Reduced Complexity Beamforming with Optimal Power Allocation in Two-Way Multi-Antenna Relay Systems

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***Abstract*—This paper introduces a new beamforming method for a two-way relay system, where two single-antenna terminals exchange messages via a multi-antenna amplify-and-forward (AF) relay operating in the half-duplex mode. The optimization problem under consideration is to design a relay beamforming matrix that maximizes the achievable weighted sum rate under the relay power constraint, given the channel state information. This problem is non-convex, and iterative optimization algorithms and suboptimal beamformers with reduced complexity have been developed in previous works. In this paper, we develop an alternative suboptimal scheme by converting the original non-convex optimization into convex optimization. This conversion is made possible through some changes of variables and an additional condition. The simulation results demonstrate that the proposed method performs almost as well as the optimal scheme and can yield larger achievable rates than conventional suboptimal beamformers.**

***Index Terms*—Two-way relay channel, convex optimization, amplify-and-forward, relay beamforming.**

I. Introduction

TWO-WAY relaying has been recognized as an efficient protocol for exchanging messages between two terminals with the help of relays that operate in the half-duplex mode. In two-way relaying, messages are exchanged via multiple-access and broadcast phases. In the multiple-access phase, the two terminals simultaneously transmit their signals to the relay over a shared channel, and in the broadcast phase, the relay broadcasts the received signal after some processing. Since the two terminals know their own transmitted signals, they can remove the self-interference from the received signal and recover the intended data.

Due to its bidirectional nature, the two-way relaying protocol can increase the spectral efficiency as compared with one-way relaying, which allows only unidirectional traffic. The achievable rates of two-way relaying with either an AF or a decode-and-forward (DF) relay are investigated in [1]. Following this analysis, considerable work has been reported towards characterizing the capacity of various two-way relaying schemes. For such research we refer to [2] and the references therein. Recent studies on two-way relaying have focused on multi-input multi-output (MIMO) and collaborative communication systems. For DF-based MIMO two-way relaying, re-encoding strategies at the relay for the broadcast phase were proposed in [3], [4]. For the case of AF MIMO relaying, the optimization of linear precoders for the multiple-access and broadcast phases based on either the achievable rate criterion or the mean-square-error criterion was investigated [2], [5]–[8]. The achievable rate regions for an AF-based two-way relay network with collaborative relay beamforming have been studied in [2], [9], [10].

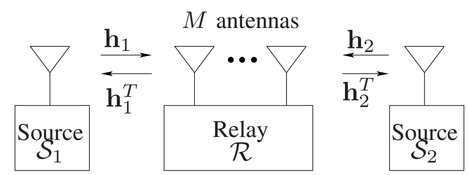

Fig. 1. A two-way relay channel where each source has a single antenna and the relay has M antennas.

In this paper, we revisit the achievable rate maximization problem for designing the relay precoding (or beamforming) matrix of a two-way AF relay system where the relay has multiple antennas and each source has a single antenna. This problem is non-convex and iterative optimization processes have been proposed in [2], [5]. In addition, to reduce the complexity in implementation, [2] and [5] also proposed suboptimal beamformers such as the maximal-ratio reception and maximal-ratio transmission (MRR-MRT) and the zero-forcing reception and zero-forcing transmission (ZFR-ZFT) [5].[1] The MRR-MRT (ZFR-ZFT) performs matched filtering (zero-forcing) both at the receiver and at the transmitter sides of the relay. It has been shown that the achievable rate region of MRR-MRT is close to that of the optimal scheme and that the MRR-MRT outperforms the ZFR-ZFT. One drawback of these suboptimal schemes is their inability to allocate power: the process for optimally allocating power has not been reported yet and equal power allocation is usually assumed.

