Digital Predistortion for Power Amplifiers in Hybrid MIMO Systems with Antenna Subarrays

Sangil Lee[†], Minhyun Kim[†], Youngwook Sirl[†], Eui-Rim Jeong[°], Soonil Hong[°], Seongjin Kim[†], and Yong H. Lee[††]

***Abstract*—We consider digital predistortion (DPD) for linearizing power amplifiers (PAs) of hybrid MIMO systems with subarrays. Designing such a DPD scheme is difficult because one DPD should linearize all PAs of a subarray whose beamforming coefficients are time-varying. We develop an adaptive DPD technique for each subarray based on direct learning method minimizing the sum of squared errors between the input and output signals of PAs. Use of the least mean square (LMS) Newton algorithm is proposed to adaptively control the DPD parameters. It is shown that the proposed DPD can simultaneously linearize multiple PAs in a subarray irrespective of its beamforming coefficients. Computer simulation results demonstrate that the proposed scheme can perform better than an alternative scheme that designs the DPD based on one of the PA outputs and uses the resulting DPD for all PAs of the subarray.**

***Index Terms*—Digital Predistortion, Power amplifier, hybrid MIMO, antenna subarray, LMS-Newton**

I. Introduction

Recently, hybrid MIMO systems consisting of analog beamformers in RF domain and digital MIMO processors in baseband have been recognized as a useful technique for reducing the cost for implementing MIMO systems. In these systems more antennas can be employed without increasing the number of costly RF chains consisting of amplifiers, mixers and analog-to-digital (AD)/digital-to-analog (DA) converters. Hybrid MIMO processors have been proposed for both current microwave communications [1]-[3] and the mm-wave communications [4]-[8].

Digital predistortion (DPD) has been widely used for compensating the nonlinearity of PAs in wireless systems [9]. In MIMO systems, a digital predistorter is needed for the PA of each antenna, and the required number of digital predistorters is equal to the number of antennas. In the case of hybrid MIMO systems, implementation of digital predistorters is challenging because of the following reasons: i) use of a digital predistorter for each PA is impossible, because a digital predistorter cannot be located in between the analog beamformer and a PA which are the devices in the RF domain, ii) the number of digital predistorters that can be implemented in the baseband is limited by the number of RF chains, and thus one predistorter should support multiple PAs, and iii) the input to the PA is one of the outputs of the analog beamformer which is a linear combination of multiple data streams and varies according to the coefficients of the analog beamformer.

In this paper, we propose a DPD technique that can compensate for nonlinearities of multiple PAs of a hybrid MIMO system with antenna subarrays [10], [11] (Fig. 1). In such hybrid MIMO systems, an antenna array is partitioned into several subarrays, each of which is driven by an RF chain. Thus, only one data stream from the digital MIMO precoder enters the analog beamformer, and the input to each PA is simply given by its data stream multiplied with a beamforming coefficient. We design an adaptive DPD technique for each subarray based on the direct learning method [12], [13] minimizing the sum of squared errors between the input and output signals of PAs. To adaptively adjust the DPD parameters, we develop an least mean squares (LMS) Newton algorithm [14] exhibiting excellent convergence characteristics. It is shown that the proposed DPD is independent of the analog beamforming coefficients if phased array beamformers, which only control the phase of each antenna, are employed. Performance of the proposed DPD is evaluated through computer simulation. The results indicate that a single digital predistorter can simultaneously support multiple PAs with slightly different characteristics.

The organization of this paper is as follows. Section II describes the system model and section III describes the proposed DPD technique. The simulation results are presented in Section IV. Finally, Section V concludes the paper.

Notations: Throughout this paper we use bold-faced upper and lower case letters to denote matrices and column vectors, respectively. Superscripts $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ denote conjugate, transpose and conjugate transpose, respectively. $\mathrm{E}[\cdot]$ represents the expectation and $\mathrm{diag}(a_1, \cdots, a_M)$ is an $M \times M$ diagonal matrix with diagonal entries $(a_1, \cdots, a_M)$.

