Closed Form of Optimum Cooperative Distributed Relay Amplifying Matrix

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***Abstract*—This paper presents a closed form of an optimal cooperative amplify-and-forward (AF) relay amplifying matrix for a distributed relay network of M-source-M-destination pairs and N relays, called a *cooperative distributed AF relay network*. The objective of this paper is to derive closed forms of minimum mean square error (MMSE)-based and zero-forcing (ZF)-based optimal AF relay amplifying matrices for the cooperative distributed AF relay network under the transmitter power constraint (TPC) at the relays, the receiver power constraint (RPC) at the destinations, and the no-power constraint (NPC) condition. Additionally, by substituting the derived optimum AF relay amplifying matrices into the original cost functions (CFs), the behavior of the optimum CFs and the total optimum signal component power (SCP) at the destinations are compared to each other for different cases. Finally, using the MMSE criterion, a novel relay selection scheme is proposed for the cooperative distributed AF relay network.**

***Index Terms*—Amplify-and-forward relay, minimum mean square error (MMSE), zero-forcing (ZF), transmit power constraint (TPC), receive power constraint (RPC), cost function (CF).**

I. Introduction

Unlike a decode-and-forward (DF) relay scheme [1]–[6] or a compress-and-forward relay scheme [7], [8], an amplify-and-forward (AF) relay scheme has been popular because the AF relays do not need to decode, encode, and compress the signals received from the sources [9]–[21]. However, the AF relays in this current paper need to know the optimum relay amplifying matrices to retransmit their received signals from multiple sources to multiple destinations. The calculations of the optimum relay amplifying matrices require a higher complexity and are assumed to be done at a central service station (CSS) such as a cloud radio access network (CRAN). The individual AF relays do not need to compute the optimum amplifying matrices. Hence, this paper still considers the AF relay network.

In the literature, *cooperative* and *noncooperative* definitions generally have not been consistent in the AF wireless relay networks; therefore, in this paper, the cooperative and noncooperative definitions will be used based on the ones in [9]–[11] (refer to definition in Section II). Although this cooperative network will increase complexity compared to a noncooperative one, the improved system performance, such as bit error rate (BER) and capacity, over a noncooperative AF relay network makes the cooperative relay network attractive. Additionally, the cooperative and noncooperative distributed relay networks can be realized by a 5G candidate system called the CRAN. If the relays are connected through optical fibers to a CSS in the CRAN, then the channel coefficients can be reported to the CSS, and the proposed optimal relay amplifying matrix can be computed at the CSS and forwarded to the relays.

Literature closely related to the proposed cooperative distributed AF relay network in this current paper was surveyed and comparisons are listed in Table I. This current paper studies cooperative and noncooperative distributed AF relay networks based on the minimum mean square error (MMSE) or zero-forcing (ZF) criteria under various power constraints, such as transmit power constraint (TPC), receive power constraint (RPC), and no power constraint (NPC). On the other hand, in [9], the authors obtained a cooperative distributed AF multiple-input multiple-output (MIMO) relay amplifying matrix in an implicit expression using the MMSE criterion under the TPC at the relays. The obtained AF relaying matrix is nondiagonal because of the cooperation availability among the relay nodes. The MIMO relay network means a network of multiple sources, multiple destinations, and multiple relay nodes, where each node has a single antenna element. On the other hand, each node in the conventional MIMO network can have multiple antenna elements, but no relay nodes exist. The authors in [11] studied the same model as the one in [9] but they presented their results under the TPC at the sources instead of the relays. In addition, in [11], the total relay transmit power is constrained to be equal to the total source transmit power during data transmission.

A noncooperative relay network instead of the cooperative one was studied in [12]–[15]. The authors in [12] presented a noncooperative distributed AF MIMO relay amplifying matrix in a closed-form expression using the MMSE criterion under

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TABLE I
Prior Works (cooperative and noncooperative distributed AF relay networks) and this current paper

References	System	Scheme	Criterion	Relay amplifying matrix		Constraint	CF / SCP behavior	Relay Selection
Current paper	MIMO	Cooperative, noncooperative	MMSE, ZF	Nondiagonal, diagonal	Closed form	TPC, RPC, NPC	CF / SCP behavior	Relay selection (MMSE)
[9]	MIMO	Cooperative	MMSE	Nondiagonal	Implicit	TPC (Relay)	-	-
[11]	MIMO	Cooperative	MMSE	Nondiagonal	Closed form	TPC (Source)	-	-
[12]	MIMO	Noncooperative	MMSE	Diagonal	Implicit	NPC	-	-
[13]	MIMO	Noncooperative	Capacity	Diagonal	Implicit	-	-	-
[14]	MIMO	Noncooperative	Minimizing total relay power	Diagonal	Implicit	SINR	-	-
[15]	SISO	Noncooperative	-	-	-	-	-	Relay selection (Relay ordering)

NPC. The obtained AF relaying matrix is diagonal because of the no cooperation among the relay nodes. The authors in [13] studied the same model as the one in [12], but they employed the sum capacity criterion instead of the MMSE, and they obtained a diagonal AF matrix in an implicit form under no constraint. The authors in [14] studied the same model as the one in [13], but they employed the minimum total relaying power criterion under the target signal-to-interference-plus-noise ratio (SINR) constraint. They also obtained a diagonal AF matrix in an implicit form. Additionally, a relay amplifying matrix was designed based on the ZF criterion in [22], [23] but not for cooperative distributed AF wireless relay networks.

In addition, there exist multiple references about antenna (not relay) selection for conventional MIMO systems. However, a few papers presented relay-selection methods for noncooperative distributed AF MIMO relay networks, but the authors in this current paper could not find any references about relay selection schemes for the cooperative distributed AF MIMO relay network. For example, the authors in [15] studied a relay selection method for noncooperative distributed relay networks by using their relay ordering for the AF single-input single-output (SISO) relay network. This current paper proposes a relay selection scheme for the cooperative as well as noncooperative distributed AF MIMO relay network, which can show significantly better performance than the one in [15] under the same environments. This relay selection for the cooperative/noncooperative distributed AF MIMO relay network is based on the fact that the smaller the MMSE value, the better the BER performance.

These existing papers have not presented the optimum AF relaying matrices in closed forms for the MIMO AF relay networks with the MMSE or ZF criteria under various power constraints. This current paper will take the same or similar relay network model in [9], [11]–[15] but will consider various practical power constraints. The main objective of this current paper is to obtain closed forms of the optimal nondiagonal relay amplifying matrices for the cooperative distributed AF MIMO wireless relay networks using MMSE- or ZF-based cost functions (CFs) under the TPC at the relays, the RPC at the destinations, and the NPC condition. Similar to [9], [11], the MMSE in this paper criterion will be mainly used for determining them. And the ZF criterion will also be used, which was not the case in [9], [11]. Additionally, a *diagonal weight matrix* can be inserted to limit the relay input powers at the relays as the RPC at the destinations during data transmission. This was not studied in [9], [11]. Finally, this paper will show that the proposed optimum relay amplifying matrix performs better than the existing ones in [9], [11], [12], [15].

In particular, the diagonal relay amplifying matrices in [13], [14] were designed by using different criterions with different constraints (e.g., SINR) but not in a closed form. Hence, in this paper, a closed form of a diagonal optimum AF relaying matrix will be derived by the MMSE criterion, and will be used for a fair comparison. Furthermore, by substituting the derived optimum relay amplifying matrices back into the original CFs, this paper can compare the behavior of the optimum CFs and the total optimum signal component power (SCP) at the destinations for the different cases, which were not investigated in [9]–[14], [19]–[23]. The TPC has traditionally been studied because of a practical limitation scenario placed at the device level. On the other hand, in [19]–[21], [24], the RPC at the destinations has been studied. The justifications for the practicality of RPC are well stated, either at subsection B of the introduction in [19] or the introduction of [24]. This current paper will also consider the RPC as well as the TPC. In addition, this current paper will take specific practical system models presented in Fig. 1 of [19] and in Fig. 1 of [24]. The main purpose of the RPC instead of the TPC is to achieve an energy-efficient network in the network level rather than the device level [19]–[21], [24]. Too much extra received power can be avoided using RPC. Since each node uses power just sufficient to meet the requirements under the RPC, the neighboring networks can also enhance their throughput and hence overall network throughput due to the fact that there may be less intercell interference. Studies on

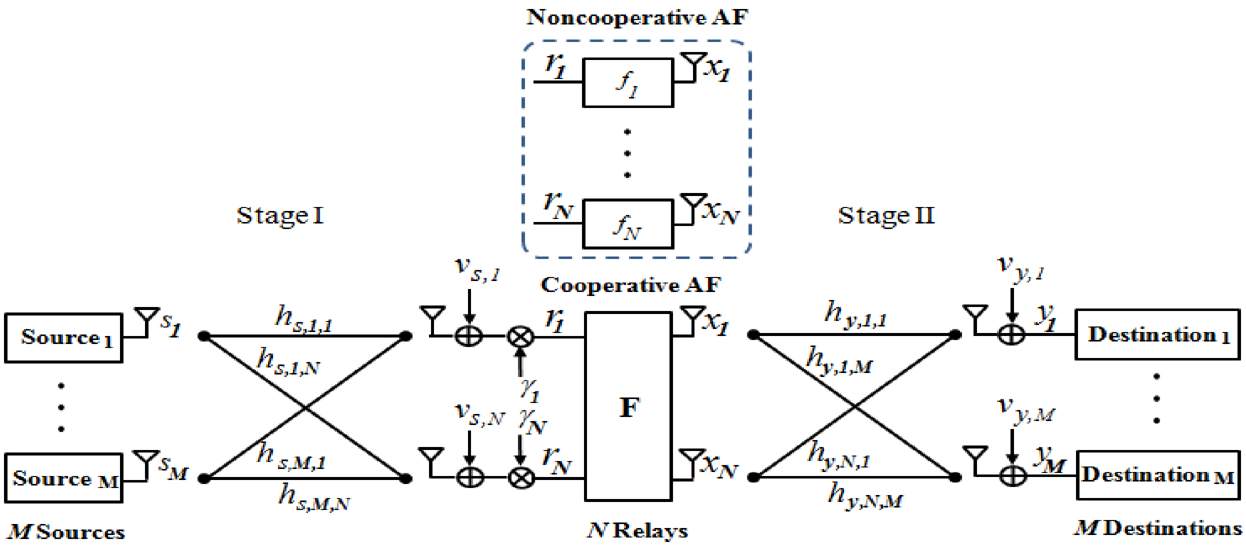

Fig. 1. Cooperative distributed AF relay network with M-sources, N-relays, and M-destinations under TPC, RPC, and NPC.

the RPC at destinations have only recently appeared and are not yet popular in the literature [19]–[21], [24]. The explicit expression for the optimum amplifying matrix under the RPC has not yet been made available in a diagonal weight matrix form [21].

Furthermore, there exist many other references related to this paper, For example, in [16], the authors studied distributed generalized ABBA (GABBA) space time codes for the AF relay network, and they also presented an opportunistic relaying scheme for the AF MIMO network in [17], [18]. But this current paper does not focus on space time codes. Moreover, there exist numerous references about the DF cooperative relay network. For example, the authors in [1]–[6] studied maximum likelihood DF schemes for MIMO relay networks. However, this current paper focuses on the AF relay network instead of the DF relay network.

The novel contributions made in this paper compared to the existing ones in [9], [11]–[15] shown in Table I are listed below:

- This paper presents closed forms of the AF relaying amplifying matrices for the cooperative and noncooperative AF relay networks, using the MMSE or ZF criteria under various power constraints, including both the receive and transmit power constraints at the relay nodes and the receive power constraints at the destination nodes.
- This current paper presents an efficient and different relay selection method from the one in [15] to select a given number of relay nodes, using the minimum cost function obtained after substitution of the optimum AF matrix into the original cost function. Then, this paper recalculates the optimum AF relaying matrix and uses it for the selected relay nodes. Significant gain, e.g., 4 dB in SNR at BER $= 10^{-2}$, can be obtained with the proposed relay selection method, compared to the one in [15].

Section II describes the system model. Section III derives the explicit optimal relay amplifying matrices $\mathbf{F}^\star$ and the diagonal weight matrices, where the superscript * denotes the optimum. Section IV analyzes the MMSE CF behavior using the explicit optimal $\mathbf{F}^\star$. Section V studies the total SCP of the received signals at the destinations using the explicit optimal $\mathbf{F}^\star$. Section VI proposes a novel relay selection scheme based on the MMSE criterion. Section VII presents simulation results. Section VIII concludes the paper.

Notation: Matrices and vectors are denoted, respectively, by uppercase and lowercase boldface characters (e.g., $\mathbf{A}$ and $\mathbf{a}$). The transpose, complex conjugate, inverse, trace, pseudo-inverse, and Hermitian of $\mathbf{A}$ are denoted, respectively, by $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, $tr(\mathbf{A})$, $\mathbf{A}^\dagger$, and $\mathbf{A}^H$. An $N \times N$ identity matrix is denoted by $\mathbf{I}_N$. Notations $|a|$, $||\mathbf{a}||$, and $||\mathbf{A}||_F$ denote the absolute value of a for any scalar, 2-norm of $\mathbf{a}$, and Frobenius-norm of $\mathbf{A}$, respectively. The real, expectation, Kronecker product, and rank operators are denoted by $\text{Re}\{\mathbf{A}\} = (\mathbf{A} + \mathbf{A}^*)/2$, $E[\,\cdot\,]$, $\otimes$, and rank($\mathbf{A}$), respectively.

II. System Models and Data Transmission

There are multiple definitions about cooperative and noncooperative AF relay networks in the literature. In this paper, the following definition is used, based on the ones in [9]–[11]:

Definition : A relay network is called a cooperative AF relay network if all global channel coefficients are available at a central node, e.g., a CRAN. Otherwise, the relay network is called a noncooperative relay network when only the local channel coefficients connected to a relay node are available at the relay node. In other words, if the relay amplifying matrix $\mathbf{F}$ is nondiagonal, then it is called a cooperative AF relay network. Otherwise, if the relay amplifying matrix $\mathbf{F}$ is diagonal, then the relay network is called a noncooperative relay network.