The main objective of this paper is to develop an alternative suboptimal scheme by converting the original nonconvex optimization to convex optimization. The conversion is made possible through some changes of variables and an additional condition. The resulting suboptimal scheme performs matched filtering and zero-forcing at the relays's receiver and transmitter sides, respectively, and *optimally* allocates power in the middle. The proposed beamforming is referred to as the MRR-ZFT. It is shown through simulation that the proposed MRR-ZFT can perform better than the MRR-MRT and that the

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[1] In [2], the MRR-MRT is referred to as the dual matched filtering.

performance gain is achieved by the optimal power allocation strategy of the proposed scheme.

Notations: The matrices and vectors are denoted by bold capital and lowercase letters, respectively: $\mathbf{S}^*, \mathbf{S}^T, \mathbf{S}^H$, tr($\mathbf{S}$), and $\mathbf{S}^+$ are the conjugate, transpose, conjugate transpose, trace and pseudo inverse of $\mathbf{S}$, respectively; $\mathbf{W} = \text{diag}[w_1, w_2]$ denotes a 2×2 diagonal matrix with (w_1, w_2) on its diagonal. $\mathbb{C}(\mathbb{R})$ is a set of complex (real) numbers; $\mathbb{C}^{M\times N}(\mathbb{R}^{M\times N})$ is a set of complex (real) M-by-N matrices; $\mathbf{I}_N$ is an N-by-N identity matrix; $\mathcal{CN}(\mathbf{x}, \mathbf{\Sigma})$ refers to a circularly symmetric complex Gaussian (CSCG) distribution with mean $\mathbf{x}$ and covariance matrix $\mathbf{\Sigma}$; and $\mathbf{A} \odot \mathbf{B}$ represents the Hadamard product of matrices $\mathbf{A}$ and $\mathbf{B}$.

II. System Model and Problem Setting

Consider a two-way relay channel where two source nodes $\mathcal{S}_1$ and $\mathcal{S}_2$ exchange their messages via a relay nodes $\mathcal{R}$ (Fig. 1). Each source has a single antenna, and the relay has M antennas, $M \geq 2$. The received baseband signal at $\mathcal{R}$ in the multi-access phase is represented as

$$\mathbf{y}_R = \mathbf{h}_1\sqrt{p_1}s_1 + \mathbf{h}_2\sqrt{p_2}s_2 + \mathbf{z}_R, \tag{1}$$

where $\mathbf{y}_R \in \mathbb{C}^{M\times 1}$ is the received signal vector; $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$, $i \in \{1,2\}$, denotes the channel vector from the source $\mathcal{S}_i$ to $\mathcal{R}$; s_i is the transmitted symbol from $\mathcal{S}_i$ and p_i denotes its power; and $\mathbf{z}_R \in \mathbb{C}^{M\times 1}$ is the receiver noise vector with $\mathbf{z}_R \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The symbols $\{s_i\}$ are assumed to be independent and $s_i \sim \mathcal{CN}(0,1)$. The received signal $\mathbf{y}_R$ is linearly processed to yield

$$\mathbf{x}_R = \mathbf{A}\mathbf{y}_R, \tag{2}$$

where $\mathbf{A} \in \mathbb{C}^{M\times M}$ is the relay beamforming matrix. In the broadcast phase, the relay broadcasts $\mathbf{x}_R \in \mathbb{C}^{M\times 1}$ in (2) and the received signals at $\mathcal{S}_1$ and $\mathcal{S}_2$ can be expressed as $y_1 = \mathbf{h}_1^T\mathbf{x}_R + z_1 = \mathbf{h}_1^T\mathbf{A}\mathbf{h}_1 s_1 + \mathbf{h}_1^T\mathbf{A}\mathbf{h}_2 s_2 + \mathbf{h}_1^T\mathbf{A}\mathbf{z}_R + z_1$ and $y_2 = \mathbf{h}_2^T\mathbf{x}_R + z_2 = \mathbf{h}_2^T\mathbf{A}\mathbf{h}_1 s_1 + \mathbf{h}_2^T\mathbf{A}\mathbf{h}_2 s_2 + \mathbf{h}_2^T\mathbf{A}\mathbf{z}_R + z_2$ where z_i, $i \in \{1,2\}$, is the independent receiver noise with $z_i \sim \mathcal{CN}(0,1)$. In y_i, $\mathbf{h}_i^T\mathbf{A}\mathbf{h}_i s_i$ is the self-interference that can be canceled out by $\mathcal{S}_i$. Thus, the achievable weighted sum rate can be written as