II. System Model

Fig. 2 shows the baseband equivalent model of the DPD scheme for a subarray of the hybrid MIMO system in Fig. 1. The input signal $x(n)$, which represents one of the output data streams of the digital MIMO precoder in Fig. 1, enters the predistorter which yields its output $y(n)$. Then $y(n)$ is multiplied with the analog beamforming vector $\mathbf{g} \in \mathbb{C}^M$, given by $\mathbf{g} = [g_1, \cdots, g_M]^T$ where M is the number of antennas in

[†] Department of Electrical Engineering, KAIST, Daejeon, Korea

[°] Department of Information And Communication, Hanbat National University, Daejeon, Korea

Email: {lsi5555, mhkim, yos, kimsjkia}@stein.kaist.ac.kr, erjeong@hanbat.ac.kr, hongsi110@nate.com, yohlee@kaist.ac.kr

[††] Corresponding author

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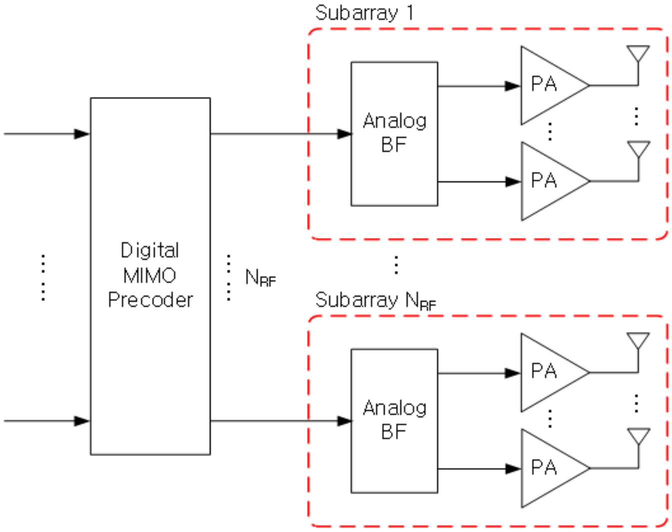


Fig. 1. A hybrid MIMO system with antenna subarrays. Here N_{RF} is the number of RF chains.

the subarray. This results in $g_j y(n)$ which is the input to the j-th PA. The output of the PA is denoted as $z_j(n)$. To feedback the PA output, $z_j(n)$ is normalized by the PA's gain, K, to yield $a_j(n) = z_j(n)/K$. The PA identification block receives both $y(n)$ and $\{a_1(n), a_2(n), \cdots, a_M(n)\}$, and identifies each PA's model parameters. The DPD adaptation block receives both $x(n)$ and the results of PA identification, and updates the parameters of the DPD.

We employ the polynomial model for PA identification [15]. Using this model, $a_j(n)$ is written as

$$\begin{aligned} a_j(n) &= \sum_{k=1}^{L} w_{k,j} |g_j y(n)|^{2(k-1)} g_j y(n) \\ &= g_j \mathbf{w}_j^T \mathbf{G}_j \mathbf{y}(n), \end{aligned} \tag{1}$$

where $2L-1$ is the maximum polynomial order for the PA, $\mathbf{w_j} = [w_{1,j}, w_{2,j}, \cdots, w_{L,j}]^T$, $\mathbf{w_j} \in \mathbb{C}^L$, is the polynomial coefficient vector of the j-th PA, $\mathbf{G}_j \in \mathbb{R}^{L\times L}$ is given by $\mathbf{G}_j = \text{diag}([1, |g_j|^2, \cdots, |g_j|^{2(L-1)}])$, and $\mathbf{y}(n) \in \mathbb{C}^L$ is given by $\mathbf{y}(n) = [y(n), y(n)|y(n)|^2, \cdots, y(n)|y(n)|^{2(L-1)}]^T$.