Figure 1 shows data transmission between M source-destination pairs through cooperative distributed N relays $(N \geq M)$. The dotted block in Fig. 1 is for a noncooperative AF relay network. All communication nodes have only one single antenna. For illustration purpose, at each relay, the transmit and receive antennas are separate, but one antenna serve both transmit and receive functions. Assume that channel coefficient matrices $\mathbf{H}_s \in \mathbf{C}^{N \times M}$, $\mathbf{H}_y \in \mathbf{C}^{M \times N}$, and the received vector $\mathbf{r}$ at the relays are available at a CSS and that the CSS can compute the optimal relay amplifying matrices $\mathbf{F}^\star \in \mathbf{C}^{N\times N}$ using the proposed ones in this paper and forward the amplified signal vector $\mathbf{x} = \mathbf{Fr}$ to the relays. This may be feasible in a 5G CRAN system. The purpose of this paper is to find the optimal $\mathbf{F}^\star$ under power constraints in closed forms. During data transmission, power can be constrained at the relays as well as globally at the destination. There are two stages: Stage I (broadcasting stage), where sources broadcast the signal vector $\mathbf{s} \in \mathbf{C}^{M\times 1} = [s_1, \cdots, s_M]^T$, and Stage II (relaying stage), where all relays retransmit their received signals to destinations after multiplying by an $N \times N$ optimal relay amplifying matrix $\mathbf{F}^\star$ for the linear processing operation. There have been extensive studies about $\mathbf{F}^\star$, but closed forms are not yet available.

The received complex signal column vector $\mathbf{y} \in \mathbf{C}^{M\times 1}$ at the destinations can be written as

$$\mathbf{y} = \mathbf{H}_y\mathbf{x} + \mathbf{v}_y = \mathbf{H}_y\mathbf{Fr} + \mathbf{v}_y = \mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s\mathbf{s} + \mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{v}_s + \mathbf{v}_y \tag{1}$$

where $\mathbf{H}_s = [\mathbf{h}_{s,1}, \mathbf{h}_{s,2}, \cdots, \mathbf{h}_{s,M}] \in \mathbf{C}^{N\times M}$ is a backward channel matrix consisting of complex channel coefficients between the sources and the relays, $\mathbf{h}_{s,m} = [h_{s,m,1}, \cdots, h_{s,m,N}]^T \in \mathbf{C}^{N\times 1}, m = 1, \cdots, M$, is a column vector representing the channel coefficients from the m-th source to all N relays, $\mathbf{H}_y = [\mathbf{h}_{y,1}, \cdots, \mathbf{h}_{y,M}]^T \in \mathbf{C}^{M\times N}$ is a forward channel matrix consisting of complex channel coefficients between the relays and the destinations, $\mathbf{h}_{y,m} = \left[h_{y,m,1}, \cdots, h_{y,m,N}\right] \in \mathbf{C}^{1\times N}$, $m = 1, \cdots, M$, is a row vector representing the channel coefficients from all relays to the m-th destination, all channel coefficients $h_{s,m,i}$ and $h_{y,m,i}$ are independent identically distributed (i.i.d.) complex Gaussian$\sim CN(0,1)$, $\mathbf{r} = \boldsymbol{\Gamma}\mathbf{H}_s\mathbf{s} + \boldsymbol{\Gamma}\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is the received signal column vector at the relays, $\boldsymbol{\Gamma} \in \mathbf{C}^{N\times N}$ is a diagonal weight matrix consisting of the positive scaling factor γ_i on the diagonal to limit the i-th relay receive power, $\mathbf{v}_s \in \mathbf{C}^{N\times 1} \sim CN(0, \sigma_{v_s}^2\mathbf{I}_N)$ is an additive white Gaussian noise (AWGN) vector at the relays, $\mathbf{x} = \mathbf{Fr} \in \mathbf{C}^{N\times 1}$ is the amplified signal vector at the relay outputs, $\mathbf{F} \in \mathbf{C}^{N\times N}$ is a relay amplifying matrix, and $\mathbf{v}_y \in \mathbf{C}^{M\times 1} \sim CN(0, \sigma_{v_y}^2\mathbf{I}_M)$ is an AWGN vector at the destinations. In addition, the signal y_m at the m-th destination is equalized by a positive scaling factor ω_m to produce the estimated $\hat{y}_m = \omega_m^{-1}y_m, m = 1, \cdots, M$. For simplicity, a common equalization factor ω is used for all $m, m = 1, \cdots, M$. In the next section, the optimal $\mathbf{F}^\star$ will be determined by using MMSE and ZF criterions. The equalization scaling factor ω and the diagonal weight matrix $\boldsymbol{\Gamma}$ are also determined.

III. Cooperative Distributed Wireless Relay Schemes

A. MMSE Relay Scheme

1) Transmit Power Constraint (TPC): The optimum $\mathbf{F}^\star$, which minimizes the MSE between the originally transmitted signal vector $\mathbf{s}$ from the sources and the equalized signal vector $\hat{\mathbf{y}}\left(\triangleq \omega^{-1}\mathbf{y}\right)$ at the destinations, is determined under TPC as

$$\mathbf{F}^\star = \arg\min_{\mathbf{F},\omega} J(\mathbf{F}) \quad \text{s.t. } E\left[||\mathbf{x}||^2\right] = p_{\text{R}} \tag{2}$$

where p_{R} is the total TPC at the relays. Here, using the definition of the MMSE [25], the CF $J(\mathbf{F}) \triangleq E[||\mathbf{s} - \hat{\mathbf{y}}||^2]$ is written as

$$J(\mathbf{F}) = \omega^{-2}p_s\|\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s\|_F^2 + \omega^{-2}\sigma_{v_s}^2\|\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\|_F^2 + Mp_s - 2\omega^{-1}p_s tr\left(\text{Re}\left[\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s\right]\right) + \omega^{-2}M\sigma_y^2 \tag{3}$$

under the assumption that the data symbols, channel coefficients, and noises are independent of each other, where $E[|s_i|^2] = p_{s_i}$, $p_{s_1} = \cdots = p_{s_M} = p_s$, $\sigma_{v_{s_1}}^2 = \cdots = \sigma_{v_{s_N}}^2 = \sigma_{v_s}^2$, $\sigma_{v_{y_1}}^2 = \cdots = \sigma_{v_{y_M}}^2 = \sigma_{v_y}^2$, $\omega_1 = \cdots = \omega_m = \omega$, $E[\mathbf{v}_s] = \mathbf{0}$, and $E[\mathbf{v}_y] = \mathbf{0}$ are assumed. Here, $\mathbf{0}$ is a vector consisting of all zero entries with appropriate dimensions. The total TPC $p_{\text{R}}\left(\triangleq E\left[||\mathbf{x}||^2\right]\right)$ at the relays can be written as

$$p_{\text{R}} = p_s\|\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s\|_F^2 + \sigma_{v_s}^2\|\mathbf{F}\boldsymbol{\Gamma}\|_F^2. \tag{4}$$

The constrained optimization problem under total TPC p_{R} at the relays can be written, with the Lagrangian multiplier λ [26], as

$$L(\mathbf{F}, \omega, \lambda) = J(\mathbf{F}) + \lambda\left(E\left[||\mathbf{x}||^2\right] - p_{\text{R}}\right). \tag{5}$$

Theorem 1: The constrained Lagrangian optimization $L(\mathbf{F}, \omega, \lambda)$ in (5) is convex or strictly quasi-convex with respect to $\mathbf{F}$ and ω. Hence, the solutions in (5) for $\mathbf{F}$ and ω are the global optimum.

Proof: The CF $J(\mathbf{F})$ in (5) is a convex function of $\mathbf{F}$ for a given ω because any linear composition with the norm is convex [26], [27]. Additionally, the convexity of $L(\mathbf{F}, \omega, \lambda)$ with respect to ω can be proven for the first and second derivatives of $L(\mathbf{F}, \omega, \lambda)$ with respect to ω for a given $\mathbf{F}$. Refer to Appendix A for the detailed proof. ■

Theorem 2: Using the constrained Lagrangian optimization $L(\mathbf{F}, \omega, \lambda)$ in (5), the closed forms of optimal $\mathbf{F}^\star$ at the relays, the optimal scale factor gain $\omega^\star$ at the destinations, the corresponding diagonal weight matrix $\boldsymbol{\Gamma}$ at the relay inputs, and the positive optimal Lagrangian multiplier $\lambda^\star$ can be written, respectively, as

$$\mathbf{F}^\star = \frac{\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\boldsymbol{\Gamma}^{-1}\sqrt{p_{\text{R}}}}{\sqrt{p_s\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}} \tag{6}$$

$$\omega^\star = \sqrt{\frac{p_{\text{R}}p_s^{-2}}{p_s\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}} \tag{7}$$

$$\boldsymbol{\Gamma}^\star = \left(M + \frac{\sigma_{v_s}^2}{p_s}\right)^{-\frac{1}{2}}\mathbf{I}_N \tag{8}$$

$$\lambda^\star = \frac{M\sigma_{v_y}^2p_s^2}{p_{\text{R}}^2}\left(tr\left(\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\mathbf{H}_y\mathbf{H}_t^H\right)\right) \tag{9}$$

where

$$\mathbf{H}_d = \left(p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_N\right)^{-1} \tag{10}$$

$$\mathbf{H}_t = \left(\mathbf{H}_y^H\mathbf{H}_y + \frac{M\sigma_{v_y}^2}{p_{\text{R}}}\mathbf{I}_N\right)^{-1}. \tag{11}$$

Proof: Refer to Appendix B. ■

Note that the optimal equalization factor $\omega^\star$ at the destinations in (7) are, in fact, the Wiener filter [28], which is given by $\omega^\star = p_y p_s^{-1} tr\big(\text{Re}[\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s]\big)^{-1}$, where $p_y \triangleq E\big[||\mathbf{y}||^2\big] = p_s\|\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\mathbf{H}_s\|_F^2 + \sigma_{v_s}^2\|\mathbf{H}_y\mathbf{F}\boldsymbol{\Gamma}\|_F^2 + M\sigma_{v_y}^2$. For a special case with no optimal scaling factor ω, i.e., $\omega = 1$ in (3), the optimal $\mathbf{F}^\star_{\text{TPC}}$ can be written from (2) to (5) as

$$\mathbf{F}^\star_{\text{TPC}} = \frac{\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\boldsymbol{\Gamma}^{-1}\sqrt{p_{\text{R}}}}{\sqrt{p_s\left\|\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}} \tag{12}$$

where the superscript † refers to the pseudo-inverse, and $\mathbf{A}^\dagger = \big(\mathbf{A}^H\mathbf{A}\big)^\dagger\mathbf{A}^H$ if $\mathbf{A} \in \mathbf{C}^{m\times n}$ is a right matrix, i.e., $n > m$ [29].

2) Receiver Power Constraint (RPC): Constraining the received power at the destinations can be considered similar to the TPC at the relays. Hence, using the CF under RPC[1], i.e., $J(\mathbf{F}_{\text{RPC}}) \triangleq E[||\mathbf{s} - \hat{\mathbf{y}}||^2]$, and the Lagrangian multiplier λ_{RPC}, the constrained Lagrangian optimization problem can be written as

$$L(\mathbf{F}_{\text{RPC}}, \lambda_{\text{RPC}}) = J(\mathbf{F}_{\text{RPC}}) + \lambda_{\text{RPC}}\big(E\big[||\mathbf{H}_y\mathbf{x}||^2\big] - p_{\text{D}}\big) \tag{13}$$

where the total SCP $p_{\text{D}}\big(\triangleq E\big[||\mathbf{H}_y\mathbf{x}||^2\big]\big)$ of the received signals at the destinations, from (1), is

$$p_{\text{D}} = p_s||\mathbf{H}_y\mathbf{F}_{\text{RPC}}\boldsymbol{\Gamma}\mathbf{H}_s||_F^2 + \sigma_{v_s}^2||\mathbf{H}_y\mathbf{F}_{\text{RPC}}\boldsymbol{\Gamma}||_F^2. \tag{14}$$

Following the steps of the previous Lagrangian optimization solution in (5), the explicit optimal $\mathbf{F}^\star_{\text{RPC}}$ for the cooperative distributed AF relay network under RPC can be written as

$$\mathbf{F}^\star_{\text{RPC}} = \frac{\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\boldsymbol{\Gamma}^{-1}\sqrt{p_{\text{D}}}}{\sqrt{p_s\left\|\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}}. \tag{15}$$

Substituting (15) into (4), the total transmit power $p_{\text{R-RPC}}$ at the relays under RPC with $p_{\text{D}} = 1$ can be written as

$$p_{\text{R-RPC}} = \frac{E\big[tr\big(\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\big(\mathbf{H}_y^\dagger\big)^H\big)\big]}{E\big[tr\big(\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\big)\big]} \approx \frac{1}{N-M} \tag{16}$$

where $E\big[tr\big(\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\big(\mathbf{H}_y^\dagger\big)^H\big)\big] \approx \frac{M}{N-M}$ using *Lemma 1* in [30] when $N \neq M$, and $E\big[tr\big(\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\big)\big] = M$, if N or M is large enough or if an input signal-to-noise ratio (SNR) is high.

3) No-Power Constraint (NPC)[2]: Although the power constraint can be intentionally avoided during data transmission, this paper applies a positive scaling factor ω_{NPC} in the MMSE CF to meet the target signal-to-noise ratio (SNR) (SNR_{TAT}) at the destinations. This modified NPC is more desirable than the NPC without a scaling factor because as long as SNR_{TAT} is met at the destinations, any redundant power in the received signal can be avoided. Hence, using the positive scaling factor ω_{NPC} and the CF $J(\mathbf{F}_{\text{NPC}}) \triangleq E\big[||\mathbf{y} - \omega_{\text{NPC}}^{-1}\mathbf{s}||^2\big]$, the optimal $\mathbf{F}^\star_{\text{NPC}}$ under NPC for the cooperative distributed AF relay system can be derived as

$$\mathbf{F}^\star_{\text{NPC}} = \omega_{\text{NPC}}p_s\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\boldsymbol{\Gamma}^{-1} \tag{17}$$

[1]The true RPC including thermal noise power should be $p_y = p_{\text{D}} + \sigma_{v_y}^2 M$. However, the RPC in this paper is simplified to p_{D} because $\sigma_{v_y}^2 M$ is a constant. Note that the SCP p_{D} in (14) includes $\sigma_{v_s}^2$.

