

for $i = 1, 2$ where $k_1 = 1$, $k_2 = \frac{p_2}{p_1+p_3}$, $l_1 = \frac{p_2+p_3}{p_1}$, $l_2 = 1$, $m_1 = \frac{(p_2+p_3)\gamma_2\ln(K)+1}{p_1\gamma_1}$, and $m_2 = \frac{p_2\gamma_2\ln(K)+1}{(p_1+p_3)\gamma_1}$.

Proof: When K is sufficiently large, the random variable $g_2 \triangleq \max_{k=1,\cdots,K}\{g_{2,k}\}$ scales as $\gamma_2\ln(K)$ with high probability [11]. By applying this result to (10a), we get

$$\bar{r}_1 \approx \mathbb{E}_{g_1}\left[\frac{1}{2}\log_2\left(1+\frac{p_1p_3g_1\gamma_2\ln(K)}{p_1g_1+(p_2+p_3)\gamma_2\ln(K)+1}\right)\right] \quad (12a)$$

$$= \frac{1}{2\ln(2)}\int_{x=0}^{\infty}\ln\left(1+\frac{p_1p_3x\gamma_2\ln(K)}{p_1x+(p_2+p_3)\gamma_2\ln(K)+1}\right)f_{g_1}(x)dx \quad (12b)$$

$$= \frac{1}{2\ln(2)}\left(\bar{E}_1\left(\frac{(p_2+p_3)\gamma_2\ln(K)+1}{p_1\gamma_1(p_3\gamma_2\ln(K)+1)}\right) - \bar{E}_1\left(\frac{(p_2+p_3)\gamma_2\ln(K)+1}{p_1\gamma_1}\right)\right). \quad (12c)$$

Using the Eq. (4.337.2) in [12], (12c) is derived from (12b). We can further approximate (12c) to (11a) for $i = 1$ by using the Lemma 1 in the Appendix B under the condition that $(p_2+p_3)\gamma_2\ln(K)+1 \ll p_1\gamma_1$. When K is sufficiently large and $(p_2+p_3)\gamma_2\ln(K)+1 \gg p_1\gamma_1$, (12c) is approximated to (11b) for $i = 1$ since $\frac{(p_2+p_3)\gamma_2\ln(K)+1}{p_1\gamma_1(p_3\gamma_2\ln(K)+1)} \approx \frac{(p_2+p_3)}{p_1p_3\gamma_1}$ and the second term in (12c) is nearly zero. The asymptotic expression of (11) for $i = 2$ can be derived by following a similar procedure as that of (11) for $i = 1$. ■

When $m_i \ll 1$ for $i = 1, 2$, the average rates of MU-TWRC increase as K increases which denotes that the MU diversity gain is fully utilized the same as that for the MU scheduling without a relay [11]. Interestingly, when $m_i \gg 1$ for $i = 1, 2$, which is satisfied as K increases, but the link condition between BS and relay is fixed, the asymptotic results of (11b) show that the average rates of MU-TWRC is bounded to a constant value irrespective to K, which means that the MU diversity gain is not utilized. Hence, the values of m_i for $i = 1, 2$ can be provided by the MU-TWRC design measures, and the following Corollary is given based on Theorem 2.

Corollary 1: If $m_1 \ll 1$ in Theorem 2 is satisfied, then the MU diversity gain is fully exploited for the MU-TWRC.

Proof: Since $m_2 < m_1$, the condition that $m_1 \ll 1$ is the sufficient condition that $m_2 \ll 1$. ■

IV. Simulation Results

The average rates of the MU-TWRC are examined through computer simulation. We assume that $p_1 = p_2 = p_3 = 1$. Fig. 1 shows the average rates of $\bar{r}_1$ and $\bar{r}_2$ against the number of MSs, K, under the various link conditions such that $\gamma_1 = 10, 30, 60$ dB and $\gamma_2 = 30$ dB. In here, it is demonstrated that

$$\bar{r}_i > \frac{1}{2}\log_2\left(\frac{1}{a_i}\right) + \frac{K}{2\ln(2)}\sum_{k=0}^{K-1}\frac{(-1)^k}{k+1}\binom{K-1}{k} \times \begin{cases} \dfrac{\frac{1}{b_i\gamma_1}\bar{E}_1\left(\frac{1}{b_i\gamma_1}\right) - \frac{k+1}{c_i\gamma_2}\bar{E}_1\left(\frac{k+1}{c_i\gamma_2}\right)}{\left(\frac{1}{b_i\gamma_1} - \frac{k+1}{c_i\gamma_2}\right)}, & \text{if } \frac{1}{b_i\gamma_1} \neq \frac{k+1}{c_i\gamma_2}, \quad (9a) \\ \left(\dfrac{1}{a_i\gamma_1} + 1\right)\bar{E}_1\left(\dfrac{1}{b_i\gamma_1}\right) - 1, & \text{if } \frac{1}{b_i\gamma_1} = \frac{k+1}{c_i\gamma_2}, \quad (9b) \end{cases}$$