The DPD characteristic is also modeled by the polynomial model. The output of the predistorter is modeled as

$$\begin{aligned} y(n) &= \sum_{k=1}^{Q} h_k^* |x(n)|^{2(k-1)} x(n) \\ &= \mathbf{h}^H \mathbf{x}(n), \end{aligned} \tag{2}$$

where $2Q-1$ is the maximum polynomial order for predistortion, $\mathbf{h} = [h_1, h_2, \cdots, h_Q]^T$, $\mathbf{h} \in \mathbb{C}^Q$ is the coefficient vector of the predistorter, and $\mathbf{x}(n) \in \mathbb{C}^Q$ is given by $\mathbf{x}(n) = [x(n), x(n)|x(n)|^2, \cdots, x(n)|x(n)|^{2(Q-1)}]^T$.

III. Proposed DPD

In this section, the PA is identified by the least squares(LS) approach and the DPD is designed by the LMS-Newton algorithm which is a modified LMS algorithm for fast convergence [14].

A. PA Identification

We use M blocks of training sequences where each block consists of N training symbols. Assuming that only one feedback path is available, the training blocks are used successively to identify one PA at a time. Suppose that the j-th training block is given to identify the j-th PA. Denote the training samples in this block by $\{y(1), \cdots, y(N)\}$. (Here to simplify the notation, we drop the block index j from the time indices of the training samples.) Then the least squares cost function for the j-th path is defined as

$$\mathcal{E}_j = \sum_{n=1}^{N} |a_j(n) - g_j \mathbf{w}_j^T \mathbf{G}_j \mathbf{y}(n)|^2. \tag{3}$$

and the least squares estimate, $\hat{\mathbf{w}}_j$, minimizing $\mathcal{E}_j$ is given by [16]

$$\hat{\mathbf{w}}_j = \frac{1}{g_j} \mathbf{G}_j^{-1} (\mathbf{Y}^H \mathbf{Y})^{-1} \mathbf{Y}^H \mathbf{a}_j, \tag{4}$$

where $\mathbf{Y} \in \mathbb{C}^{N\times L}$ is given by $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \cdots, \mathbf{y}(N)]^T$, and $\mathbf{a} \in \mathbb{C}^N$ is given by $\mathbf{a} = [a_j(1), a_j(2), \cdots, a_j(N)]^T$.

Due to the time sharing of the feedback path by M feedback signals, we need a longer training period proportional to M. An alternative to the proposed feedback policy is to feedback the sum of M feedback signals. It can be seen that using this feedback signal, the PA identification block can also be designed in the least squares sense. However, in this work, we exclude the use of this feedback policy which requires a shorter training period, because it needs addition in the RF domain and is vulnerable to delays in the feedback paths.

B. DPD Design

Given the estimates of PA parameters, $\{\mathbf{w}_1^T, \cdots, \mathbf{w}_M^T\}$, and the input to the DPD, $x(n)$, we design the DPD block minimizing the expected sum of squared error given by

$$\begin{aligned} \text{E}\left[\sum_{j=1}^{M} |e_j(n)|^2\right] &\triangleq \text{E}\left[\sum_{j=1}^{M} |g_j x(n) - a_j(n)|^2\right] \\ &= \text{E}\left[\sum_{j=1}^{M} |g_j|^2 |x(n) - \mathbf{w}_j^T \mathbf{G}_j \mathbf{y}(n)|^2\right] \\ &\triangleq \text{E}\left[\sum_{j=1}^{M} |g_j|^2 |\bar{e}_j(n)|^2\right], \end{aligned} \tag{5}$$

where $\bar{e}_j(n) = x(n) - \mathbf{w}_j^T \mathbf{G}_j \mathbf{y}(n)$. This expected sum error can be iteratively minimized by conventional adaptive algorithms such as the LMS algorithm. In this case, however, use of the simple LMS algorithm is not recommended, because the entries of the observation vector, $\mathbf{x}(n)$ defined in (2), are highly correlated, and the simple LMS algorithm is suffered by