[2]The NPC in this paper is a modified NPC because a positive scaling factor ω_{NPC} is applied at the destinations to meet the target SNR_{TAT}.

where $\mathbf{A}^\dagger = \big(\mathbf{A}^H\mathbf{A}\big)^{-1}\mathbf{A}^H$ is also used, and ω_{NPC} is defined as

$$\omega_{\text{NPC}} = \left(M\text{SNR}_{\text{TAT}}\frac{\sigma_{v_y}^2}{\sigma_s^2}\right)^{-\frac{1}{2}}. \tag{18}$$

In (18), note that ω_{NPC} would be 1 if the SNR_{TAT} is set to $M^{-1}\frac{\sigma_s^2}{\sigma_{v_y}^2}$ at the destinations. In this paper, $\omega_{\text{NPC}} = 1$ is selected for simulation in Section VII because if the channels are AWGN, then the SNR at the destinations would be $\frac{\sigma_s^2}{M\sigma_{v_y}^2}$. In addition, using the optimal $\mathbf{F}^\star_{\text{NPC}}$ in (17) and the positive scaling factor ω_{NPC} in (18) with $p_{\text{D}} = 1$, the power usage $p_{\text{R-NPC}}$ at the relays for N and M large enough or a high input SNR can be shown as

$$p_{\text{R-NPC}} = \frac{1}{N-M}. \tag{19}$$

In other words, using the optimal $\mathbf{F}^\star_{\text{NPC}}$, similar to the RPC case, the power usage at the relays under modified NPC drops by $\frac{1}{N-M}$.

B. ZF Relay Scheme

The ZF optimization under TPC at the relays can be written as

$$\mathbf{F}^\star_{\text{ZF}} = \arg\min_{\mathbf{F}_{\text{ZF}}} J(\mathbf{F}_{\text{ZF}}) \quad \text{s.t. } E\big[||\mathbf{x}||^2\big] = p_{\text{R}} \tag{20}$$

where the CF $J(\mathbf{F}_{\text{ZF}}) = E\big[||\mathbf{s} - \hat{\mathbf{s}}||^2\big]$ and $\hat{\mathbf{s}} = \mathbf{H}_y\mathbf{F}_{\text{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\mathbf{s}$ [31], which is a noise-free signal vector in (1). And the constrained Lagrangian optimization can be written as

$$L(\mathbf{F}_{\text{ZF}}, \lambda_{\text{ZF}}) = J(\mathbf{F}_{\text{ZF}}) + \lambda_{\text{ZF}}\big(E\big[||\mathbf{x}||\big]^2 - p_{\text{R}}\big). \tag{21}$$

Theorem 3: The optimal $\mathbf{F}^\star_{\text{ZF}}$ under TPC and the optimal positive scaling factor gain $\omega^\star_{\text{ZF}}$ at the destinations can be written, respectively, as

$$\mathbf{F}^\star_{\text{ZF}} = \frac{\mathbf{H}_y^\dagger\mathbf{H}_s^\dagger\boldsymbol{\Gamma}^{-1}\sqrt{p_{\text{R}}}}{\sqrt{p_s\|\mathbf{H}_y^\dagger\|_F^2 + \sigma_{v_s}^2\|\mathbf{H}_y^\dagger\mathbf{H}_s^\dagger\|_F^2}} \tag{22}$$

$$\omega^\star_{\text{ZF}} = \sqrt{\frac{p_{\text{R}}}{p_s\|\mathbf{H}_y^\dagger\|_F^2 + \sigma_{v_s}^2\|\mathbf{H}_y^\dagger\mathbf{H}_s^\dagger\|_F^2}}. \tag{23}$$

Proof: Refer to Appendix C. ■

Note that $\boldsymbol{\Gamma}$ in (12), (15), (17), and (22) is the same as in (8). Also note that the total number of computations including multiplications, additions, and divisions to compute the proposed optimum relaying amplifying matrices in (6) and (22) for the MMSE and ZF schemes are, respectively, $8N^3 + 26N^2M - 2NM - N^2 + 7N + 8$ and $\frac{5}{3}N^3 + 22N^2M - 3NM - 3N^2 + \frac{N}{3} + 2$ [32]. Hence, the complexity to calculate the MMSE AF relay amplifying matrix is approximately 4.8 times higher than that of the ZF relay amplifying matrix by considering only the highest order of N.

In the literature survey, the authors of this paper found that the $\mathbf{F}^\star$ in (13) of [9] is the most explicit one, which is given by

$$\mathbf{F}^\star_{\text{EXT}} = p_s\big(\mathbf{H}_y^H\mathbf{H}_y + \lambda_{\text{EXT}}\mathbf{I}_N\big)^{-1}\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d. \tag{24}$$

However, (24) is expressed in terms of the Lagrangian multiplier λ_{EXT}, which is introduced for the TPC at the relays. The

λ_{EXT} is searched separately using a complicated equation (14) of [9] as

$$p_{\text{R}} = p_s^2 tr\left(\left(\mathbf{H}_y^H \mathbf{H}_y + \lambda_{\text{EXT}} \mathbf{I}_N\right)^{-1} \mathbf{H}_p \left(\mathbf{H}_y^H \mathbf{H}_y + \lambda_{\text{EXT}} \mathbf{I}_N\right)^{-1}\right) \quad (25)$$

where $\mathbf{H}_p = \mathbf{H}_y^H \mathbf{H}_s^H \mathbf{H}_d \mathbf{H}_s \mathbf{H}_y$. Hence, the $\mathbf{F}^\star$ in (13) of [9] (or $\mathbf{F}_{\text{EXT}}^\star$ in (24)) is an implicit rather than explicit expression.

For more performance comparison, the $\mathbf{F}_{\text{MF}}^\star$ for the cooperative AF MIMO relay network based on the matched filter (MF) criterion [33] under the TPC can be written as

$$\mathbf{F}_{\text{MF}}^\star = \frac{\mathbf{H}_y^H \mathbf{H}_s^H \sqrt{p_{\text{R}}}}{\sqrt{tr\left(\mathbf{H}_g \mathbf{H}_d^{-1} \mathbf{H}_g^H\right)}} \quad (26)$$

using the linearity of the trace function, i.e., $a_1 tr(\mathbf{A}_1) + a_2 tr(\mathbf{A}_2) = tr(a_1 \mathbf{A}_1 + a_2 \mathbf{A}_2)$, where $\mathbf{H}_g \triangleq \mathbf{H}_y^H \mathbf{H}_s^H$ is the relay transceiver matched to the channels from the sources to the destinations.

Finally, using the SNR at the destination, the achievable rate for an AF SISO relay network (i.e., $\mathbf{H}_s \rightarrow \mathbf{h}_s = [h_{s,1}, \cdots, h_{s,N}]^T$ and $\mathbf{H}_y \rightarrow \mathbf{h}_y = [h_{y,1}, \cdots, h_{y,N}]$) can be calculated as

$$\mathcal{R} = \frac{1}{2} \log_2(1 + \text{SNR}) \quad (27)$$

where

$$\text{SNR} = \frac{p_s |\mathbf{h}_y \mathbf{F}_{\text{AR}} \mathbf{h}_s|^2}{\sigma_{v_s}^2 ||\mathbf{h}_y \mathbf{F}_{\text{AR}}||^2 + \sigma_{v_y}^2}. \quad (28)$$

From (27) and (28), the maximum achievable rate optimization problem at the destination can be written as

$$\mathbf{F}_{\text{AR}}^\star = \arg \max_{\mathbf{F}_{\text{AR}}} \text{SNR}. \quad (29)$$

Using the Kronecker product and vectorization operators and applying the generalized Rayleigh quotient, the maximum sum rate optimization can be rewritten as

$$\max_{\mathbf{F}_{\text{AR}}} \frac{\mathbf{f}_{\text{AR}}^H \mathbf{Z} \mathbf{f}_{\text{AR}}}{\mathbf{f}_{\text{AR}}^H \mathbf{G} \mathbf{f}_{\text{AR}}} = \mathbf{eig}_{\max}\left(\mathbf{G}^{-1}\mathbf{Z}\right) \quad (30)$$

where $\mathbf{f}_{\text{AR}} \left(\in \mathbf{C}^{N^2 \times 1}\right) \triangleq \text{vec}(\mathbf{F}_{\text{AR}})$, $\mathbf{Z}\left(\triangleq \mathbf{z}^H \mathbf{z}\right)$ is an $N^2 \times N^2$ positive definitive and Hermitian matrix with rank one, $\mathbf{z}$ is a $1 \times N^2$ row vector defined as $\mathbf{z} \triangleq \sqrt{p_s}(\mathbf{h}_s^T \otimes \mathbf{h}_y)$, $\mathbf{G} \triangleq \sigma_{v_s}^2 \mathbf{H}_x + \sigma_{v_y}^2 p_r^{-1} \mathbf{W})$ is also an $N^2 \times N^2$ positive definitive and Hermitian matrix with rank N^2, $\mathbf{H}_x$ is an $N^2 \times N^2$ matrix defined as $\mathbf{H}_x \triangleq \left(\mathbf{I}_N \otimes \mathbf{h}_y^H \mathbf{h}_y\right)^H$, $\mathbf{W}$ is an $N^2 \times N^2$ matrix defined as $\mathbf{W} \triangleq \left((\mathbf{h}_s \mathbf{h}_s^H)^T + \sigma_{v_s}^2 \mathbf{I}_N\right) \otimes \mathbf{I}_N$, and $\mathbf{eig}_{\max}(\mathbf{G}^{-1}\mathbf{Z})$ is the maximum eigenvalue of $\mathbf{G}^{-1}\mathbf{Z}$. Additionally, using Cholesky factorization and the given conditions of $\mathbf{Z}$ and $\mathbf{G}$, $\mathbf{f}_{\text{AR}}$ can be written as

$$\mathbf{f}_{\text{AR}} = \varpi \boldsymbol{\upsilon}_{\max}(\mathbf{G}^{-1}\mathbf{Z}) = \varpi \mathbf{G}^{-1} \mathbf{z}^H \quad (31)$$

where $\boldsymbol{\upsilon}_{\max}(\mathbf{G}^{-1}\mathbf{Z})$ is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{G}^{-1}\mathbf{Z}$, and ϖ is a positive scaling factor to adjust the transmit power usage p_r at the relays. Using (31) and the relay transmission power consumption, ϖ can be written as

$$\varpi = \frac{\sqrt{p_r}}{\sqrt{\mathbf{z}\mathbf{G}^{-1}\mathbf{W}\mathbf{G}^{-1}\mathbf{z}^H}}. \quad (32)$$

Finally, substituting (32) into (31), the closed form of the optimal $\mathbf{f}_{\text{AR}}^\star$ under the relay transmission power constraint can be written as

$$\mathbf{f}_{\text{AR}}^\star = \frac{\mathbf{G}^{-1}\mathbf{z}^H \sqrt{p_r}}{\sqrt{\mathbf{z}\mathbf{G}^{-1}\mathbf{W}\mathbf{G}^{-1}\mathbf{z}^H}} \quad (33)$$

with the corresponding optimal relay amplifying matrix $\mathbf{F}_{\text{AR}}^\star$ given by $\mathbf{F}_{\text{AR}}^\star = reshape(\mathbf{f}_{\text{AR}}^\star, N, N)$, where the *reshape* operator denotes the reshape of an $N^2 \times 1$ vector $\mathbf{f}_{\text{AR}}^\star$ to an $N \times N$ nondiagonal matrix.

For a cooperative AF MIMO relay network, it is not a simple optimization problem but a joint optimization to find one common amplifying matrix $\mathbf{F}$ that maximizes the sum rate given by

$$\mathcal{R} = \frac{1}{2} \sum_{m=1}^{M} \log_2(1 + \text{SINR}_m) \quad (34)$$

where

$$\text{SINR}_m = \frac{p_s |\mathbf{h}_{y,m} \mathbf{F} \mathbf{h}_{s,m}|^2}{\sum_{k=1, k \neq m}^{M} |\mathbf{h}_{y,m} \mathbf{F} \mathbf{h}_{s,k}|^2 + \sigma_{v_s}^2 ||\mathbf{h}_{y,m} \mathbf{F}||^2 + \sigma_{v_{ym}}^2}. \quad (35)$$

Hence, a future challenging problem would be to find an optimum cooperative AF relay amplifying matrix that maximizes the sum rate.

IV. CF Behavior

The smaller the CF value, the smaller the MSE because the cost function $J(\mathbf{F})$ in this paper is defined as the MSE in (3), and vice versa [34]. That is, the system BER performance either improves or degrades according to the MSE values. Additionally, the number of relays (*N*) affects the MSE values for a given M. Similarly, the number of sources (*M*) affects the MSE values for a given *N*. Hence, in this section, the CF behavior will be mathematically investigated by varying the parameters *M* and *N*.

Proposition 1: The CF $J(\mathbf{F}^\star)$ in (3) under TPC decreases as N increases for a given M.