$$\begin{aligned} R(\mathbf{A}) = \frac{c_1}{2}\log_2\left(1 + \frac{p_2\mathbf{h}_1^T\mathbf{A}\mathbf{h}_2\mathbf{h}_2^H\mathbf{A}^H\mathbf{h}_1^*}{\mathbf{h}_1^T\mathbf{A}\mathbf{A}^H\mathbf{h}_1^* + 1}\right) \\ + \frac{c_2}{2}\log_2\left(1 + \frac{p_1\mathbf{h}_2^T\mathbf{A}\mathbf{h}_1\mathbf{h}_1^H\mathbf{A}^H\mathbf{h}_2^*}{\mathbf{h}_2^T\mathbf{A}\mathbf{A}^H\mathbf{h}_2^* + 1}\right), \end{aligned} \tag{3}$$

where c_1 and c_2 are nonnegative weights. Our objective is to maximize $R(\mathbf{A})$ in (3) with respect to (w.r.t.) $\mathbf{A}$ under the transmit power constraint at the relay $\mathcal{R}$: from (1) and (2), the constraint can be expressed as

$$\begin{aligned} p_R(\mathbf{A}) &= \text{tr}(\mathbb{E}[\mathbf{x}_R\mathbf{x}_R^H]) \\ &= \text{tr}\{\mathbf{A}(\mathbf{H}_{\text{UL}}\mathbf{P}_s\mathbf{H}_{\text{UL}}^H + \mathbf{I}_M)\mathbf{A}^H\} \leq P_R, \end{aligned} \tag{4}$$

where $\mathbf{H}_{\text{UL}} = [\mathbf{h}_1, \mathbf{h}_2] \in \mathbb{C}^{M\times 2}$, $\mathbf{P}_s \in \mathbb{R}^{2\times 2}$ is a diagonal matrix with (p_1, p_2) on its diagonal and P_R is the maximum available power of $\mathcal{R}$. Let us define $\mathbf{e}_i = \mathbf{A}^H\mathbf{h}_i^* \in \mathbb{C}^{M\times 1}$, $i \in \{1,2\}$, and

$$\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2] = \mathbf{A}^H\mathbf{H}_{\text{UL}}^* \in \mathbb{C}^{M\times 2}. \tag{5}$$

Then, (3) can be rewritten as

$$R(\mathbf{E}) = \sum_{i=1}^{2}\frac{c_i}{2}\log\left[1 + \frac{\mathbf{e}_i^H\mathbf{T}_i\mathbf{e}_i}{\mathbf{e}_i^H\mathbf{e}_i + 1}\right], \tag{6}$$

where $\mathbf{T}_1 = p_2\mathbf{h}_2\mathbf{h}_2^H \in \mathbb{C}^{M\times M}$ and $\mathbf{T}_2 = p_1\mathbf{h}_1\mathbf{h}_1^H \in \mathbb{C}^{M\times M}$. If $\mathbf{E}$ is given, then $\mathbf{A}$ can be obtained by evaluating

$$\mathbf{A} = \left(\mathbf{H}_{\text{UL}}^T\right)^+\mathbf{E}^H, \tag{7}$$

where $\left(\mathbf{H}_{\text{UL}}^T\right)^+$ is the pseudo inverse of $\mathbf{H}_{\text{UL}}$.[2] Substituting $\mathbf{A}$ in (4) by $\left(\mathbf{H}_{\text{UL}}^T\right)^+\mathbf{E}^H$ in (7), $p_R(\mathbf{A})$ becomes the same as