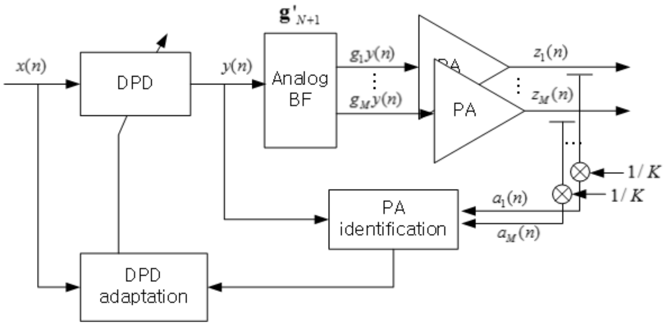


Fig. 2. Digital predistortion structure for an antenna subarray.

slow convergence. To accelerate we adopt the LMS-Newton algorithm [14] which is updated by

$$\mathbf{h}(n+1) = \mathbf{h}(n) - \mu \hat{\mathbf{R}}^{-1}(n)\hat{\mathbf{g}}_{\mathbf{h}}(n), \tag{6}$$

where μ is a step size, $\hat{\mathbf{R}}$ and $\hat{\mathbf{g}}_{\mathbf{h}}(n)$, respectively, are estimates of the correlation matrix, $\mathbf{R}(n) = E[\mathbf{x}(n)\mathbf{x}(n)^H]$, and the gradient vector $\mathbf{g}_{\mathbf{h}}(n)$ given by

$$\mathbf{g}_{\mathbf{h}}(n) = \frac{\partial}{\partial \mathbf{h}^*} \mathrm{E}\left[\sum_{j=1}^{M} |e_j(n)|^2\right]. \tag{7}$$

In (6), $\hat{\mathbf{R}}^{-1}(n)$ can be recursively estimated by

$$\begin{aligned}\hat{\mathbf{R}}^{-1}(n) = \frac{1}{1-\alpha}\Big[&\hat{\mathbf{R}}^{-1}(n-1) \\ &- \frac{\hat{\mathbf{R}}^{-1}(n-1)\mathbf{x}(n)\mathbf{x}^T(n)\hat{\mathbf{R}}^{-1}(n-1)}{\frac{1-\alpha}{\alpha} + \mathbf{x}^T(n)\hat{\mathbf{R}}^{-1}(n-1)\mathbf{x}(n)}\Big],\end{aligned} \tag{8}$$

where $0 < \alpha \leq 0.1$ [14]. Following the LMS approach, the gradient vector is estimated by

$$\begin{aligned}\hat{\mathbf{g}}_{\mathbf{h}}(n) &= \sum_{j=1}^{M} |g_j|^2 \frac{\partial |\bar{e}_j(n)|^2}{\partial \mathbf{h}^*} \\ &= \sum_{j=1}^{M} |g_j|^2 \left(\frac{\partial \bar{e}_j(n)}{\partial \mathbf{h}^*} \cdot \bar{e}_j^*(n) + \bar{e}_j(n) \cdot \frac{\partial \bar{e}_j^*(n)}{\partial \mathbf{h}^*}\right) \\ &= \sum_{j=1}^{M} |g_j|^2 \mathbf{x}(n)\left[-(\mathbf{w}_j^T \mathbf{G}_j \mathbf{v}_1(n)) \cdot \bar{e}_j^*(n)\right. \\ &\qquad \left. - \bar{e}_j(n) \cdot (\mathbf{w}_j^H \mathbf{G}_j \mathbf{v}_2(n))\right]\end{aligned} \tag{9}$$

where $\mathbf{v}_1(n) = [1, 2|y(n)|^2, \cdots, L|y(n)|^{2(L-1)}]^T$, and $\mathbf{v}_2(n) = [0, (y^*(n))^2, \cdots, (L-1)(y^*(n))^2|y(n)|^{2(L-2)}]^T$.