Proof: Using the optimal $\mathbf{F}^\star$ in (6) and the linearity of the trace function, the MMSE CF $J(\mathbf{F}^\star)$ in (3) can be written as

$$J(\mathbf{F}^\star) = p_s \sum_{i=1}^{M} \delta_i - 2p_s^2 \sum_{i=1}^{M} \phi_i + M\left(p_s + \frac{\sigma_{v_y}^2}{\omega^2}\right) \quad (36)$$

where δ_i and ϕ_i are the i-th positive eigenvalues of $\mathbf{H}_q \in \mathbf{C}^{M \times M}$ and $\mathbf{H}_b \in \mathbf{C}^{M \times M}$ with $\mathbf{H}_q \triangleq \mathbf{H}_y \mathbf{H}_t \mathbf{H}_p \mathbf{H}_t \mathbf{H}_y^H$ and $\mathbf{H}_b \triangleq \mathbf{H}_y \mathbf{H}_t \mathbf{H}_p \mathbf{H}_y^\dagger$, which are conjugate symmetric with $\text{rank}(\mathbf{H}_q) = M$ and $\text{rank}(\mathbf{H}_b) = M$, respectively, i.e., all eigenvalues of $\mathbf{H}_q$ and $\mathbf{H}_b$ are positive. Recall that $\mathbf{H}_t = \left(\mathbf{H}_y^H \mathbf{H}_y + M\sigma_{v_y}^2 p_{\text{R}}^{-1} \mathbf{I}_N\right)^{-1}$ and $\mathbf{H}_p = \mathbf{H}_y^H \mathbf{H}_s^H \mathbf{H}_d \mathbf{H}_s \mathbf{H}_y$. It can be analytically shown that the difference between $\sum_{i=1}^{M} \delta_i$ and $2\sum_{i=1}^{M} \phi_i$ in (36) gets smaller as N increases for a given M and p_s, e.g., $p_s = 1$. In addition, $M\left(p_s + \sigma_{v_y}^2 \omega^{-2}\right)$ decreases as N increases. This is because the optimal scaling factor $\omega^\star$ in (7) increases as N increases. Hence, the $J(\mathbf{F}^\star)$ value in (36) decreases as N increases. ■

In summary, the system BER performance improves as N increases for a given M. On the other hand, it can be clearly seen that the overall MMSE CF $J(\mathbf{F}^\star)$ value in (36) increases as M increases for a given N. Hence, the BER performance degrades as M increases for a given N. Additionally, using the optimum equalization scaling factor ω in (7), the original CF $J(\mathbf{F})$ in (3) can be rewritten as

$$J(\mathbf{F}) = p_s tr\big((p_s \mathbf{H}^H \mathbf{H}_q^{-1} \mathbf{H} + \mathbf{I}_{\text{N}})^{-1}\big) \quad (37)$$

where $\mathbf{H} = \mathbf{H}_y \mathbf{F} \mathbf{\Gamma} \mathbf{H}_s$ and $\mathbf{H}_q = \sigma_{v_s}^2 \mathbf{H}_y \mathbf{F} \mathbf{\Gamma} \mathbf{\Gamma}^H \mathbf{F}^H \mathbf{H}_y^H + \sigma_{v_y}^2 \mathbf{I}_{\text{M}}$. From (37), when $M = 1$, it can be seen that the minimizing MSE is equivalent to the SNR maximization as

$$J(\mathbf{F}) = (1 + \text{SNR})^{-1}. \quad (38)$$

Here, the optimal relay amplifying matrix and the SNR at the destination in (38) can be determined using (1) and (6) when $M = 1$. In particular, using (38), the achievable rate $\mathcal{R}$ can be written as $\mathcal{R} = -\frac{1}{2}\log_2 J(\mathbf{F})$ when $M = 1$.

Proposition 2: The CF $J(\mathbf{F}_{\text{RPC}}^\star)$ in (13) under RPC converges to a constant as N increases.

Proof: Using the optimal $\mathbf{F}_{\text{RPC}}^\star$ in (15), the MMSE CF $J(\mathbf{F}_{\text{RPC}}^\star)$ in (13) can be rewritten as

$$J(\mathbf{F}_{\text{RPC}}^\star) = p_{\text{D}} + M p_s + M \sigma_{v_y}^2 - 2 p_s \left(p_{\text{D}} \sum_{i=1}^{M} \psi_i \right)^{-\frac{1}{2}} \quad (39)$$

where ψ_i is the i-th positive eigenvalue of $\mathbf{H}_o \triangleq \mathbf{H}_s \mathbf{H}_s^H \mathbf{H}_d \in \mathbf{C}^{N \times N}$, which is a conjugate symmetric with rank$(\mathbf{H}_o) = M$, i.e., all eigenvalues of $\mathbf{H}_o$ are nonnegative, and the number of zero-eigenvalues is $(N - M)$. From (39), $J(\mathbf{F}_{\text{RPC}}^\star)$ converges because $\sum_{i=1}^{M} \psi_i$ converges to M as N increases for a given M, p_{D}, p_s, and $\sigma_{v_y}^2$. Additionally, it can be easily seen that the $J(\mathbf{F}_{\text{RPC}}^\star)$ in (39) increases as M increases for a given N. In particular, p_{D} and p_s can be constrained to 1 when the RPC is applied. Hence, $J(\mathbf{F}_{\text{RPC}}^\star)$ in (39) converges to

$$J(\mathbf{F}_{\text{RPC}}^\star) = \big(\sqrt{M} - 1\big)^2 + M \sigma_{v_y}^2. \quad (40)$$

■

In summary, the system BER performance converges as N increases for a given M, while it degrades as M increases for a given N.

Proposition 3: The CF $J(\mathbf{F}_{\text{NPC}}^\star)$ under NPC with $\mathbf{F}_{\text{NPC}}^\star$ in (17) converges to a constant as N increases.

Proof: Using the optimal $\mathbf{F}_{\text{NPC}}^\star$ in (17), the MMSE CF $J(\mathbf{F}_{\text{NPC}}^\star)$ can be rewritten as

$$J(\mathbf{F}_{\text{NPC}}^\star) = \omega_{\text{NPC}}^2 p_s^3 \sum_{i=1}^{M} \theta_i + \omega_{\text{NPC}}^2 p_s^2 \sigma_{v_s}^2 \sum_{i=1}^{M} \varphi_i - 2 p_s^2 \sum_{i=1}^{M} \psi_i + M \sigma_{v_y}^2 + M \omega_{\text{NPC}}^{-2} p_s \quad (41)$$

where θ_i and φ_i are the i-th positive eigenvalues of $\mathbf{H}_o \mathbf{H}_o^H$ and $\mathbf{H}_o \mathbf{H}_d$, which are conjugate symmetric with rank$(\mathbf{H}_o \mathbf{H}_o^H) = M$ and rank$(\mathbf{H}_o \mathbf{H}_d) = M$, respectively. Assume that N is large enough for a given M and $\sigma_{v_y}^2 = \sigma_{v_s}^2 = \sigma_v^2$. In addition, ω_{NPC} and p_s are normalized to 1, respectively. Then, $J(\mathbf{F}_{\text{NPC}}^\star)$ in (41) can be rewritten as

$$J(\mathbf{F}_{\text{NPC}}^\star) = M \sigma_v^2 \Big(1 + \frac{1}{N - M}\Big) \quad (42)$$

because $\sum_{i=1}^{M} \theta_i = M$, $\sum_{i=1}^{M} \varphi_i = M(N-M)^{-1}$, and $N > M$ for N to be large enough for a given M. Hence, $J(\mathbf{F}_{\text{NPC}}^\star)$ decreases as N increases for a given M. But, if N is large enough, then $J(\mathbf{F}_{\text{NPC}}^\star)$ converges to a constant, i.e., $M\sigma_v^2$, as $J(\mathbf{F}_{\text{RPC}}^\star)$ does in (39). On the other hand, $J(\mathbf{F}_{\text{NPC}}^\star)$ in (41) increases as M increases for a given N, similar to $J(\mathbf{F}_{\text{RPC}}^\star)$ in (39). ■

Proposition 4: For the ZF relay scheme, the CF $J(\mathbf{F}_{\text{ZF}}^\star)$ in (21) under TPC increases as N increases.

Proof: Using the optimal $\mathbf{F}_{\text{ZF}}^\star$ in (22), the CF $J(\mathbf{F}_{\text{ZF}}^\star)$ in (21) can be rewritten as

$$J(\mathbf{F}_{\text{ZF}}^\star) = M p_s \big(\omega_{\text{ZF}} - 1\big)^2. \quad (43)$$

Hence, $J(\mathbf{F}_{\text{ZF}}^\star)$ increases as N increases for a given M because ω_{ZF} increases as N increases, unlike $J(\mathbf{F}^\star)$ in (36). In general, the CF value decreases as the MSE decreases if the CF is defined as the MMSE [34]. However, in a ZF scheme, only the noise-free signal vector is used to apply the CF. As a result, as N increases, $J(\mathbf{F}_{\text{ZF}}^\star)$ increases.[3] ■

Similar to $J(\mathbf{F}^\star)$, $J(\mathbf{F}_{\text{RPC}}^\star)$, and $J(\mathbf{F}_{\text{NPC}}^\star)$, $J(\mathbf{F}_{\text{ZF}}^\star)$ in (43) increases as M increases for a given N. Note that $J(\mathbf{F}^\star)$ in (36) is smaller than $J(\mathbf{F}_{\text{RPC}}^\star)$ in (39). As a result, the BER performance applying $\mathbf{F}^\star$ in (6) is smaller than the one applying $\mathbf{F}_{\text{RPC}}^\star$ in (15), in particular, for N to be large enough. This analytical result will be verified through the simulation in Section VII. However, comparing $J(\mathbf{F}^\star)$ in (36) with $J(\mathbf{F}_{\text{ZF}}^\star)$ in (43) is not fair because they have different CFs. Hence, to provide a fair comparison, the SCP of the received signals at the destinations will be considered in the next section.

V. SCP of Received Signals at Destinations

The larger the SCP of the received signals at the destinations, the better the BER performance for a given noise power. Based on this relationship, the BER performance will be examined in this section.

Proposition 5: The SCP at the destinations under the TPC using the MMSE with the scaling factor ω including 1 increases as N increases.

Proof: Using (1) and the explicit optimal $\mathbf{F}^\star$ in (6) under the TPC, the total SCP $p_{\text{D}_{\text{MMSE}}}$ of the received signals at the destinations can be written as

$$p_{\text{D}_{\text{MMSE}}} = K_{\text{MMSE}} p_{\text{R}} \quad (44)$$

where $K_{\text{MMSE}} \triangleq E\big[tr\big(\mathbf{H}_y \mathbf{H}_t \mathbf{H}_p \mathbf{H}_t \mathbf{H}_y^H\big) tr\big(\mathbf{H}_t \mathbf{H}_p \mathbf{H}_t\big)^{-1}\big] = N - M + E[\varepsilon_{\text{MMSE}}]$ with $0 < E[\varepsilon_{\text{MMSE}}] < 0.5M$. It is observed that K_{MMSE} increases as N increases for a given M. In addition, as N goes to ∞, $\varepsilon_{\text{MMSE}}$ in K_{MMSE} gets smaller. Hence, if N is large enough, K_{MMSE} can be approximated as $N - M$. For a high SNR, both $\mathbf{H}_t = \big(\mathbf{H}_y^H \mathbf{H}_y + M \sigma_{v_y}^2 p_{\text{R}}{}^{-1} \mathbf{I}_N\big)^{-1}$ and $\mathbf{H}_d = \big(p_s \mathbf{H}_s \mathbf{H}_s^H + \sigma_{v_s}^2 \mathbf{I}_N\big)^{-1}$ can be approximated as

[3]Even if $J(\mathbf{F}_{\text{ZF}}^\star)$ increases as N increases, the BER does not get worse because $J(\mathbf{F}_{\text{ZF}}^\star)$ represents only the SCP with a noise-free condition for the ZF case. In addition, $\text{SNR}_{\text{TOTAL}} \triangleq \frac{p_s ||\mathbf{H}_{\text{TOTAL}}||_F^2}{||\mathbf{v}_{\text{TOTAL}}||^2} = \frac{\omega_{\text{ZF}}^\star p_s}{\omega_{\text{ZF}}^\star \sigma_{v_s}^2 (N-M)^{-1} + \sigma_{v_y}^2}$ at the destinations increases as N increases for a given M, p_s, $\sigma_{v_s}^2$, and $\sigma_{v_y}^2$, where $\mathbf{H}_{\text{TOTAL}} \triangleq \mathbf{H}_y \mathbf{F}^\star \mathbf{\Gamma} \mathbf{H}_s \in \mathbf{C}^{M \times M} = \sqrt{\omega_{\text{ZF}}^\star} \mathbf{I}_{\text{M}}$ and $\mathbf{v}_{\text{TOTAL}} = \mathbf{H}_y \mathbf{F}^\star \mathbf{\Gamma} \mathbf{v}_s + \mathbf{v}_y = \sqrt{\omega_{\text{ZF}}^\star} \mathbf{H}_s^\dagger \mathbf{v}_s + \mathbf{v}_y \in \mathbf{C}^{M \times 1}$ in (1). As a result, the BER improves as N increases, although $J(\mathbf{F}_{\text{ZF}}^\star)$ increases as N increases.