$$p_R(\mathbf{E}) = \text{tr}\left\{\mathbf{E}^H\mathbf{\Theta}\mathbf{E}\mathbf{\Psi}\right\} \leq P_R, \tag{8}$$

where $\mathbf{\Theta} = \mathbf{H}_{\text{UL}}\mathbf{P}_s\mathbf{H}_{\text{UL}}^H + \mathbf{I}_M$ and $\mathbf{\Psi} = \left(\mathbf{H}_{\text{UL}}^T\mathbf{H}_{\text{UL}}^*\right)^+ = \left\{\left(\mathbf{H}_{\text{UL}}^H\mathbf{H}_{\text{UL}}\right)^{-1}\right\}^* \in \mathbb{C}^{2\times 2}$. To simplify further the objective function, we express $\mathbf{e}_i$ and $\mathbf{T}_i$ in terms of unit norm vectors. Specifically, let $\mathbf{e}_i = w_i\mathbf{z}_i$ for $w_i \in \mathbb{R}$ and $\mathbf{z}_i \in \mathbb{C}^{M\times 1}$ with $||\mathbf{z}_i|| = 1$. Then,

$$\mathbf{E} = \mathbf{Z}\mathbf{W}, \tag{9}$$

where $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2] \in \mathbb{C}^{M\times 2}$ and $\mathbf{W} = \text{diag}[w_1, w_2]$. For $\mathbf{T}_i$, let $\mathbf{T}_i = \sigma_i\mathbf{u}_i\mathbf{u}_i^H$, where $\sigma_i > 0$ and $\mathbf{u}_i \in \mathbb{C}^{M\times 1}$ with $\|\mathbf{u}_i\| = 1$ for $i \in \{1,2\}$, i.e., $\mathbf{u}_1 = \frac{1}{\|\mathbf{h}_2\|}\mathbf{h}_2$, $\mathbf{u}_2 = \frac{1}{\|\mathbf{h}_1\|}\mathbf{h}_1$. Then, (6) and (8) can be reformulated as

$$R(\mathbf{Z}, \mathbf{W}) = \sum_{i=1}^{2}\frac{c_i}{2}\log\left(1 + \frac{\sigma_i w_i^2\left|\mathbf{z}_i^H\mathbf{u}_i\right|^2}{w_i^2 + 1}\right) \tag{10a}$$

and

$$p_R(\mathbf{Z}, \mathbf{W}) = \text{tr}\left\{\mathbf{W}\widetilde{\mathbf{\Theta}}(\mathbf{Z})\mathbf{W}\mathbf{\Psi}\right\} \leq P_R, \tag{10b}$$

where $\widetilde{\mathbf{\Theta}}(\mathbf{Z}) = \mathbf{Z}^H\mathbf{\Theta}\mathbf{Z}$.

Now the problem is to find $\{\mathbf{Z}, \mathbf{W}\}$ maximizing (10a) subject to (10b). In the following section, we shall show that this problem can be converted into a convex optimization problem if we set $\mathbf{z}_i = \mathbf{u}_i$, $i \in \{1,2\}$ or equivalently,

$$\mathbf{Z} = \begin{bmatrix} \frac{1}{||\mathbf{h}_2||}\mathbf{h}_2 & \frac{1}{||\mathbf{h}_1||}\mathbf{h}_1 \end{bmatrix} = \mathbf{H}_{\text{UL}}\begin{bmatrix} 0 & \frac{1}{||\mathbf{h}_1||} \\ \frac{1}{||\mathbf{h}_2||} & 0 \end{bmatrix}. \tag{11}$$

Under this condition, the optimization problem in (10) is simplified as

$$\begin{aligned} \max_{\mathbf{W}} R(\mathbf{W}) &= \sum_{i=1}^{2}\frac{c_i}{2}\log\left[1 + \frac{\sigma_i w_i^2}{w_i^2 + 1}\right] && \text{(12a)} \\ \text{s.t.} \quad p_R(\mathbf{W}) &= \text{tr}\left\{\mathbf{W}\widetilde{\mathbf{\Theta}}(\mathbf{U})\mathbf{W}\mathbf{\Psi}\right\} \leq P_R, && \text{(12b)} \end{aligned}$$

where $\widetilde{\mathbf{\Theta}}(\mathbf{U}) = \mathbf{U}^H\mathbf{\Theta}\mathbf{U}$ with $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2]$. Furthermore, the condition in (11) presents an interesting structure to the beamforming matrix $\mathbf{A}$, as shown below.