Using (9) in (6), we get

$$\begin{aligned}\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\hat{\mathbf{R}}^{-1}(n)\sum_{j=1}^{M}|g_j|^2\mathbf{x}(n)\left[\bar{e}_j^*(n)\right. \\ \left. \cdot (\mathbf{w}_j^T\mathbf{G}_j\mathbf{v}_1(n)) + \bar{e}_j(n)\cdot(\mathbf{w}_j^H\mathbf{G}_j\mathbf{v}_2(n))\right]\end{aligned} \tag{10}$$

where $\hat{\mathbf{R}}^{-1}(n)$ is given by (8). The proposed LMS-Newton algorithm is summarized in Algorithm 1.

Algorithm 1 The proposed LMS-Newton algorithm

Require: $\{x(n)\},\ \mathbf{h}(1),\ \hat{\mathbf{R}}(0),\ g_j,\ \mathbf{w}_j,\ \mathbf{G}_j$

for $n \leq N_{\mathrm{DPD}}$ **do**

$\quad \mathbf{x}(n) = [x(n), x(n)|x(n)|^2, \cdots, x(n)|x(n)|^{2(Q-1)}]^T$

$\quad y(n) = \mathbf{h}^H\mathbf{x}(n)$

$\quad \mathbf{y}(n) = [y(n), y(n)|y(n)|^2, \cdots, y(n)|y(n)|^{2(L-1)}]^T$

$\quad \hat{\mathbf{R}}^{-1}(n) = \frac{1}{1-\alpha}\left[\hat{\mathbf{R}}^{-1}(n-1) - \frac{\hat{\mathbf{R}}^{-1}(n-1)\mathbf{x}(n)\mathbf{x}^T(n)\hat{\mathbf{R}}^{-1}(n-1)}{\frac{1-\alpha}{\alpha}+\mathbf{x}^T(n)\hat{\mathbf{R}}^{-1}(n-1)\mathbf{x}(n)}\right]$

$\quad$ **for** $j \leq M$ **do**

$\quad\quad a_j(n) = g_j\mathbf{w}_j^T\mathbf{G}_j\mathbf{y}(n)$

$\quad\quad e_j(n) = g_jx(n) - a_j(n)$

$\quad\quad \hat{\mathbf{g}}_{\mathbf{h}j}(n) = |g_j|^2\mathbf{x}(n)\left[-(\mathbf{w}_j^T\mathbf{G}_j\mathbf{v}_1(n))\cdot\bar{e}_j^*(n) - \bar{e}_j(n)\cdot(\mathbf{w}_j^H\mathbf{G}_j\mathbf{v}_2(n))\right]$

$\quad$ **end for**

$\quad \mathbf{h}(n+1) = \mathbf{h}(n) - \mu\hat{\mathbf{R}}^{-1}(n)\sum_{j=1}^{M}\hat{\mathbf{g}}_{\mathbf{h}j}(n)/M$

end for

return h

When phased array beamformers are employed in the hybrid MIMO system, $|g_j|^2 = 1$ and $\mathbf{G}_j$ becomes the identity matrix. Therefore, (10) reduces to

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu\hat{\mathbf{R}}^{-1}(n)\sum_{j=1}^{M}\mathbf{x}(n)d_j(n). \tag{11}$$

where $d_j(n) = \bar{e}_j^*(n)\cdot(\mathbf{w}_j^T\mathbf{v}_1(n)) + \bar{e}_j(n)\cdot(\mathbf{w}_j^H\mathbf{v}_2(n))$, and the update is independent of the bemforming gain.