$\mathbf{H}_t \approx \left(\mathbf{H}_y^H\mathbf{H}_y\right)^{-1}$ and $\mathbf{H}_d \approx \left(\mathbf{H}_s\mathbf{H}_s^H\right)^{-1}$. For example, for SNR = 20 dB, if the value of $\frac{M\sigma_{v_y}^2}{p_R}$ is less than 0.2 percentile of the *i*-th diagonal element of $E\left[\mathbf{H}_y^H\mathbf{H}_y\right]$ or $E\left[\mathbf{H}_s\mathbf{H}_s^H\right]$, i.e., $\frac{M\sigma_{v_y}^2}{p_R} \le 0.002E\left[[\mathbf{H}_y^H\mathbf{H}_y]_{ii}\right]$, then the approximations are valid when $N \ge 5M$. Here, $[\mathbf{H}_y^H\mathbf{H}_y]_{ii}$ is the i-th diagonal entry of $\mathbf{H}_y^H\mathbf{H}_y$. This is because $\frac{M\sigma_{v_y}^2}{p_R} = M\mathrm{SNR}^{-1} \le 0.002E\left[[\mathbf{H}_y^H\mathbf{H}_y]_{ii}\right] = 0.002N$ when $p_s = p_R = 1$. Hence, using the properties of the pseudo-inverse, $tr\left(\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\right)$ and $tr\left(\mathbf{H}_y\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\mathbf{H}_y^H\right)$ can be simplified, respectively, as

$$tr\left(\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\right) = tr\left((\mathbf{H}_y\mathbf{H}_y^H)^{-1}\right) \tag{45}$$

$$tr\left(\mathbf{H}_y\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\mathbf{H}_y^H\right) = M \tag{46}$$

where $\mathbf{H}_p = \mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\mathbf{H}_y$. Hence, $K_{\text{MMSE}} \triangleq E\big[tr\left(\mathbf{H}_y\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\mathbf{H}_y^H\right)tr\left(\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\right)^{-1}\big]$ can be written as

$$K_{\text{MMSE}} = ME\left[tr\left((\mathbf{H}_y\mathbf{H}_y^H)^{-1}\right)\right]. \tag{47}$$

To determine K_{MMSE} in (47), a useful lemma in [30] will be applied, as follows:

Lemma 1 : Let $\mathbf{Y} \sim CN(\mathbf{0}, \mathbf{I}_{\text{N}} \otimes \mathbf{I}_{\text{M}})$ with $N > M$. Then,

$$tr\left(E\left[(\mathbf{Y}\mathbf{Y}^H)^{-1}\right]\right) = \frac{M}{N-M} \tag{48}$$

where $\mathbf{0}$ is an $N \times M$ matrix consisting of all zero entries. Hence, using *Lemma 1*, K_{MMSE} in (47) can be written as

$$K_{\text{MMSE}} = N - M. \tag{49}$$

Note that K_{MMSE} in (49) can be easily verified using the $M = 1$ case. In other words, $K_{\text{MMSE}} = N - 1$ when $M = 1$ (i.e., $\mathbf{H}_s \to \mathbf{h}_s$ and $\mathbf{H}_y \to \mathbf{h}_y$) and $N > 1$ because $tr\left(\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\right) = tr\left((\mathbf{h}_y\mathbf{h}_y^H)^{-1}\right) = \frac{1}{||\mathbf{h}_y||^2}$ and $tr\left(\mathbf{h}_y\mathbf{H}_t\mathbf{H}_p\mathbf{H}_t\mathbf{h}_y^H\right) = 1$ with $\mathbf{h}_y = [h_{y,1}, \cdots, h_{y,N}]$ and $\mathbf{h}_s = [h_{s,1}, \cdots, h_{s,N}]^T$. Here, $E\left[||\mathbf{h}_y||^{-2}\right] = \frac{1}{N-1}$ is used. As the SNR increases, the minimum N required for valid approximations of $\mathbf{H}_t$ and $\mathbf{H}_d$ can be smaller than M. From (44), it can be concluded that the system performance enhances as N increases. In other words, the BER performance improves by $10\log_{10}(K_{\text{MMSE}})$ when $p_R = 1$. ■

Proposition 6: The SCP at the destinations under the TPC using ZF increases as N increases.

Proof: Using (1) and the optimal $\mathbf{F}_{\text{ZF}}^\star$ in (22), the total SCP $p_{\text{D}_{\text{ZF}}}$ of the received signals at the destinations can be written as

$$p_{\text{D}_{\text{ZF}}} = K_{\text{ZF}}p_{\text{R}} \tag{50}$$

where $K_{\text{ZF}} \triangleq E\big[tr\left(p_s\mathbf{I}_M + \sigma_{v_s}^2\mathbf{H}_s^\dagger\left(\mathbf{H}_s^\dagger\right)^H\right)tr\left(\mathbf{H}_y^\dagger\left(p_s\mathbf{I}_M + \sigma_{v_s}^2\mathbf{H}_s^\dagger\left(\mathbf{H}_s^\dagger\right)^H\right)\left(\mathbf{H}_y^\dagger\right)^H\right)^{-1}\big] = N - M + E[\varepsilon_{\text{ZF}}]$. Here, the average ε_{ZF} is in $-1 < E[\varepsilon_{\text{ZF}}] < 1$. Hence, K_{ZF} increases as N increases for a given M and p_{R}, like K_{MMSE}. Hence, as N increases, the SCP $p_{\text{D}_{\text{ZF}}}$ increases, i.e., the BER performance enhances by $10\log_{10}(K_{\text{ZF}})$. ■

Additionally, it can be theoretically observed that $K_{\text{MMSE}} > K_{\text{ZF}} \simeq K_{\text{TPC}}$. Here, K_{TPC} is the SCP under TPC with $\mathbf{F}_{\text{TPC}}^\star$ in (12) as $K_{\text{TPC}} \triangleq E\big[tr\left(\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right)tr\left(\mathbf{H}_y^\dagger\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\left(\mathbf{H}_y^\dagger\right)^H\right)^{-1}\big] = N - M + E[\varepsilon_{\text{TPC}}]$ with $\left|E[\varepsilon_{\text{TPC}}]\right| < 1$. Therefore, the BER performance with $\mathbf{F}^\star$ in (6) is the best compared to the one with $\mathbf{F}_{\text{TPC}}^\star$ in (12) and $\mathbf{F}_{\text{ZF}}^\star$ in (22). In particular, it can be predicted that the BER performance with $\mathbf{F}_{\text{ZF}}^\star$ in (22) is similar to the one with $\mathbf{F}_{\text{TPC}}^\star$ in (12) due to $K_{\text{ZF}} \simeq K_{\text{RPC}}$.

Proposition 7: The SCP at the destinations under NPC converges to a constant for a given M and p_{R} as N increases.

Proof: Unlike $p_{\text{D}_{\text{MMSE}}}, p_{\text{D}_{\text{TPC}}}$, and $p_{\text{D}_{\text{ZF}}}$, the SCP $p_{\text{D}_{\text{NPC}}}$ applying $\mathbf{F}_{\text{NPC}}^\star$ in (17) has a different theoretical result as

$$p_{\text{D}_{\text{NPC}}} = K_{\text{NPC}}p_{\text{R}} \tag{51}$$

where $K_{\text{NPC}} \triangleq E\left[tr\left(\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right)\right] = M - E[\varepsilon_{\text{NPC}}]$ with $0 < E[\varepsilon_{\text{NPC}}] < 1$. Note that K_{NPC} is $M - E[\varepsilon_{\text{NPC}}]$ for a given M and p_{R}, e.g., $p_{\text{R}} = 1$, regardless of increasing N because the term $\mathbf{H}_s\mathbf{H}_s^H$ inside $\mathbf{H}_d = \left(p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_{\text{N}}\right)^{-1}$ is the dominant term for N to be large enough. Hence, the SCP $p_{\text{D}_{\text{NPC}}}$ converges to $M - E[\varepsilon_{\text{NPC}}]$ if N is large enough. ■

In addition, if $\sigma_{v_s}^2$ approaches 0, then ε_{NPC} approaches 0. Hence, K_{NPC} can be equal to M when $E[\varepsilon_{\text{NPC}}] = 0$. Finally, if p_{D} in (15) is constrained to 1 using the optimal $\mathbf{F}_{\text{RPC}}^\star$ in (15) and $M = 1$, then $p_{\text{D}} = 1 > p_{\text{D}_{\text{NPC}}} = 1 - E[\varepsilon_{\text{NPC}}]$. However, if $M \ge 2$, then $p_{\text{D}} = 1 < p_{\text{D}_{\text{NPC}}} \approx M$. Hence, the NPC shows a better BER performance compared to the RPC when $M \ge 2$.

In summary, from *Propositions 5* to *7*, the MMSE-based $\mathbf{F}^\star$ under the TPC with equalization factor ω shows the best BER (i.e., the highest SCP at the destinations), and $\mathbf{F}_{\text{RPC}}^\star$ under RPC has the worst BER, especially, for N to be large enough.

VI. Relay Selection

System performance, such as BER, can be improved by applying an efficient relay selection scheme. The SNR value of the distributed AF SISO relay network can be used for the relay selection [15]. Additionally, the SINR value can be used for the relay selection scheme of the cooperative distributed AF relay network. However, this is not a convenient method for the cooperative distributed AF relay system because it is not easy to find the closed-form of one common amplifying matrix, similar to the sum rate case. As analyzed in previous sections, the smaller the MMSE CF value, the better the BER performance. As a result, the MMSE CF criterion will be applied, where the MMSE CF calculations will be performed at a CSS.

Using (37), among all MMSE CF values at the destinations, the relays will be selected for data transmission from the minimum value. Let n relays out of N be orderly selected from the smallest MMSE CF value among all calculated MMSE CF values and the subscript n denote the number of selected relays, where $n = 1, \cdots, N$. For instance, when all N relays are selected, the CF $J\left(\mathbf{F}^\star\right)$ in (37) will be used. Only the channels corresponding to the selected relays will participate in data transmission. The n relays with the minimum MMSE CF, $J_{\min}\left(\mathbf{F}^\star\right)$, among all MMSE CF combination values for the MMSE relay scheme will be used as

$$J_{\min}\left(\mathbf{F}^\star\right) = \arg\min_{1\le n\le N} J\left(\mathbf{F}^\star\right)_{(n)} \tag{52}$$

where

$$J\left(\mathbf{F}^\star\right)_{(n)} = p_s tr\left((p_s\mathbf{H}_{(n)}^H\mathbf{H}_{q_{(n)}}^{-1}\mathbf{H}_{(n)} + \mathbf{I}_{\text{N}})^{-1}\right). \tag{53}$$

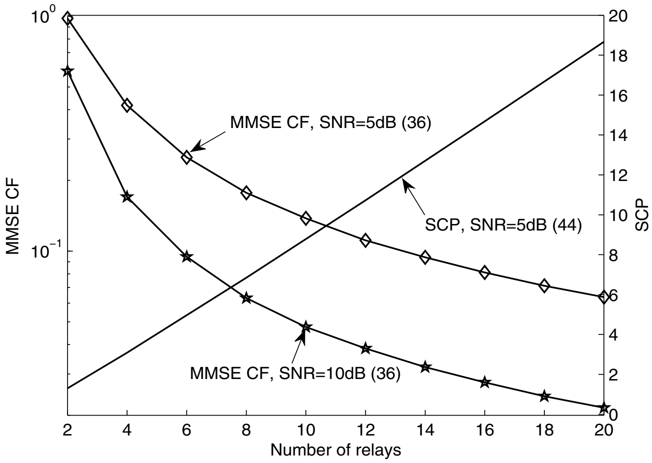

Fig. 2. MMSE CF $J(\mathbf{F}^\star)$ in (36) and SCP in (44) versus number of relays for cooperative distributed AF relay networks under TPC with $M = 2$ using the MMSE relay scheme.

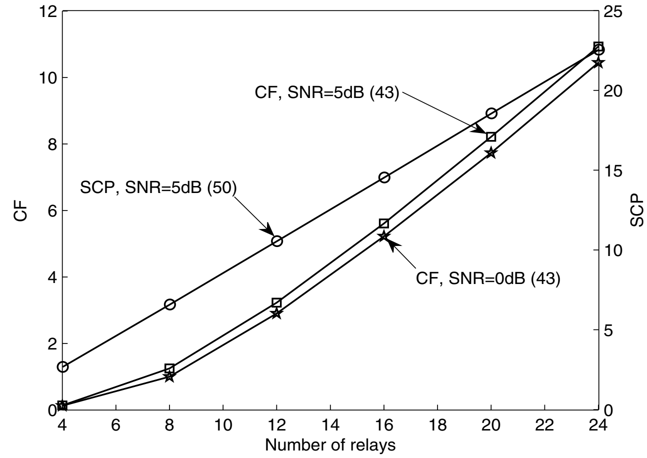

Fig. 3. CF $J(\mathbf{F}_{\text{ZF}}^\star)$ in (43) and SCP in (50) versus number of relays for cooperative distributed AF relay networks under TPC with $M = 2$ using the ZF relay scheme.

Note that the matrix size of $\mathbf{F}_n^\star$ inside $J(\mathbf{F}^\star)_{(n)}$ of (53) is $(n \times n)$ instead of $(N \times N)$, where the corresponding $\mathbf{F}_n^\star$ can be written as

$$\mathbf{F}_{(n)}^\star = \frac{\mathbf{H}_{t_{(n)}}\mathbf{H}_{y_{(n)}}^H\mathbf{H}_{s_{(n)}}^H\mathbf{H}_{d_{(n)}}\mathbf{\Gamma}_{(n)}^{-1}\sqrt{p_{\text{R}}}}{\sqrt{tr\left(\mathbf{H}_{t_{(n)}}\mathbf{H}_{y_{(n)}}^H\mathbf{H}_{s_{(n)}}^H\mathbf{H}_{d_{(n)}}\mathbf{H}_{s_{(n)}}\mathbf{H}_{y_{(n)}}\mathbf{H}_{t_{(n)}}^H\right)}}. \quad (54)$$

Here, $\mathbf{\Gamma}_{(n)} \in \mathbf{C}^{n\times n}$ in (54) can be written as

$$\mathbf{\Gamma}_{(n)} = \text{diag}\left(\left(\kappa_1 + \frac{\sigma_{v_s}^2}{p_s}\right)^{-\frac{1}{2}}, \cdots, \left(\kappa_n + \frac{\sigma_{v_s}^2}{p_s}\right)^{-\frac{1}{2}}\right) \quad (55)$$

where $\kappa_j = \sum_{k=1}^{M} |h_{s,j,k}|^2,\ j = 1, \cdots, n$. Additionally, the sizes of $\mathbf{H}_{s_{(n)}}$ and $\mathbf{H}_{y_{(n)}}$ are $(n \times M)$ and $(M \times n)$ instead of $(N \times M)$ and $(M \times N)$, respectively, where $n \geq M$. A significantly higher gain can be achieved using the proposed relay selection scheme than the one in [15] because the optimum AF relaying matrix is recalculated and used for the selected relays. However, the complexity of the proposed relay selection scheme would be higher than the one in [15] because the cooperative AF relay network is used, i.e., the nondiagonal AF relay amplifying matrix **F**, whereas the noncooperative AF relay network is employed, i.e., a diagonal **F** in [15].