Observation 1. When $\mathbf{z}_i = \mathbf{u}_i$, $i \in \{1,2\}$, $\mathbf{A}$ can be represented as

$$\mathbf{A} = \underbrace{\left(\mathbf{H}_{\text{UL}}^T\right)^+}_{\text{Zero forcing}}\underbrace{\begin{bmatrix} 0 & \frac{w_1}{||\mathbf{h}_1||} \\ \frac{w_2}{||\mathbf{h}_2||} & 0 \end{bmatrix}}_{\text{Power allocation}}\underbrace{\mathbf{H}_{\text{UL}}^H}_{\text{Matched filtering}}. \tag{13}$$

[2] Solving (5) for $\mathbf{A}$ is an underdetermined problem, and the solution in (7) minimizes $p_R(\mathbf{A})$ in (4).

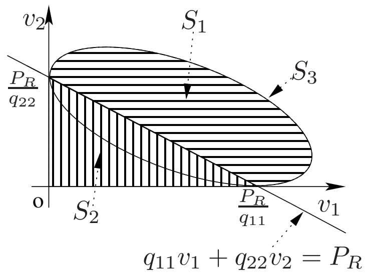


Fig. 2. The constraint set for (17) is given by the union of S_1 (marked horizontally) and S_2 (marked vertically). This constraint set can be reduced to S_3, which is the inside of an ellipse.

This expression for $\mathbf{A}$ can be obtained by using (9) and (11) in (7). The beamforming matrix of the proposed MRR-ZFT is given by (13). It performs matched filtering and zero-forcing at the relay's receiver and transmitter sides, respectively, and power allocation in the middle. Now the remaining problem is to find the optimal power allocation parameters (w_1, w_2) by solving (12).

III. Power allocation via convex optimization

In this section, the optimization problem in (12) is converted into a convex form. By replacing w_i^2 in (12a) with v_i, (12a) is rewritten as

$$R(\mathbf{v}) = \frac{1}{2}\sum_{i=1}^{2} c_i \log_2\left(1 + \frac{\sigma_i v_i}{v_i + 1}\right), \quad (14)$$

and by examining the Hessian of $R(\mathbf{v})$, we can verify that $R(\mathbf{v})$ is a concave function of $\mathbf{v} = [v_1, v_2]^T$. In addition, it can be seen that for $\beta > 1$,

$$R(\beta\mathbf{v}) \geq R(\mathbf{v}). \quad (15)$$

Now by directly calculating the right-hand-side (RHS) of (12b), we can show that $p_R(\mathbf{W}) = q_{11}w_1^2 + 2q_r w_1 w_2 + q_{22}w_2^2 \leq P_R$, where (q_{11}, q_r, q_{22}) are real numbers with $q_{11}, q_{22} > 0$. Using $v_i = w_i^2$, so $w_i = \text{sgn}(w_i)\sqrt{v_i}$, we obtain

$$q_{11}v_1 + 2\text{sgn}(w_1 w_2) q_r \sqrt{v_1}\sqrt{v_2} + q_{22}v_2 \leq P_R, \quad (16)$$

where $\text{sgn}(\cdot)$ represents the sign of its argument. In (16), $\text{sgn}(w_1 w_2)$ is not fixed yet. We choose to set $\text{sgn}(w_1 w_2) = -\text{sgn}(q_r)$, resulting in a new power constraint

$$q_{11}v_1 - 2\text{sgn}(q_r) q_r \sqrt{v_1}\sqrt{v_2} + q_{22}v_2 \leq P_R. \quad (17)$$

The reason is as follows. Since $\text{sgn}(q_r)q_r = |q_r| \geq 0$, the left-hand-side (LHS) of (17) is always less than or equal to that of (16) for any $\mathbf{v}$. This fact, together with (15), suggests that if a solution maximizing the rate subject to (16) is found for some $\text{sgn}(w_1 w_2)$, the rate can be further increased by scaling up the solution until (17) is satisfied with equality.