IV. Simulation results

The performance of the proposed DPD is demonstrated by computer simulation. We consider a subarray consisting of 4-antennas. Transmitted symbols are modulated by 16 quadrature amplitude modulation (16-QAM) and pulse-shaped by a square root raised cosine filter with roll-off factor of 0.25. The sampling rate of the pulse shaping filter (PSF) is 10 times the symbol rate. To model the PA, we use the Saleh model [17] in which the PA response is

$$\varphi(y(n)) = \frac{1.1\alpha y(n)}{1+0.3|y(n)|^2}\exp\left(j\frac{0.8\beta|y(n)|^2}{1+3|y(n)|^2}\right). \tag{12}$$

Here we assume that $\alpha = \beta \in \{1, 1.02, 1.04, 1.06\}$ for the four PAs. The i-th PA is referred to as PAi for $i \in \{1,2,3,4\}$, and α values for PA1, PA2, PA3 and PA4 are set at 1, 1.02, 1.04, and 1.06, respectively. The ideal gain of the PA is assumed to be 1. 7th order polynomial model is used for both the PA identification and DPD model($L = Q = 4$). For DPD adaptation, the step size μ is 0.6. We assume a phased array beamformer whose coefficients vary at every 1,000 symbol period.

In the simulation, we compare the proposed scheme with an alternative scheme, called the PAi-centric DPD, that designs the DPD only for PAi, using the feedback of the PAi output, and uses the resulting DPD for the four PAs. The PAi-centric

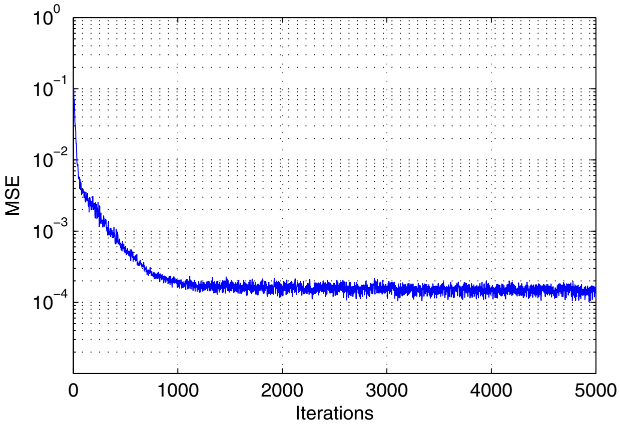


Fig. 3. Learning curve for the proposed DPD.

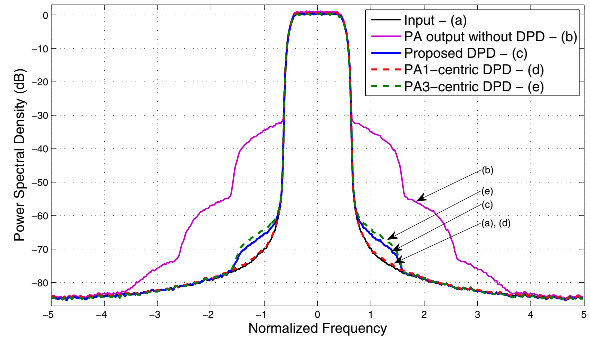
TABLE I
SPECTRAL REGROWTH AT NORMALIZED FREQUENCY 1.

	PA1-centric	PA3-centric	Proposed
PSD of PA1	0.31 dB	5.64 dB	4.25 dB
PSD of PA2	2.80 dB	2.55 dB	1.03 dB
PSD of PA3	6.23 dB	0.32 dB	0.88 dB
PSD of PA4	9.63 dB	2.77 dB	4.54 dB

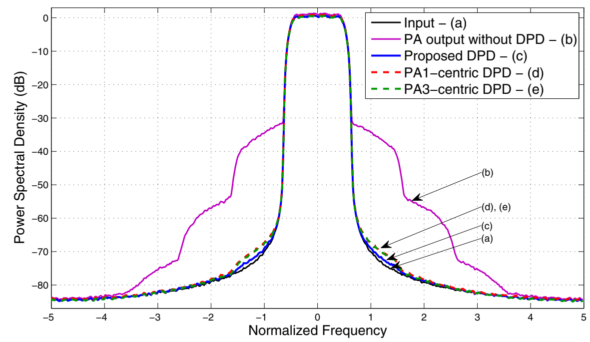
DPD is also adapted by the proposed LMS-Newton method (in this case, M=1 in (11)). Due to the LMS-Newton adaptation, the proposed and PAi-centric methods are independent of the beamforming coefficients.