VII. Simulation Results

Monte Carlo simulation results for cooperative AF relay schemes under TPC, RPC, and NPC are performed. The BER performance using the optimum relay amplifying matrices is evaluated with $p_{\text{R}} = 1$ and $p_{\text{D}} = 1$. The BER is a good criterion because the MMSE and ZF criterions are applied in this paper. Thus, the system BER is defined as the average BER over M users. The complex channel matrices $\mathbf{H}_s$ and $\mathbf{H}_y$ are generated from zero-mean and unit variance i.i.d. complex Gaussian random variables. All nodes have the same noise power, i.e., $\sigma_{v_{s_1}}^2 = \cdots = \sigma_{v_{s_N}}^2 = \sigma_{v_s}^2 = \sigma_{v_{y_1}}^2 = \cdots = \sigma_{v_{y_M}}^2 = \sigma_{v_y}^2$. The originally transmitted signals at the sources are assumed to be modulated using quadrature phase shift keying with unit power, i.e., $p_s = 1$.

Figure 2 shows the MMSE CF $J(\mathbf{F}^\star)$ in (36) and SCP in (44) versus the number of relays (N) for the cooperative

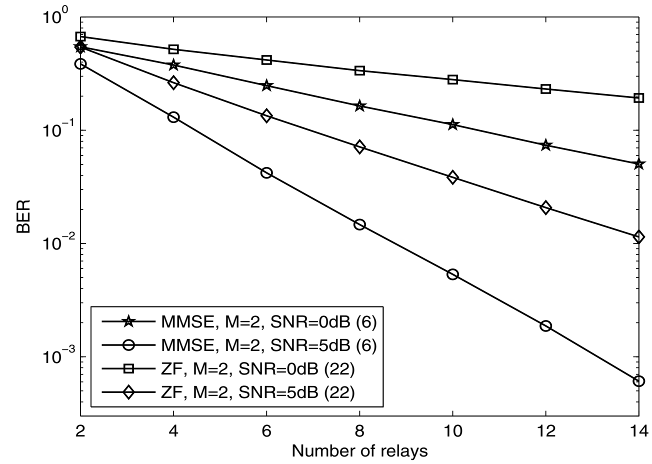

Fig. 4. BER performance versus number of relays (N) for cooperative distributed AF relay networks under TPC with different SNRs as a parameter and $M = 2$ using both MMSE and ZF relay schemes in (6) and (22), respectively.

distributed AF relay networks under TPC with $M = 2$ using the MMSE relay scheme. Simulation results agree well with the analysis. As analyzed in *Proposition 1*, it is observed in Fig. 2 that the CF value decreases as N increases for a given $M = 2$. The smaller the CF value, the smaller the MSE. Additionally, it is also observed in Fig. 2 that the total SCP $p_{\text{D}_{\text{MMSE}}}$ of the received signal increases as N increases for a given $M = 2$, as expected from *Proposition 5*.

Figure 3 provides the CF $J(\mathbf{F}_{\text{ZF}}^\star)$ in (43) and SCP in (50) versus the number of relays (N) for the cooperative distributed AF relay networks under TPC with $M = 2$ using the ZF relay scheme. Unlike the MMSE CF $J(\mathbf{F}^\star)$, it can be seen in Fig. 3 that the CF $J(\mathbf{F}^\star)$ increases as N increases, as analyzed in *Proposition 4*. In addition, as analyzed in *Proposition 6*, it is observed in Fig. 3 that the SCP $p_{\text{D}_{\text{ZF}}}$ increases as N increases.

Figure 4 shows the BER performance versus the number of relays ($N = 2 \sim 14$) for the cooperative distributed AF relay networks under TPC with different input SNR $\left(\triangleq \frac{p_s}{\sigma_{v_s}^2} = \frac{p_s}{\sigma_{v_y}^2} = \frac{1}{\sigma_{v_s}^2} = \frac{1}{\sigma_{v_y}^2}\right)$ as a parameter and $M = 2$ using the both MMSE and ZF relay schemes in (6) and (22), respectively. As

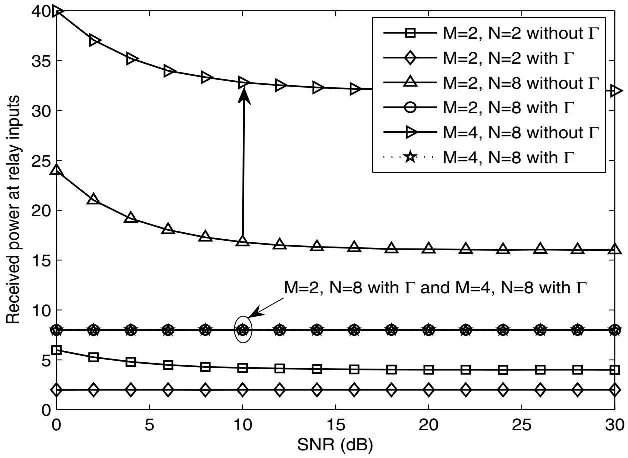

Fig. 5. Total received power at relay inputs with/without $\mathbf{\Gamma}$ versus input SNR for AF relay networks with a different number of relays $N = 2$, 8, and the number of source-destination pairs $M = 2, 4$.

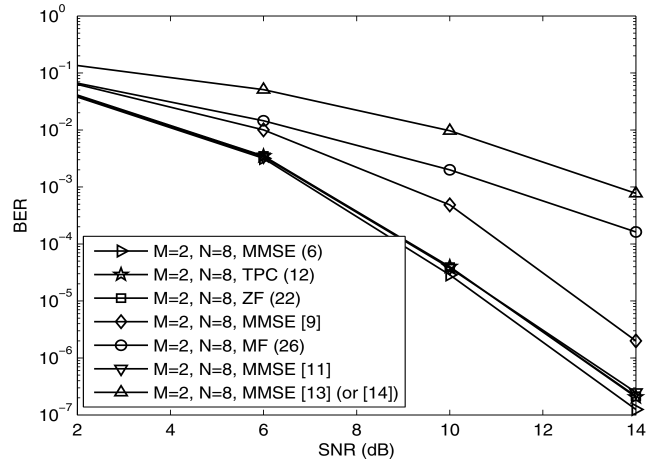

Fig. 6. BER versus input SNR for $M = 2$ and $N = 8$ cooperative AF relay networks using $\mathbf{F}^\star$ in (6), $\mathbf{F}^\star_{\text{TPC}}$ in (12), $\mathbf{F}^\star_{\text{ZF}}$ in (22), and $\mathbf{F}^\star_{\text{MF}}$ in (26), including existing ones $\mathbf{F}^\star_{\text{EXT}}$ under TPC in (13) of [9] and the one in [11]. Noncooperative one presented in [13] (or [14]) is also shown.

expected from Figs. 2 and 3, the BER performances in the both MMSE and ZF relay schemes improve by $10\log_{10}(K_{\text{MMSE}})$ and $10\log_{10}(K_{\text{ZF}})$, respectively, as N increases when $p_{\text{R}} = 1$ because the SCPs $p_{\text{D}_{\text{MMSE}}}$ and $p_{\text{D}_{\text{ZF}}}$ increase as N increases for a given M.

Figure 5 presents the total received signal power at the relay inputs with/without $\mathbf{\Gamma}$ versus an input SNR for the cooperative distributed AF relay networks with a different number of relays $N = 2$, 8, and the number of source-destination pairs $M = 2, 4$. Similar to the case of the RPC at the destinations, the received power constraint at the relay inputs reduces the total received power at the relay inputs using a diagonal weighting matrix $\mathbf{\Gamma}$ applied at the relay inputs (or the source outputs). For example, with the same number of $N = 8$, the total received signal power at the relay inputs increases significantly (e.g., 150% indicated by a vertical arrow) as M increases from 2 to 4 when $\mathbf{\Gamma}$ is not applied. However, in contrast to this, when $\mathbf{\Gamma}$ is applied, the total received signal power at the relay inputs is always proportional to N, regardless of increasing M. However, if $\mathbf{\Gamma}$ is not applied for a given M, the total received signal power at the relay inputs increases considerably as N increases.

Figure 6 shows BER performance versus input SNR for $M = 2$ cooperative AF relay networks using $\mathbf{F}^\star$ in (6), $\mathbf{F}^\star_{\text{TPC}}$ in (12), $\mathbf{F}^\star_{\text{ZF}}$ in (22), and $\mathbf{F}^\star_{\text{MF}}$ in (26), with $N = 8$, including the existing $\mathbf{F}^\star_{\text{EXT}}$ under power constraints at the relays in (24), which is in (13) of [9], and the one in [11] under power constraints at the sources. Also, the BER of the *noncooperative* distributed AF relay networks in [13], [14] is shown in Fig. 6, using the *diagonal* $\mathbf{F}^\star_{\text{Diag}}$ newly derived by the the authors of this paper under the TPC based on the MMSE criterion. The diagonal matrix is written as

$$\mathbf{F}^\star_{\text{Diag}} = \frac{\text{diag}\big(\mathbf{H}_1^{-1}\mathbf{h}_{ys}\big)\sqrt{p_{\text{R}}}}{\big|\big|(\mathbf{H}_2 \odot \mathbf{I}_N)^{1/2}\mathbf{H}_1^{-1}\mathbf{h}_{ys}\big|\big|^2} \tag{56}$$

where $\mathbf{H}_1 = (\mathbf{H}_y^H\mathbf{H}_y) \odot \mathbf{H}_2^* + \sigma^2_{v_y} M p_{\text{R}}^{-1}(\mathbf{H}_2 \odot \mathbf{I}_N)$, $\mathbf{H}_2 = p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma^2_{v_s}\mathbf{I}_N$, and $\mathbf{h}_{ys} = \text{diag}(\mathbf{H}_y^H\mathbf{H}_s^H)$. BER performance improves in Fig. 6 as expected for all schemes as N increases. Additionally, due to $K_{\text{MMSE}} > K_{\text{ZF}} \simeq K_{\text{TPC}}$, e.g., $K_{\text{MMSE}} = 0.3081 > K_{\text{ZF}} = 0.2671$ when $N = 4$, BER performance using $\mathbf{F}^\star$ in (6) is the best, compared to the other two methods. Furthermore, the proposed MMSE relay scheme shows approximately 2 dB and 0.4 dB better in SNR at BER $= 10^{-3}$ and BER $= 10^{-6}$ for $M = 2$ and $N = 8$ than the existing ones in [9] and [11], respectively. And the proposed MMSE relay scheme also shows better performance, e.g., 4 dB better in SNR at BER $= 10^{-3}$ for $M = 2$ and $N = 8$, than the MF one. In particular, due to $K_{\text{ZF}} \simeq K_{\text{TPC}}$, the almost identical BER performance is observed in Fig. 6, as analyzed in *Proposition 6*. In general, the cooperative relay network requires a higher complexity over the noncooperative ones. However, the cooperative relay networks can achieve significantly lower MMSE, lower BER, higher capacity, lower outage probabilities, and higher throughput. For example, as shown in Fig. 6, an $(M = 2, N = 8)$ AF cooperative relay network under the TPC shows 6.3 dB gain in SNR at BER $= 10^{-3}$ over the noncooperative one in [13] (or [14]) under the same environments.

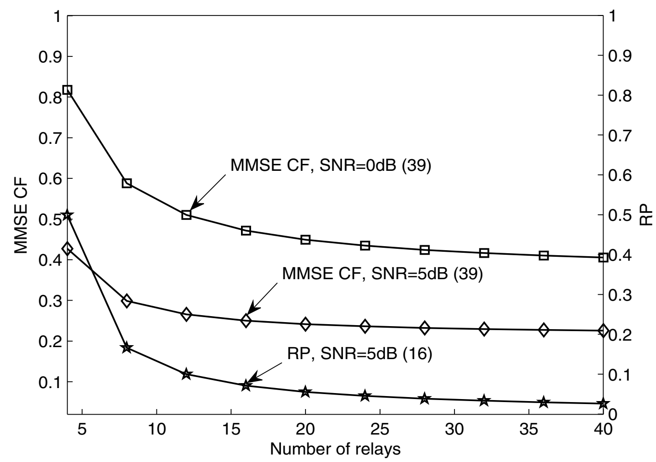

Fig. 7. MMSE CF $J(\mathbf{F}^\star_{\text{RPC}})$ in (39) and relay power (RP) in (16) versus number of relays for cooperative distributed AF relay networks under RPC with $M = 2$.

Figure 7 provides the MMSE CF $J(\mathbf{F}^\star_{\text{RPC}})$ in (39) and

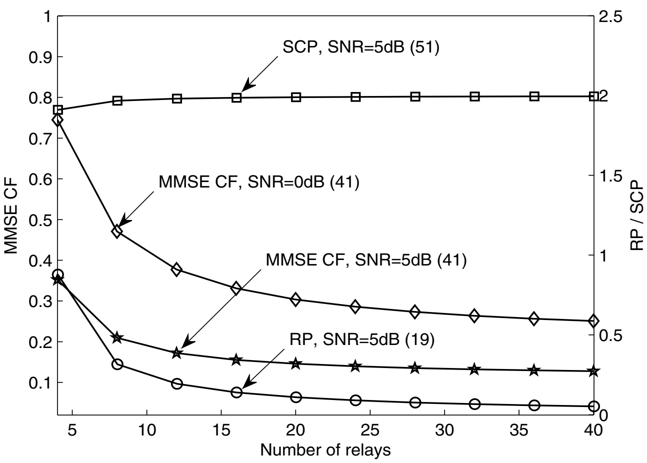

Fig. 8. MMSE CF $J(\mathbf{F}^{\star}_{\text{NPC}})$ in (41), relay power (RP) in (19), and SCP in (51) versus number of relays for cooperative distributed AF relay networks under NPC with $M = 2$.