To characterize the new power constraint, (17) is rewritten as

$$q_{11}v_1 + q_{22}v_2 - P_R \leq 2\text{sgn}(q_r) q_r \sqrt{v_1}\sqrt{v_2}. \quad (18)$$

Note that the RHS of (18) is non-negative for all q_r. If the LHS of (18) is also non-negative, then squaring both sides of (18) yields $q_{11}^2 v_1^2 + 2(q_{11}q_{22} - 2q_r)^2 v_1 v_2 + q_{22}^2 v_2^2 - 2P_R q_{11} v_1 - 2P_R q_{22} v_2 + P_R^2 \leq 0$ or in a matrix form

$$\mathbf{v}^T\mathbf{Q}_1\mathbf{v} - 2P_R\mathbf{1}^T\mathbf{Q}_2\mathbf{v} + P_R^2 \leq 0, \quad (19)$$

where $\mathbf{Q}_1 = \begin{bmatrix} q_{11}^2 & q_{11}q_{22} - 2q_r^2 \\ q_{11}q_{22} - 2q_r^2 & q_{22}^2 \end{bmatrix}$, $\mathbf{Q}_2 = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix}$ and $\mathbf{1} = [1, 1]^T$. On the other hand, if the LHS of (18) is negative then (18) is satisfied automatically for all $v_i \geq 0$ $(i \in \{1, 2\})$ because the RHS of (18) is always non-negative. Let S_1 denote the set of $\mathbf{v}$ satisfying (19) under the non-negativity condition of the LHS of (18): $S_1 = \{\mathbf{v} | f(\mathbf{v}) \leq 0, \mathbf{q}^T\mathbf{v} \geq P_R\}$, where $f(\mathbf{v})$ represents the LHS of (19) and $\mathbf{q} = [q_{11}, q_{22}]^T$. For the case where the LHS of (18) is negative, we define S_2 as follows: $S_2 = \{\mathbf{v} | v_1 \geq 0, v_2 \geq 0, \mathbf{q}^T\mathbf{v} \leq P_R\}$. The union of S_1 and S_2, $S_1 \cup S_2$, represents the set of vectors $\mathbf{v}$ satisfying (17). To examine the convexity of $S_1 \cup S_2$, we define $S_3 = \{\mathbf{v} | f(\mathbf{v}) \leq 0\}$. Referring to (19), $f(\mathbf{v}) = 0$ is an ellipse and S_3 is convex. As shown in Fig. 2, two boundary points of S_3 are tangential to the v_1- and v_2- axes at $\mathbf{v} = \left[\frac{P_R}{q_{11}}, 0\right]^T$ and $\mathbf{v} = \left[0, \frac{P_R}{q_{22}}\right]^T$. Next, we show that $S_1 \cup S_2$ can be shrunk to S_3 in our optimization. For any $\mathbf{v} \in S_2$ it is possible to find $\mathbf{v}' = \beta\mathbf{v} \in S_3$ for $\beta \geq 1$, and, from (15), $R(\mathbf{v}') \geq R(\mathbf{v})$. Therefore, $\mathbf{v} \in S_2 \setminus S_3$ can never be the optimal solution maximizing the achievable rate in (14), and $S_1 \cup S_2$ can be replaced with S_3. Summarizing these results, the optimization problem is written as follows:

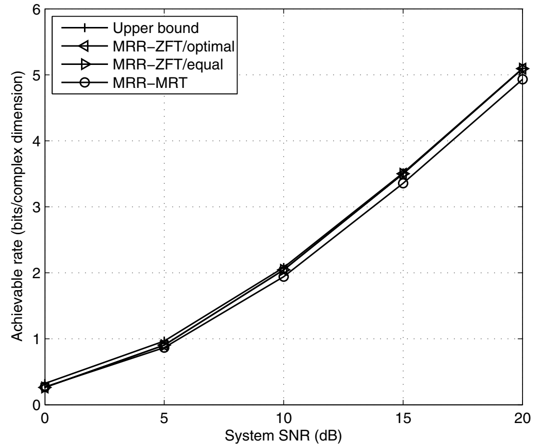
$$\max_{\mathbf{v}} R(\mathbf{v}) = \frac{1}{2}\sum_{i=1}^{2} c_i \log_2\left(1 + \frac{\sigma_i v_i}{v_i + 1}\right) \quad (20)$$
$$\text{subject to } \mathbf{v}^T\mathbf{Q}_1\mathbf{v} - 2P_R\mathbf{1}^T\mathbf{Q}_2\mathbf{v} + P_R^2 \leq 0,$$

where the objective function is identical to (14) and the power constraint represents S_3. This optimization problem is convex. After obtaining the optimal solution, say $\mathbf{v}^o = [v_1^o, v_2^o]^T$, the corresponding vector $\mathbf{w}$, denoted as $\mathbf{w}^o$, can be determined as $\mathbf{w}^o = [\sqrt{v_1^o}, -sgn(q_r)\sqrt{v_2^o}]^T$ since $sgn(w_1 w_2) = -\text{sgn}(q_r)$. Finally, the beamforming matrix $\mathbf{A}$ is obtained by using $\mathbf{w}^o$ in (13).

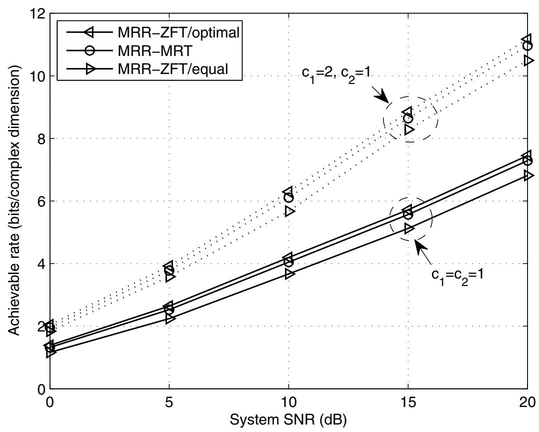
IV. Simulation Results

In this section, we compare the achievable sum rates of the proposed MRR-ZFT with optimal/equal power allocations and the MRR-MRT with equal power allocation in [2], [5]. Two types of channels have been generated: they are the normalized channels with $||\mathbf{h}_1|| = ||\mathbf{h}_2|| = 1$, which are identical to those in [5], and the channels without normalization. The normalized channels are as follows: $M = 4$; $\mathbf{h}_1$ is a normalized CSCG vector given by $\mathbf{h}_1 = \mathbf{h}_1'/||\mathbf{h}_1'||$, where $\mathbf{h}_1' \sim \mathcal{CN}(\mathbf{0}, I_M)$; $\mathbf{h}_2 = \sqrt{\rho}\mathbf{h}_1 + \sqrt{1-\rho}\mathbf{h}_w$, where $\rho = ||\mathbf{h}_1^H\mathbf{h}_2||^2$; and $\mathbf{h}_w$ is also a normalized CSCG vector with $||\mathbf{h}_w|| = 1$ and $\mathbf{h}_1^H\mathbf{h}_w = 0$. The non-normalized channels have been obtained by dropping the normalization steps when generating $\mathbf{h}_1$ and $\mathbf{h}_w$. The transmit powers of $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{R}$ are assumed to be the same $(p_1 = p_2 = P_R)$.