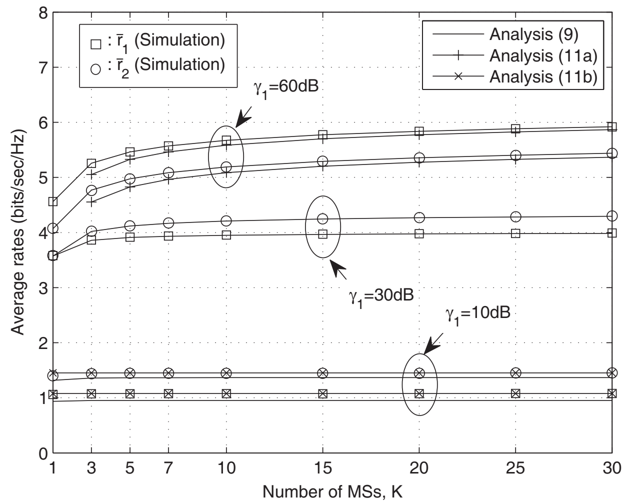

Fig. 1. Average rates of $\bar{r}_1$ and $\bar{r}_2$ against the number of MSs, K, when $\gamma_2 = 30\,\text{dB}$.

the analysis results of (9) and the simulation results match well over the various K and link conditions. When $\gamma_1 = 60\,\text{dB}$ and K ranges from 5 to 30, the simulation results of $\bar{r}_1$ and $\bar{r}_2$ asymptotically follow the analysis of (11a) since m_1 ranges from 0.0032 to 0.0068. When $\gamma_1 = 10\,\text{dB}$, the simulation results of $\bar{r}_1$ and $\bar{r}_2$ asymptotically follow the analysis of (11b) since $m_i \gg 1$ for $i = 1, 2$.

V. Conclusion

The performances of the MU-TWRC are analyzed. The closed-form expressions of the lower bounds and the asymptotic expressions of the average rates are derived. Based on the analysis, we provide the design guideline for the MU-TWRC to fully utilize the MU diversity gain.

Appendix

A. Derivation of the closed-form expression of J in (10c)

$$J = \frac{K}{\gamma_1\gamma_2}\sum_{k=0}^{K-1}(-1)^k\binom{K-1}{k}\underbrace{\int_{x=0}^{\infty}\int_{y=0}^{\infty}\tilde{J}e^{-\frac{x}{\gamma_1}}e^{-\frac{(k+1)y}{\gamma_2}}dxdy}_{J'} \quad (13)$$

where $\tilde{J} = \ln(1 + p_1 x) - \ln\left(1 + \frac{p_1 x}{1+(p_2+p_3)y}\right)$.

$$J' = \gamma_1\int_{y=0}^{\infty}\left(\bar{E}_1\left(\frac{1}{p_1\gamma_1}\right) - \bar{E}_1\left(\frac{1+(p_2+p_3)y}{p_1\gamma_1}\right)\right)e^{-\frac{(k+1)y}{\gamma_2}}dy \quad (14)$$

Using the Eq. (4.337.2) in [12] and utilizing the equality that $E_1(x) = -\mathrm{E}_i(-x)$, J' of (14) is derived from J' in (13). When $\frac{1}{p_1\gamma_1} \neq \frac{k+1}{(p_2+p_3)\gamma_2}$, (9a) for $i = 1$ is derived after some manipulation by applying the integral by parts to (14). When $\frac{1}{p_1\gamma_1} = \frac{k+1}{(p_2+p_3)\gamma_2}$, (9b) for $i = 1$ is derived by applying the integral by parts and the Eq. (6.221) in [12].

B. Lemma 1

When $0 < x \ll 1$ and $0 < y \ll 1$, the following approximation

$$\bar{E}_1(x) - \bar{E}_1(y) \approx \ln\left(\frac{y}{x}\right) \quad (15)$$

is satisfied.

Proof: By applying the Taylor's series expansion and the first order approximation, the left-hand-side in (15) is approximated as follows:

$$\bar{E}_1(x) - \bar{E}_1(y) = e^x E_1(x) - e^y E_1(y) \quad (16a)$$

$$\approx (1+x)(-\gamma - \ln(x) + x) - (1+y)(-\gamma - \ln(y) + y) \quad (16b)$$

$$\approx \ln(y/x) + (2-\gamma)(x-y) \quad (16c)$$

where $\gamma \approx 0.577215664\cdots$ is the Euler gamma constant. When $0 < x \ll 1$, the approximation from (16a) to (16b) is satisfied since $e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} \approx 1 + x$ and $E_1(x) = -\gamma - \ln x + \sum_{k=1}^{\infty}\frac{(-1)^{k+1}x^k}{k\cdot k!} \approx -\gamma - \ln x + x$. Also, the approximation from (16b) to (16c) is given since $x\ln(x) = x\sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x-1)^k}{k} \approx x(x-1)$ when $0 < x \ll 1$. Due to $x - y \approx 0$ when $0 < x \ll 1$ and $0 < y \ll 1$, the approximation from (16c) to the RHS in (15) is satisfied. ■

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