The learning curve of the proposed DPD is obtained by evaluating the mean square errors (MSEs) over 100 trials, and the result is shown in Fig. 3. The proposed DPD converges within around 1000 iterations and has a mean square error of 10^{-4} after convergence. Fig. 4 (a)-(d) compare the output power spectral densities (PSDs) of the proposed and the PAi-centric schemes with $i \in \{1, 3\}$ at the ouputs of PA1, PA2, PA3 and PA4, respectively.

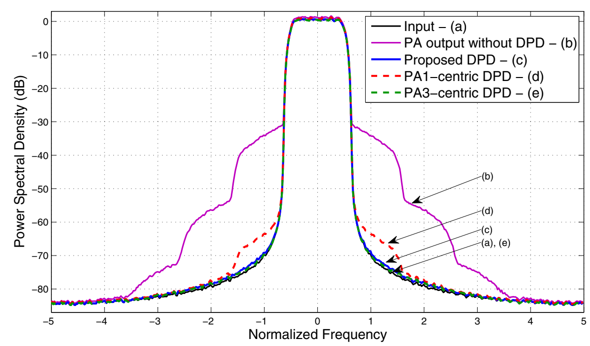
To compare the residual spectral regrowth (RSR) more precisely, we obtain the values of RSR at the normalized frequency 1 of Fig. 4 (a)-(d) and list the results in Table I. As expected, the PAi-centric method $(i \in \{1, 3\})$ outperforms the others when it is used for PAi, but its performance is degraded for the other PAs. The PA1-centric causes the largest RSR for PA4 because PA1 has the smallest value of α $(\alpha = 1)$, while PA4 has the largest α $(\alpha = 1.06)$. Comparing the PA1-centric and PA3-centric methods when they are applied to PAs other than the one that they are designed for, the latter tends to perform better because its value $(\alpha = 1.04)$ is neither the maximum nor the minimum among the four values of α. These results indicate that use of the PAi-centric method needs caution: its performance can be inconsistent depending on the choice of "i". There is no such inconsistency for the proposed scheme. Due to the sum of squared error minimization, the proposed DPD is close to the DPD corresponding to the average value of α: referring to Table I, its RSR values for PA2 and PA3 having median values of α are close to each other; similarly, the RSR values for PA1 and PA4 having the minimum and the maximum values of α, respectively, are close to each other.



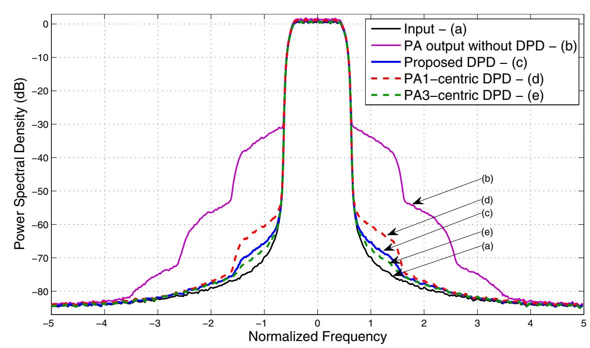
(a)



(b)



(c)



(d)

Fig. 4. Output PSD of (a)PA1, (b)PA2, (c)PA3 and (d)PA4.

V. Conclusion

A DPD technique for hybrid MIMO systems with antenna subarrays was proposed. We developed a direct learning method that adaptively adjusts the DPD parameters using the LMS-Newton algorithm. When a phased array beamformer is employed, the proposed DPD can simultaneously linearize multiple PAs irrespective of the beamforming coefficients. The performance of the proposed DPD was examined through computer simulation. The results indicate that the proposed DPD is useful for linearizing PAs of subarrays in hybrid MIMO systems.

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