Fig. 3(a) shows the achievable sum rate $(c_1 = c_2 = 1)$ versus the system signal-to-noise ratio (SNR) for the normalized channels. Here the system SNR is given by P_R due to unit-variance noises. Referring to Fig. 3(a), for the normalized channels the equal power allocation turned out to be optimal, and the MRR-ZFT with optimal and equal power allocations



(a)



(b)

Fig. 3. Sum rate versus system SNR with $p_1 = p_2 = P_R$, $M = 4$ and $\rho = \frac{1}{3}$. (a) Normalized channels ($||\mathbf{h}_1|| = ||\mathbf{h}_2|| = 1$) and $(c_1, c_2) = (1, 1)$. (b) Channels without normalization and $(c_1, c_2) \in \{(1, 1), (2, 1)\}$. Here for MRR-ZFT, "optimal" and "equal" stand for the optimal and equal power allocations. In (a), the curves for MRR-ZFT/optimal and MRR-ZFT/equal are identical.

exhibited identical performance.[3] In mid to high SNR regime the achievable sum rate curve of the proposed scheme nearly coincide with the upper bound which was derived in [5]. The gap is 0.0005 bits/complex dimension (bpcd) at SNR=20dB. On the other hand, MRR-MRT exhibits a gap of 0.161 bpcd at high SNR. The MRR-ZFT outperformed the MRR-MRT when SNR$\geq$1.4 dB, but their performance was reversed for SNR$<$1.4 dB. This happened because ZFT performs worse than MRT for noisy cases.

Fig. 3(b) shows the achievable weighted sum rates for the non-normalized channels when the weights $(c_1, c_2) \in \{(1, 1), (2, 1)\}$. In this case, equal power allocation is not optimal, and the MRR-ZFT with equally allocated power performed the worst. For both weights the MRR-ZFT with optimal power allocation outperformed the MRR-MRT — this was true for all SNR values in between 0 dB and 20 dB.

[3]This happened because $||\mathbf{h}_1|| = ||\mathbf{h}_2|| = 1$, $p_1 = p_2 = P_R$ and $c_1 = c_2 = 1$. Under these conditions, σ_1 becomes identical to σ_2 in (12a) as well as $c_1 = c_2$. Also, q_{11} and q_{22} in (17) become equal. Due to the symmetry, the solution to this optimization becomes $w_1 = w_2$, i.e., equal power allocation.

Finally, we comment on the computational complexities for implementing the suboptimal beamformers. The MRR-MRT needs $\mathcal{O}(M^2)$ multiplications for evaluating the beamforming matrix $\mathbf{A}$, and the same is true for MRR-ZFT if its power allocation matrix is given. Due to the convex optimization for power allocation, implementation of the MRR-ZFT is more complex than the MRR-MRT, but much simpler than the conventional optimal scheme.

V. Concluding Remarks

We considered the achievable rate maximization problem for designing the relay beamforming matrix for a two-way relay system, where two single-antenna terminals exchange their messages via a multi-antenna AF relay operating in the half-duplex mode. By converting the non-convex optimization problem into convex optimization, a suboptmal relay beamforming method called the MRR-ZFT has been developed. The simulation results indicated that the proposed method can perform better than the conventional suboptimal beamformers such as the MRR-MRT in mid to high SNR regime.

Due to the zero-forcing operation, the effective end-to-end channel of the MRR-ZFT consists of two parallel subchannels; as a result, the power allocation matrix in (13) is skew-diagonal and optimal power allocation parameters can be found. The same is true for the ZFR-ZFT in [5]; the proposed convexification can be extended for the optimal power allocation of the ZFR-ZFT beamformer in a straightforward manner. On the other hand, in the case of MRR-MRT, the effective channel cannot be parallelized and the optimal power allocation matrix is a general 2×2 matrix that is not skew-diagonal. Therefore, the proposed convexification process cannot be applied to MRR-MRT.

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