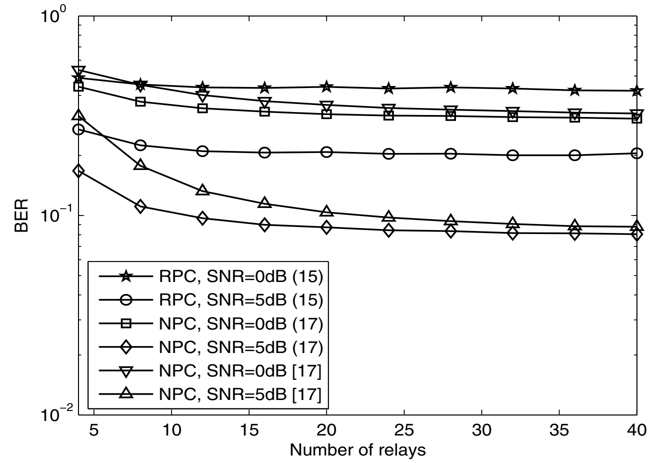

Fig. 9. BER performance versus number of relays (N) for cooperative distributed AF relay networks under RPC and NPC with different input SNRs and $M = 2$ using (15) and (17), respectively. Noncooperative results under NPC are also presented in [12].

relay power in (16) versus the number of relays (N) for the cooperative distributed AF relay networks under RPC with $M = 2$ and $p_{\text{D}} = 1$. As analyzed in *Proposition 2*, it can be seen in Fig. 7 that the MMSE CF decreases as N increases within a certain N. However, if N is large enough, the MMSE CF under RPC converges to a constant as N increases (*Proposition 2*). As a result, their BER performance converges as N increases. This will be verified in Fig. 9. In addition, it is observed in Fig. 7 that the total power usage at the relays drops by $(N - M)^{-1}$.

Figure 8 presents the MMSE CF $J(\mathbf{F}^{\star}_{\text{NPC}})$ in (41), relay power in (19), and SCP in (51) versus the number of relays (N) for the cooperative distributed AF relay networks under NPC with $M = 2$. The positive scaling factor ω_{NPC} is assumed to be 1. As analyzed in *Proposition 3*, it is observed in Fig. 8 that the MMSE decreases but converges as N increases because $p_{\text{D}_{\text{NPC}}}$ in (51) converges (*Proposition 7*), i.e., $K_{\text{NPC}} \simeq M$, for N to be large enough. In addition, similar to the RPC, it is also observed in Fig. 8 that the total power usage at the relays drops approximately by $(N - M)^{-1}$, as analyzed in (19). Finally, it can be seen in Fig. 8 that total relay power usage at the relays converges approximately to M as N increases, as analyzed in *Proposition 7*.

Figure 9 presents the BER performance versus the number of relays ($N = 4 \sim 40$) for the cooperative distributed AF relay networks under RPC and NPC with different input SNRs and $M = 2$ using (15) and (17), respectively. The noncooperative results under the NPC in [12] are also presented for comparison. As analyzed in Figs. 7 and 8, their BER performances converge as N increases for a given M. Additionally, it can be seen that the NPC shows a better BER performance than the RPC when N becomes larger, due to $p_{\text{D}} = 1 < p_{\text{D}_{\text{NPC}}} \approx M$ when $M \geq 2$. However, the RPC is desirable in the green energy environment as long as the minimum required SNR can be met. Finally, it can be seen that the BER performance of the cooperative AF relay networks under the NPC shows a better BER performance than that of the noncooperative one in [12] under the same power constraint due to the relay cooperation.

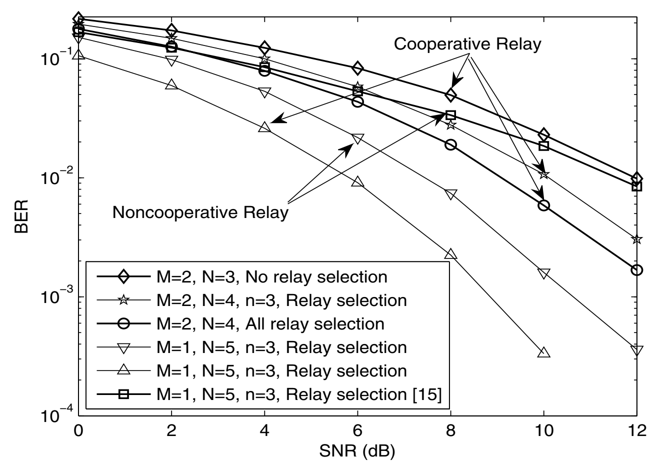

Fig. 10. BER versus input SNR with relay selection $n = 3, 4$ out of $N = 4$ relays and $M = 2$ for cooperative AF MIMO relay networks, and $n = 3$ out of $N = 5$ relays and $M = 1$ for cooperative and noncooperative AF SISO relay networks. No relay selection cases presented with $N = 3$ and $M = 2$.

Figure 10 shows a comparison of BER performance versus an input SNR with relay selection $n = 3, 4$ out of $N = 4$ relays for cooperative distributed AF MIMO relay networks based on the MMSE criterion using $\mathbf{F}^{\star}_{(n)}$ in (54) and $M = 2$, and $n = 3$ out of $N = 5$ relays and $M = 1$ for the cooperative and the noncooperative AF SISO relay networks, respectively. No relay selection cases are presented with $N = 3$ and $M = 2$. As expected, it is observed in Fig. 10 that the BER performance of relay selection gets better than those cases of no relay selection with the same number of relays used with $N = 3$ when $M = 2$ because the best n relays that have the smallest MMSE CF value participates in data transmission. Power consumption and bandwidth efficiency of the best n relay selection scheme are better than those of the all-relay selection. In addition, through the best n relay selection scheme, a higher diversity order in the cooperative distributed AF relay system can be achieved. Moreover, in AF SISO relay networks, the BER performance of the cooperative relay selection scheme is the best compared to two noncooperative schemes considered for a given N because of the relays' cooperation. Finally, the noncooperative relay selection scheme newly in (56) derived by the authors' of this paper using [13], [14] based on the MMSE CF criterion outperforms, e.g., 4 dB better in SNR at BER $= 10^{-2}$, the existing noncooperative one in [15] based on the relay ordering when $n = 3$ relays out of $N = 5$ relays are selected.

VIII. Conclusion

The optimal AF relay amplifying matrices based on the MMSE and the ZF criterions were found in closed forms for cooperative and noncooperative distributed AF MIMO relay networks under the following: (1) TPC, (2) RPC, and (3) NPC. It was shown that the constrained MMSE Lagrangian optimization is convex with respect to the optimal $\mathbf{F}^\star$ and $\omega^\star$. Hence, $\mathbf{F}^\star$ and $\omega^\star$ are global optimums.

It was observed that the BER gets smaller as the MMSE CF decreases, i.e., N increases, except for the ZF case. It was also observed that the proposed MMSE-based relay scheme shows the best BER performance among all relay schemes considered. It was observed through both theoretical and simulation results that the gain of diversity order can be obtained as N increases in cooperative distributed AF relay networks when the TPC is applied. However, when the RPC and NPC with scaling factor $\omega_{\text{NPC}} = 1$ are applied, the BER performance improves but converges as N increases because the CF converges as N increases.

It was also found that the SCP of the received signal at the destinations increases proportionally to $N-M$ under the TPC. This causes the BER performance to improve by the amount of the SCP as N increases. In addition, the proposed scheme with the explicit optimal relay amplifying matrix shows a better performance than the existing ones in [9] and [11]. It was observed that the BER improves but converges as N increases because the SCP converges approximately to M as N increases under RPC and NPC. Finally, the novel relay selection scheme using the total MMSE CF criterion outperforms the existing one in [15]. The results in this paper can be useful for designing an energy-efficient relay network.

Appendix

A. Proof of Theorem 1

To prove *Theorem 1*, it is necessary to show that $\alpha L(\mathbf{F}_1,\omega,\lambda) + (1-\alpha)L(\mathbf{F}_2,\omega,\lambda)$ is greater than or equal to $L(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2,\omega,\lambda)$ for any relay amplifying matrices $\mathbf{F}_1$ and $\mathbf{F}_2$ in $\mathbf{F}$ and any $\alpha \in [0,\ 1]$ with any nonnegative Lagrangian multiplier λ. Let $f_1(L) \triangleq \alpha L(\mathbf{F}_1,\omega,\lambda) + (1-\alpha)L(\mathbf{F}_2,\omega,\lambda)$ and $f_2(L) \triangleq L(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2,\omega,\lambda)$. Then, $f_1(L)$ and $f_2(L)$ can be written, respectively, as

$$\begin{aligned} f_1(L) &= \alpha J(\mathbf{F}_1) + \alpha\lambda\left(E\left[||\mathbf{x}||^2\right] - p_{\text{R}}\right) + J(\mathbf{F}_2) - \alpha J(\mathbf{F}_2) \\ &\quad + \lambda\left(E\left[||\mathbf{x}||^2\right] - p_{\text{R}}\right) - \alpha\lambda\left(E\left[||\mathbf{x}||^2\right] - p_{\text{R}}\right) \qquad (57) \end{aligned}$$

$$f_2(L) = J(\alpha\mathbf{F}_1 + (1-\alpha)\mathbf{F}_2) + \lambda\left(E\left[||\mathbf{x}||^2\right] - p_{\text{R}}\right). \qquad (58)$$

Using the cyclic properties of the trace function and the linearity of the trace function, i.e., $tr(\mathbf{A}_1 + \mathbf{A}_2) = tr(\mathbf{A}_1) + tr(\mathbf{A}_2)$ and $tr(\xi\mathbf{A}) = \xi tr(\mathbf{A})$, $f(L) \triangleq f_1(L) - f_2(L)$ can be written as

$$\begin{aligned} f(L) &= \alpha(1-\alpha)\omega^{-2}\left(p_s||\mathbf{H}_y\mathbf{F}_0\mathbf{\Gamma}\mathbf{H}_s||_F^2 + \sigma_{v_s}^2||\mathbf{H}_y\mathbf{F}_0\mathbf{\Gamma}||_F^2\right) \\ &\quad + \lambda\alpha(1-\alpha)\left(p_s||\mathbf{F}_0\mathbf{\Gamma}\mathbf{H}_s||_F^2 + \sigma_{v_s}^2||\mathbf{F}_0\mathbf{\Gamma}||_F^2\right) \geq 0 \qquad (59) \end{aligned}$$

where $\mathbf{F}_0 = \mathbf{F}_1 - \mathbf{F}_2$. It can be clearly seen that $f(L)$ in (59) is always greater than or equal to 0 for any $\alpha \in [0,\ 1]$ and any nonnegative Lagrangian multiplier λ. Hence, the constrained Lagrangian optimization $L(\mathbf{F},\omega,\lambda)$ in (5) is convex with respect to $\mathbf{F}$ for a given ω.

In addition, to show that $L(\mathbf{F},\omega,\lambda)$ in (5) is convex with respect to ω, take the partial derivative of $L(\mathbf{F},\omega,\lambda)$ in (5) with respect to $\{\mathbf{F}^*,\omega,\lambda\}$, respectively. Then, use the cyclic permutation of the trace function, i.e., $tr\left(\mathbf{A}_1\mathbf{A}_2\mathbf{A}_3\right) = tr\left(\mathbf{A}_2\mathbf{A}_3\mathbf{A}_1\right)$, and the linear and nonlinear properties of the complex matrix derivative [35]. They can be written as

$$\begin{aligned} \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial \mathbf{F}^*} &= \omega^{-2}p_s\mathbf{H}_y^H\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\mathbf{H}_s^H\mathbf{\Gamma}^H + \lambda\sigma_{v_s}^2\mathbf{F}\mathbf{\Gamma}\mathbf{\Gamma}^H \\ &\quad + \lambda p_s\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\mathbf{H}_s^H\mathbf{\Gamma}^H + \omega^{-2}\sigma_{v_s}^2\mathbf{H}_y^H\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\mathbf{\Gamma}^H \\ &\quad - \omega^{-1}p_s\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{\Gamma}^H = \mathbf{0}_N \qquad (60) \end{aligned}$$

$$\begin{aligned} \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial \omega} &= -2\omega^{-3}p_s\|\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\|_F^2 - 2\omega^{-3}\sigma_{v_s}^2\|\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\|_F^2 \\ &\quad + 2\omega^{-2}p_s tr\left(\text{Re}\left[\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\right]\right) \\ &\quad - 2\omega^{-3}M\sigma_y^2 = 0 \qquad (61) \end{aligned}$$

$$\frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial \lambda} = p_s\|\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\|_F^2 + \sigma_{v_s}^2\|\mathbf{F}\mathbf{\Gamma}\|_F^2 - p_{\text{R}} = 0 \qquad (62)$$

where $\mathbf{0}_N$ is an $N \times N$ matrix consisting of all zero entries. From (60), the optimal $\mathbf{F}^\star$ can be written as

$$\mathbf{F}^\star = \omega p_s\left(\mathbf{H}_y^H\mathbf{H}_y + \lambda\omega^2\mathbf{I}_N\right)^{-1}\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{\Gamma}^{-1} \qquad (63)$$

where $\mathbf{H}_d = \left(p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_N\right)^{-1}$. Define an $N \times N$ matrix $\mathbf{H}$ as

$$\mathbf{H} \triangleq \omega p_s\mathbf{F}^\star\mathbf{\Gamma}\mathbf{H}_s\mathbf{H}_y. \qquad (64)$$

Using (63) and $\left(p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_N\right)^{-H} = \left(p_s\mathbf{H}_s\mathbf{H}_s^H + \sigma_{v_s}^2\mathbf{I}_N\right)^{-1}$, $\mathbf{H}$ in (64) can be rewritten as

$$\mathbf{H} = \mathbf{F}^\star\mathbf{\Gamma}\mathbf{H}_d^{-1}\mathbf{\Gamma}(\mathbf{F}^\star)^H\left(\mathbf{H}_y^H\mathbf{H}_y + \lambda\omega^2\mathbf{I}_N\right). \qquad (65)$$

Equation (65) is a key to showing that $L(\mathbf{F},\omega,\lambda)$ in (5) is convex with respect to ω. Using the cyclic properties of the trace function and the linearity of the trace function, i.e., $tr(\mathbf{A}_1 + \mathbf{A}_2) = tr(\mathbf{A}_1) + tr(\mathbf{A}_2)$ and $tr(a\mathbf{A}) = atr(\mathbf{A})$, the $tr\left(\mathbf{H}\right)$ can be written as

$$tr\left(\mathbf{H}\right) = p_{\text{D}} + p_{\text{R}}\lambda\omega^2 \qquad (66)$$

with the fact that $tr\left(\text{Re}[\mathbf{H}]\right) = tr\left(\mathbf{H}\right)$ if $tr\left(\mathbf{H}\right) = tr\left(\mathbf{H}^H\right)$ and $p_{\text{R}} = p_s\|\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\|_F^2 + \sigma_{v_s}^2\|\mathbf{F}\mathbf{\Gamma}\|_F^2$ obtained from (62), where the total SCP $p_{\text{D}}(\triangleq ||\mathbf{H}_y\mathbf{x}||^2)$ of the received signals at the destinations using (1) is defined as

$$p_{\text{D}} = p_s\|\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\mathbf{H}_s\|_F^2 + \sigma_{v_s}^2\|\mathbf{H}_y\mathbf{F}\mathbf{\Gamma}\|_F^2 + M\sigma_{v_y}^2. \qquad (67)$$

Finally, using (66), $\frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial \omega}$ in (61) can be rewritten as

$$\frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial \omega} = 2\omega^{-1}\lambda p_{\text{R}} - 2\omega^{-3}M\sigma_{v_y}^2. \qquad (68)$$

The second derivative of $L(\mathbf{F},\omega,\lambda)$ in (5) can be written as

$$\frac{\partial^2 L(\mathbf{F},\omega,\lambda)}{\partial^2 \omega} = -2\omega^{-4}\lambda p_{\text{R}}\left(\omega - \sqrt{3}\omega_o\right)\left(\omega + \sqrt{3}\omega_o\right) \qquad (69)$$

where $\omega_o = \sqrt{M\sigma_{v_y}^2 \lambda^{-1} p_{\mathrm{R}}^{-1}}$. Using (69), it can be shown that

$$\begin{cases} \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial\omega} < 0 \text{ and } \frac{\partial^2 L(\mathbf{F},\omega,\lambda)}{\partial^2\omega} > 0, & \text{for } 0 < \omega < \omega_o \\ \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial\omega} = 0 \text{ and } \frac{\partial^2 L(\mathbf{F},\omega,\lambda)}{\partial^2\omega} > 0, & \text{for } \omega = \omega_o \\ \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial\omega} > 0 \text{ and } \frac{\partial^2 L(\mathbf{F},\omega,\lambda)}{\partial^2\omega} > 0, & \text{for } \omega_o < \omega < \sqrt{3}\omega_o \\ \frac{\partial L(\mathbf{F},\omega,\lambda)}{\partial\omega} > 0 \text{ and } \frac{\partial^2 L(\mathbf{F},\omega,\lambda)}{\partial^2\omega} \in \mathrm{R}, & \text{for } \sqrt{3}\omega_o < \omega. \end{cases} \quad (70)$$

From (70), it is observed that the proposed constrained Lagrangian optimization $L(\mathbf{F},\omega,\lambda)$ in (5) is convex or strictly quasi-convex with respect to ω [27] for a given $\mathbf{F}$. ■

B. Proof of Theorem 2

It is observed that $\omega^\star = \omega_o$ from (68) in *Theorem 1*, which is written as

$$\omega^\star = \sqrt{\frac{M\sigma_{v_y}^2}{\lambda p_{\mathrm{R}}}}. \quad (71)$$

Hence, using (71), $\lambda\omega^2$ can be obtained as

$$\lambda(\omega^\star)^2 = \frac{M\sigma_{v_y}^2}{p_{\mathrm{R}}}. \quad (72)$$

Using (72), the optimal $\mathbf{F}^\star$ in (63) can be simply rewritten as

$$\mathbf{F}^\star = \omega p_s \mathbf{H}_t \mathbf{H}_y^H \mathbf{H}_s^H \mathbf{H}_d \boldsymbol{\Gamma}^{-1} \quad (73)$$

where $\mathbf{H}_t = \left(\mathbf{H}_y^H \mathbf{H}_y + M\sigma_{v_y}^2 p_{\mathrm{R}}{}^{-1}\mathbf{I}_N\right)^{-1}$ and $\mathbf{H}_d = \left(p_s \mathbf{H}_s \mathbf{H}_s^H + \sigma_{v_s}^2 \mathbf{I}_N\right)^{-1}$. Substituting (73) into (62), the explicit optimal equalization scaling factor $\omega^\star$ can be rewritten as

$$\omega^\star = \sqrt{\frac{p_R p_s^{-2}}{p_s\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}}. \quad (74)$$

Hence, using (72) and (74), the explicitly optimal Lagrangian multiplier $\lambda^\star$ can be written as

$$\lambda^\star = \frac{M\sigma_{v_y}^2 p_s^2}{p_{\mathrm{R}}^2}\Big(tr\big(\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\mathbf{H}_y\mathbf{H}_t^H\big)\Big). \quad (75)$$

Finally, substituting (74) into (73), the explicit optimal $\mathbf{F}^\star$ for the cooperative distributed AF relay network under TPC can be written as

$$\mathbf{F}^\star = \frac{\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\boldsymbol{\Gamma}^{-1}\sqrt{p_{\mathrm{R}}}}{\sqrt{p_s\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\mathbf{H}_s\right\|_F^2 + \sigma_{v_s}^2\left\|\mathbf{H}_t\mathbf{H}_y^H\mathbf{H}_s^H\mathbf{H}_d\right\|_F^2}}. \quad (76)$$

In addition, the i-th entry γ_i of the diagonal weight matrix $\boldsymbol{\Gamma} \triangleq \mathrm{diag}\left(\gamma_1, \cdots, \gamma_N\right)$ is obtained from

$$E\left[|\gamma_i r_i|^2\right] = E\left[|s_i|^2\right] \quad (77)$$

where r_i is the received signal at the i-th relay and can be written as

$$r_i = \sum_{k=1}^{M} h_{s,i,k} s_k + v_{s,i}. \quad (78)$$

Hence, using (77), γ_i can be written as

$$\gamma_i = \left(M + \frac{\sigma_{v_s}^2}{p_s}\right)^{-\frac{1}{2}}. \quad (79)$$

Note that $z \triangleq \sum_{i=1}^{M} |h_{s,i,k}|^2$ is a chi-square distribution with $2M$ degrees of freedom and $E[z] = M$, and the probability density function $p(z) = \frac{1}{\Gamma(M)} z^{M-1} e^{-z}$ [36], where $\Gamma(M) = \int_0^\infty t^{M-1} e^{-t} dt$. ■

C. Proof of Theorem 3

Similar to the analysis in [31], using the scaling factor ω_{ZF}, the $\mathbf{F}_{\mathrm{ZF}}$ in (21) can be rewritten as

$$\mathbf{F}_{\mathrm{ZF}} = \omega_{\mathrm{ZF}} \hat{\mathbf{F}}_{\mathrm{ZF}}. \quad (80)$$

Hence, using (21) and (80), the modified constrained Lagrangian optimization $L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})$ can be written as

$$\begin{aligned} L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}}) &= E\left[||\mathbf{s} - \omega_{\mathrm{ZF}}^{-1}\hat{\mathbf{s}}||^2\right] + \lambda_{\mathrm{ZF}}\left(E\left[||\omega_{\mathrm{ZF}}\hat{\mathbf{F}}_{\mathrm{ZF}}\mathbf{r}||^2\right] - p_R\right) \\ &= Mp_s + p_s tr\Big(\mathbf{H}_s^H\boldsymbol{\Gamma}^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\mathbf{H}_y^H\mathbf{H}_y\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\Big) \\ &\quad - 2p_s tr\Big(\mathrm{Re}\big[\mathbf{H}_y\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\big]\Big) - \lambda_{\mathrm{ZF}} p_R \\ &\quad + \omega_{\mathrm{ZF}}^2\lambda_{\mathrm{ZF}}\sigma_{v_s}^2 tr\big(\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\big) \\ &\quad + \omega_{\mathrm{ZF}}^2\lambda_{\mathrm{ZF}} p_s tr\Big(\mathbf{H}_s^H\boldsymbol{\Gamma}^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\Big). \end{aligned} \quad (81)$$

The partial derivatives of $L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})$ in (81) with respect to $\hat{\mathbf{F}}_{\mathrm{ZF}}^*$, ω_{ZF}, and λ_{ZF} can be written, respectively, as

$$\begin{aligned} \frac{\partial L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})}{\partial \hat{\mathbf{F}}_{\mathrm{ZF}}^*} &= p_s\mathbf{H}_y^H\mathbf{H}_y\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\mathbf{H}_s^H\boldsymbol{\Gamma}^H - p_s\mathbf{H}_y^H\mathbf{H}_s^H\boldsymbol{\Gamma}^H \\ &\quad + \omega_{\mathrm{ZF}}^2\lambda_{\mathrm{ZF}} p_s\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\mathbf{H}_s^H\boldsymbol{\Gamma}^H \\ &\quad + \omega_{\mathrm{ZF}}^2\lambda_{\mathrm{ZF}}\sigma_{v_s}^2\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H = \mathbf{0}_N \end{aligned} \quad (82)$$

$$\begin{aligned} \frac{\partial L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})}{\partial \omega_{\mathrm{ZF}}} &= 2\omega_{\mathrm{ZF}}\lambda_{\mathrm{ZF}} p_s tr\big(\mathbf{H}_s^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\hat{\mathbf{F}}_{\mathrm{ZF}}\mathbf{H}_s\big) \\ &\quad + 2\omega_{\mathrm{ZF}}\lambda_{\mathrm{ZF}}\sigma_{v_s}^2 tr\big(\hat{\mathbf{F}}_{\mathrm{ZF}}\hat{\mathbf{F}}_{\mathrm{ZF}}^H\big) = 0 \end{aligned} \quad (83)$$

$$\begin{aligned} \frac{\partial L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})}{\partial \lambda_{\mathrm{ZF}}} &= \omega_{\mathrm{ZF}}^2 p_s tr\big(\mathbf{H}_s^H\boldsymbol{\Gamma}^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s\big) - p_{\mathrm{R}} \\ &\quad + \omega_{\mathrm{ZF}}^2\sigma_{v_s}^2 tr\big(\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\boldsymbol{\Gamma}^H\hat{\mathbf{F}}_{\mathrm{ZF}}^H\big) = 0. \end{aligned} \quad (84)$$

To determine λ_{ZF}, using (84), (83) is simply rewritten as

$$\frac{\partial L(\hat{\mathbf{F}}_{\mathrm{ZF}}, \omega_{\mathrm{ZF}}, \lambda_{\mathrm{ZF}})}{\partial \omega_{\mathrm{ZF}}} = 2\omega_{\mathrm{ZF}}^{-1}\lambda_{\mathrm{ZF}} p_{\mathrm{R}} = 0. \quad (85)$$

From (85), it is clearly seen that λ_{ZF} can be 0 because $\omega_{\mathrm{ZF}} > 0$ and $p_R > 0$. Hence, using (82), $\hat{\mathbf{F}}_{\mathrm{ZF}}$ can be written as

$$\hat{\mathbf{F}}_{\mathrm{ZF}} = \mathbf{H}_y^\dagger\mathbf{H}_s^\dagger\boldsymbol{\Gamma}^{-1} \quad (86)$$

where $\mathbf{A}^\dagger = \mathbf{A}^H\left(\mathbf{A}\mathbf{A}^H\right)^\dagger$, if $\mathbf{A} \in \mathbf{C}^{m\times n}$ is either a left or square matrix, i.e., $m \le n$, and $\mathbf{A}^\dagger = \left(\mathbf{A}^H\mathbf{A}\right)^\dagger\mathbf{A}^H$, if $\mathbf{A} \in \mathbf{C}^{m\times n}$ is a right matrix, i.e., $n < m$. Using (84), the ω_{ZF} can be written as

$$\omega_{\mathrm{ZF}} = \sqrt{\frac{p_{\mathrm{R}}}{p_s||\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}\mathbf{H}_s||_F^2 + \sigma_{v_s}^2||\hat{\mathbf{F}}_{\mathrm{ZF}}\boldsymbol{\Gamma}||_F^2}}. \quad (87)$$

Substituting (86) into (87), the optimal $\omega_{\mathrm{ZF}}^\star$ can be obtained as

$$\omega_{\mathrm{ZF}}^\star = \sqrt{\frac{p_{\mathrm{R}}}{p_s||\mathbf{H}_y^\dagger||_F^2 + \sigma_{v_s}^2||\mathbf{H}_y^\dagger\mathbf{H}_s^\dagger||_F^2}}. \quad (88)$$

Finally, using (80), (86), and (88), the optimal $\mathbf{F}^{\star}_{\text{ZF}}$ for the cooperative distributed AF relay network under TPC based on the ZF criterion can be expressed as

$$\mathbf{F}^{\star}_{\text{ZF}} = \frac{\mathbf{H}_y^{\dagger}\mathbf{H}_s^{\dagger}\mathbf{\Gamma}^{-1}\sqrt{p_{\text{R}}}}{\sqrt{p_s\|\mathbf{H}_y^{\dagger}\|_F^2 + \sigma_{v_s}^2\|\mathbf{H}_y^{\dagger}\mathbf{H}_s^{\dagger}\|_F^2}}. \quad (89)$$

■

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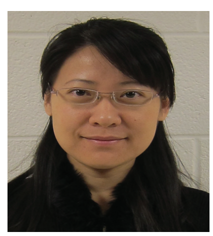

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